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HOW MUCH CAN WE TRUST SOME MOMENT TENSORS OR AN ATTEMPT OF SEISMIC MOMENT ERROR ESTIMATION

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ABSTRACT
During routine processing of selected events of an active KTB experiment it has appeared doubts concerning data reliability and consequently the reliability of results based on them. In the paper 3 events are studied in detail, full seismic moment tensors, as well as their errors, are determined (by non-linear inversion of P/SH waves ratios). It is shown that for the processed low constrained data moment tensor (MT) can be determined, however the relative error is of order of first tens of percent; the results also considerably depend on the way of data picking, used medium model, way of Cost function construction, etc. Any subsequent geophysical interpretation therefore should takes into account this uncertainty. MTs are finally decomposed into DC and non-DC parts, MTs errors are also transformed.

KEYWORDS: seismic moment tensor inversion, error estimation, seismic moment decomposition

INTRODUCTION

During routine amplitude picking of a data set coming from an active experiment at KTB deep borehole, some doubts appeared concerning reliability of the data, and consequently of the results based on them. The picked P and S wave amplitudes were input data for a seismic moment inversion. The aim of the presented work is to discuss different effects which can influence the results - inverted seismic moment tensor, eliminate these effects if their nature allows it, and estimate errors of the results. As the seismic moment (MT) is finally decomposed into three parts: Volumetric (ISO), Compensated Linear Vector Dipole (CLVD) and Double Couple (DC), errors of theses parts are also estimated.

DESCRIPTION OF DATA SET

The events under interest come from an active experiment: fluid pumping into the KTB borehole – for details see e.g. Baish et al. (2002). The seismograms were recorded by a network consisting of 40 stations (Fig. 1), there were available records of cca 150 events with magnitude ranging from -1.2 to 1.1. Our analysis was restricted only to data from surface stations and to the events with depth about 5 km (events were also located around depth 9 km, but they were not subject of this analysis). In the present study we analyzed the reliability of determined seismic moment tensor of 3 events from the set; two stronger and one of medium magnitude (M_w = 1.02, 0.55, 0.22).

Initially data examination showed, that this unique data set suffer from some shortages: signal level is not too high and therefore for many records the data are often rather noisy (up to tens of percent of observed amplitudes), waveforms are often rather complicated, seismograms are hardly enough sampled and the amplitudes cannot be recovered absolutely – only their relative values are available.

Discuss here briefly the influence of theses shortages on the data processing. The noisy data were filtered and filtration improved the readability of the data, especially for weaker events. Due to complex waveforms it was not always possible to identify the maximal amplitudes of the P and especially of the S waves directly in the seismograms. We were using particle motion diagrams and were looking for maximal displacement vector. As the seismograms (despite of maximal acceptable filtrations) contain still some low frequency noise, which is superposed over the signal, we measured (again in particle motion diagram) also amplitude of onset point.[1] Amplitudes used for inversion are then given as subtraction of onset and maximal amplitudes.

There is no way to improve hardly sufficient sampled seismograms: sampling frequency was 200 Hz.[2]

As the absolute values of amplitudes are not available, ratio of P and S waves amplitudes is used as input data for seismic moment inversion. This is an old well-known approach, however, as it is discussed later, it brings principal problems to the estimation of solution's errors.

[1] The seismograms were processed by program SeisGram2K, written by Lomax (2005).
[2] Notice here, that in only few tens of kilometers distant West Bohemia seismic network WEBNET, which usually record farther and stronger events than KTB network, operates with 250 Hz sampling frequency (Horálek et al., 2000).

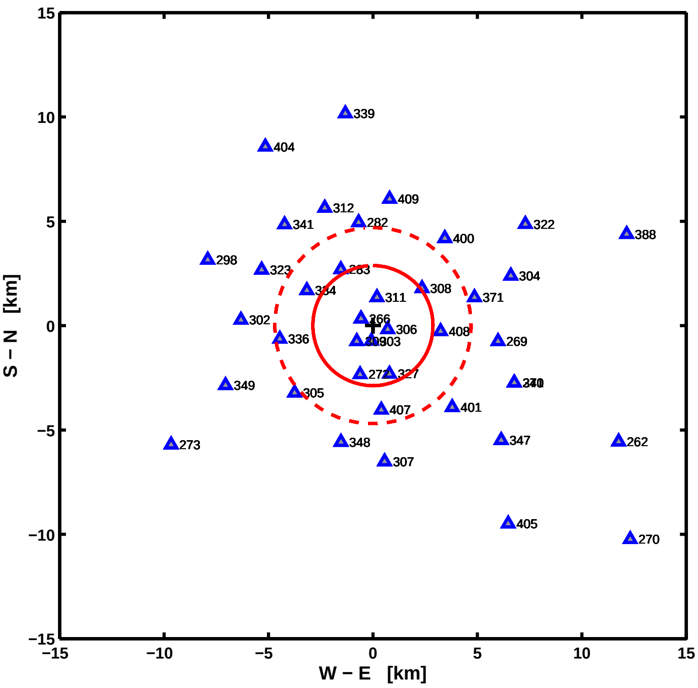

Fig. 1 Map of KTB network (only surface stations are displayed). The full circle corresponds to the 30° incident angle for homogeneous medium model, the dashed one to the Málek's gradient model – both for 5km depth epicenter (see the text); the borehole is situated in the origin.

INVERSION OF SEISMIC MOMENT

Well-known equation (see e.g. Jost and Herrmann, 1989)

$$U = G * MT \tag{1}$$

where *U* is displacement, *G* is Green's function and *MT* is seismic moment tensor, serve as a base for inversion. If amplitudes are input data for the inversion, the equation(s) is/are linear and problem has (under well-defined condition) an exact solution. Also errors, or uncertainty respectively, of such solution can be exactly determined (see e.g. Menke, 1989). Once our input data are only ratios of P and S waves, the problem becomes non-linear. From mathematical point of view, the situation is different. Even if there are numerous methods for solving such type of problem, there is no (mathematical) proof that they lead to the (best or global respectively) solution of the problem – the methods often use a sort of random search and there is only some probability of gaining the (best global) solution. Also there is no exact theory for estimation of errors.

On the basis of our previous test (Kolář, 2000), we used Boender's method for the inversion (Boender et al., 1982; Csendes, 1988). This method combines a sort of random global search with a local linear method. The method is able to determine more than one local minimum; the solution of the inversion is then taken as the lowest of them. The quality of any tested parameter's combination is express by one number: Cost function.

DATA PREPROCESSING AND INVERSION

As already mentioned above, the investigated seismograms are noisy. Principal amount of noise is eliminated by band pass filtration (2.5 – 40.0 Hz, order 4), however sometimes long period noise persist in seismograms, therefore we took P or S wave amplitudes as a difference between amplitude of P or S wave onset and their maximal amplitude. These amplitudes were measured in standard ZNE coordinate system. In some cases we observed time shift between time position of maxima of S waves on N and E components. Therefore we repeated measurement in ZRT (Z-vertical, Radial, Transversal) coordinates system and read absolute maximal values on R and T components, generally at different time moment.[3] Both data sets were inverted then. For an ideal data set the results should be identical, however for processed real data they differ – see below.

[3] The time shift of maxima, which can be observed in some processed seismograms, can have several causes: e.g. anisotropy of medium, effect of complex free surface coefficients, etc. In the present work, we do not investigate the origin of this effect.

To compute the Green's functions we adopted two medium models: a homogeneous half space with $v_P = 5.6$ km/s and $v_P/v_S = 1.8$ and model designed by Málek et al. (2000) consisting of 400 m thick low velocity layer over a half space with a gradient. This last model was originally created for West Bohemia region. We can see several reasons to adopt this model: West Bohemia seismic region is located only about 50 km far from KTB area, geological conditions are roughly similar in both localities, some gradient could be anyway expected also in the KTB area, no other enough suitable model is available, Málek's gradient model is fairly suitable for testing computation.[4]

The Green's functions including corresponding free surface conversion coefficients were computed by program SEIS83 – Červený and Pšencík (1984).

It is well known fact, that free surface coefficients of SV waves have singularities if incident angle is greater than about 30° (see e.g. Červený, 2005), therefore SV waves recorded at farther stations are not suitable for inversion. Critical distance in our case (for events with depths of 5 km) is about 2.9 km for the homogenous model; for Málek's gradient model it is slightly greater – about 4.7 km, see Fig. 1. Therefore we used only P and SH amplitudes for inversion, the under critical SV amplitudes were also excluded to keep the data set homogeneous.

We are looking for seismic moment tensor, i.e. for values of 6 parameters. Inversion method use so called "Cost function" (Cf) to express quality of any tested parameters set. A standard way of Cf construction e.g. in form of

$$\mathrm{Cf} = \sum \left(\mathrm{P/SH}_{\mathrm{synt}} - \mathrm{P/SH}_{\mathrm{observ}}\right)^2 \ , \tag{2}$$

cannot be used, as due to asymmetry of the quadratic function, the ratios greater than 1 would be emphasize more than the ratios less than 1 (even if the physical importance of both could be the same in our task). Therefore we extended (2) into

$$\begin{aligned} \mathrm{Cf} &= \sum \left(\mathrm{P/SH}_{\mathrm{synt}} - \mathrm{P/SH}_{\mathrm{observ}}\right)^2 + \\ &+ \sum \left(\mathrm{SH/P}_{\mathrm{synt}} - \mathrm{SH/P}_{\mathrm{observ}}\right)^2 \end{aligned} . \tag{3}$$

As the inversion of the real data processed with norm (3) appeared to be still rather unstable (see below), we incorporated also the signs of first P waves motion in a following way

$$\begin{aligned} \mathrm{Cf} &= \left(-\frac{1}{n}\sum\left(\mathrm{Sig}_{\mathrm{synt}}\,\mathrm{Sig}_{\mathrm{observ}}\right) + 1.1\right)\cdot \\ &\left(\sum\left(\mathrm{P/SH}_{\mathrm{synt}} - \mathrm{P/SH}_{\mathrm{observ}}\right)^2 + \sum\left(\mathrm{SH/P}_{\mathrm{synt}} - \mathrm{SH/P}_{\mathrm{observ}}\right)^2\right) \end{aligned} , \tag{4}$$

where n is a number of stations, Sig is 1 for positive first motion and -1 for negative one; the new factor express the correlation of synthetic and observed first motions. Such approach is analogical to the construction of the cost function used by Kolář (2003). Note that our definition of Cost function (3) already implicitly incorporates information of motion orientation, extension from (3) to (4) amplify the requirement of the fit of orientation of first P waves motion. When Cf is constructed e.g. as a sum of logarithms (approach used e.g. by Vavryčuk et al., 2007) information about orientation is lost and should be added inevitably (as it was also done in the quoted work).

As we use only relative amplitudes, the MT cannot be determined absolutely, solutions vary by a multiplicative constant. Consequently, the scalar seismic moment cannot be determined neither.

Our program was successfully tested by retrieving the moment tensor from synthetic data generated for the same station configuration as the real one. We did not test at all influence of number and distribution of stations neither influence of artificially added noise – such tests were performed by Jechumtálová and Šílený (2005).

ERROR ESTIMATION

As already mentioned above, there is no exact theoretical way to estimate error of solution of non-linear problems. We adopted idea proposed by Kolář (2003). During the inversion we calculated posteriori probability density function

$$\mathrm{PPD} = \exp\left(1/2\varepsilon^2\right), \tag{5}$$

where ε is the misfit function, i.e. the value of the cost function Cf for every tested parameter combination (Šílený, 1998). The highest PPD value at a certain moment during the inversion (corresponding to the best solution until that moment) is stored into matrix M_{PPD} of *n* x *k* dimension. *n* is the number of inverted parameters (n = 6 in our case), *k* is an arbitrary number of the interval of definition of the parameter; we choose k = 150. After the inversion, the stored PPD values are integrated for each parameter separately over its definition interval, normalized and we determine the length of the interval corresponding to 67% of the whole area.[5] This length we assume to represent the uncertainty of the inverted parameters; below presented numerical values are half of these interval lengths.[6]

Determined MTs are finally decomposed into their ISO, CLVD and DC parts (see e.g. review Julian et. al, 1998), following definition by Vavryčuk (2001), formulas (7) and (8):

[4] There are several velocity models designed directly for KTB, see e.g. Geissler (2004). Unfortunately all these models are rather rough in description of near-surface structure. And a low velocity surface layer can considerably affect the seismic ray incident angle at stations (J. Horálek, personal communication).

[5] Value 67% is a convention when dealing with Gaussian distribution and we adopt the same value (even if our distribution is probably non Gaussian).

[6] To ensure that all cells of M_{PPD} matrix are filled, we run every inverted parameter through all sub-intervals keeping the others parameters on their best solution.

Table 1 Values of average error for 3 processed events and for all considered models: ZNE versus ZRT coordinates system, homogeneous ‘hm’ versus gradient ‘gr’ medium model and Cost function Cf calculated by formula (3) (marked ‘X’) versus Cf calculated by formula (4) (marked ‘XS’). It follows from the table that combination ZRT coordinates and homogenous model (bold rows) yield the smallest average error value (with only one exception – event 65259); this configuration is then considered to be the most confident.

event	048290	058266	065259
ZNE hm X	0.276	0.135	0.234
ZNE gr X	0.139	0.103	0.185
ZRT hm X	**0.078**	**0.093**	**0.154**
ZRT gr X	0.142	0.141	0.274
ZNE hm XS	0.168	0.140	0.136
ZNE gr XS	0.121	0.104	0.309
ZRT hm XS	**0.122**	**0.057**	**0.180**
ZRT gr XS	0.140	0.125	0.496

$$\mathrm{ISO} = \frac{1}{3}\,\mathrm{trace}(\mathrm{MT}) / \mathrm{M}_{|\max|}$$

$$\mathrm{CLVD} = -2\ \mathrm{M}^*_{|\min|} \,/\, \mathrm{M}^*_{|\max|} \left(1 - |\mathrm{ISO}|\right) \qquad (6)$$

$$\mathrm{DC}\ = 1 - |\mathrm{ISO}| - |\mathrm{CLVD}|\ ,$$

where $M_{|max|}$ denotes that eigenvalue of MT which has the maximum absolute value (and analogously for $M_{|min|}$); symbol M^* denotes deviatoric part of MT; absolute values of ISO, CLVD and DC range in interval <0 ; 1>. This definition of MT decomposition has following basis features: the value of DC is always positive, and the ISO and CLVD are positive for tensile source, but negative for compressive one; the sum of all 3 absolute values is always a unit.

From point of view of error estimation, the decomposition is exact, however rather complicated operation (cubic equation must be solved to find eigenvalues of MT). Therefore simple rules for error propagation cannot be applied to determine errors of MT decomposition. We applied following approach: we decompose all moment tensors (represented here as 6 element vector m_i) created in following way

$$m_i = m_{ibest} + / - m_{ierror}\ , \quad \text{for i =1-6,} \qquad (7)$$

where m_{ibes} is a solution and m_{ierror} is its estimated error. There are totally 64 (i.e. 2^6) possible combination in this case. The error of moment tensor decomposition is then maximum, or minimum respectively, from all considered combination. For ISO, CLVD and DC calculated errors are explicit values any time, however there are sometimes problems with error of DC orientation – due to geometrical effects, sometime the error interval cannot be exactly determined, especially when some values of the two possible solutions are close one to each other or the angles switch over their definition period (e.g. 360^o for strike, etc.).

RESULTS OF INVERSION

For each processed event we invert two data sets (readings from ZRT or ZNT coordinate systems) and we use two medium models (homogeneous or gradient); inversions are performed with Cost function given by formula (3) or (4). Due to different Cf definition and different tested medium models the quality of solutions cannot be simply compared by value of (final) Cost function. Instead we prefer solution with the smallest average error.

The average errors are given in Table 1. It follows from the table, that combination ZRT data and homogeneous model yields systematically the smallest average errors – therefore we consider this combination as the most confident solutions. The determined solutions are plotted in Fig. 2 (for Cf given by (3)) and in Fig. 3 (for Cf given by (4)). It follows from the figures that solutions with Cf given by formula (3) are unstable with comparison to the solution with Cf given by formula (4). The second solutions we therefore consider to be the most confident. The numerical results for ZRT, homogeneous model and Cf given by (4) are given in Table 2 (MT values) and in Table 3 (MT decomposition). The most confident solutions are plotted in details in Figure 4.

Numerical results in the Tables 2 and 3 are given with their errors. We can conclude that determined errors quantitatively indirectly depend on number of station readings. Number of readings then implicitly depends on event magnitude, i.e. stronger events, recorded by more stations, are inverted more reliable and vice versa. However even MT for the weakest event 065259 (with 9 stations) is determined with average error less than +/- 0.2, which corresponds to relative error less than +/- 10%[7]; the biggest error for this event (component m_{11}) is then +/- 0.44, which corresponds to relative error little bit more than +/- 20%.

[7] In our inversion the MT components are seek for on interval <-1; 1> ; relative error we then evaluate as a fraction of an absolute error over this interval: $err_{RELAT} = 100\ err_{ABS}/|def\_int|$.

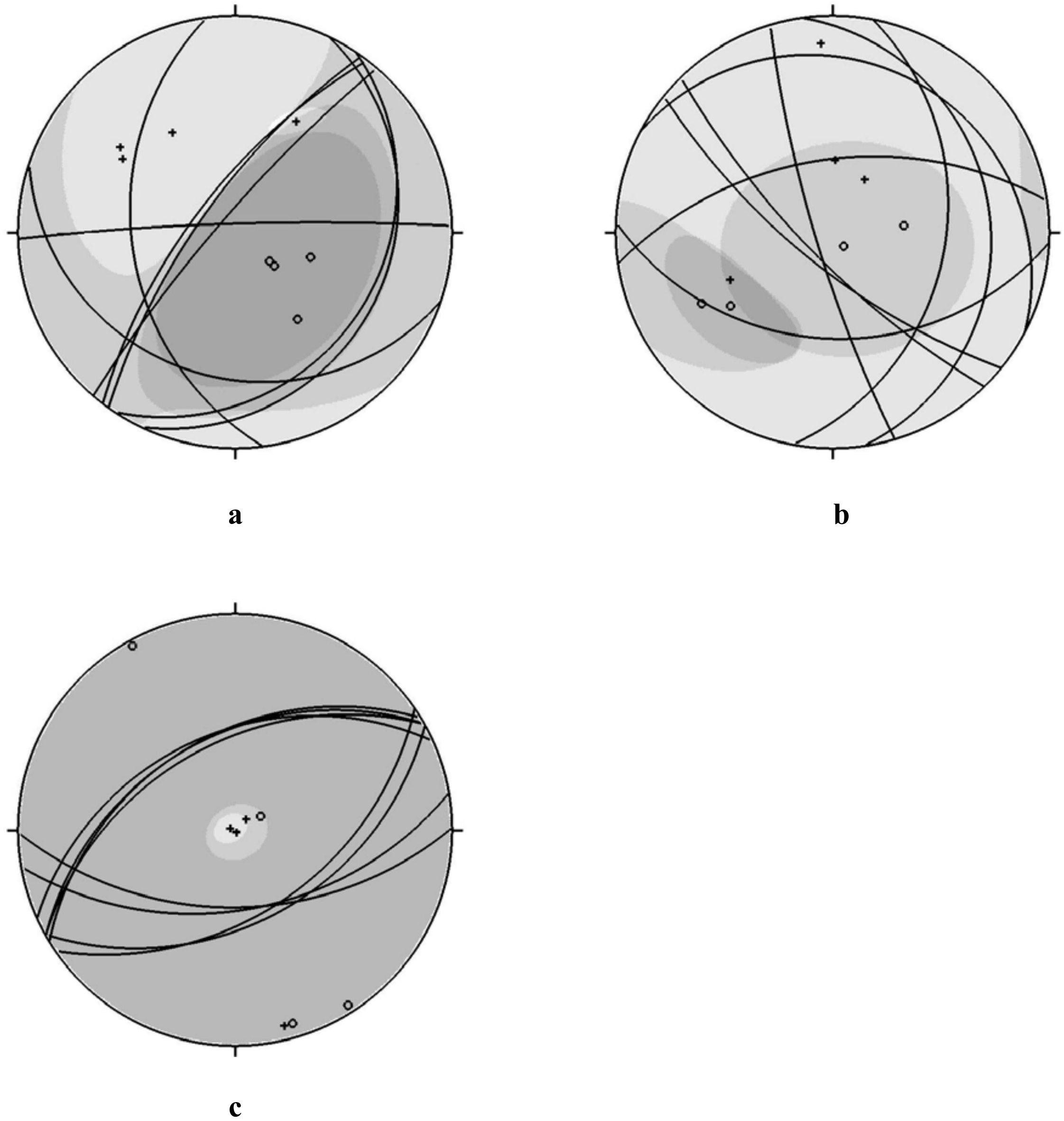

Fig. 2 All solutions for Cf given by (3) (solutions which average errors are given in Table 1, upper part), P axes are marked by ‘+’, T axes by ‘o’. Event 048290 – Fig. 2a, 058266 – Fig. 2b, 065259 – Fig. 2c.

Table 2 Determined MT for 3 processed events (including their errors) for most confident model (ZTR coordinate system, homogeneous medium model, Cf given by (4). Number of used stations and magnitudes Mw are given also. Note that from amplitudes ratio MT values cannot be determined absolutely – they differ by a multiplicative constant. Therefore it is also impossible to determine scalar moment of the events. Given m_i values range in interval <-1; 1> (i.e. arbitrary chosen range of parameters in the inversion) and MT are not normalized to enable easy comparison of errors of different events and/or solutions.

event	stats.	Mw	m11	m12	m13	m22	m23	m33	aver. error
048290	29	1.02	-0.93	0.73	-0.49	0.34	0.76	0.66	0.12
			+/- 0.14	+/- 0.14	+/- 0.09	+/- 0.17	+/- 0.11	+/- 0.10	
058266	31	0.55	-0.19	0.25	-0.11	0.20	0.14	-0.14	0.06
			+/- 0.06	+/- 0.14	+/- 0.03	+/- 0.05	+/- 0.03	+/- 0.03	
065259	9	0.22	0.67	-0.42	-0.42	1.00	0.34	-0.19	0.18
			+/- 0.44	+/- 0.16	+/- 0.16	+/- 0.09	+/- 0.11	+/- 0.12	

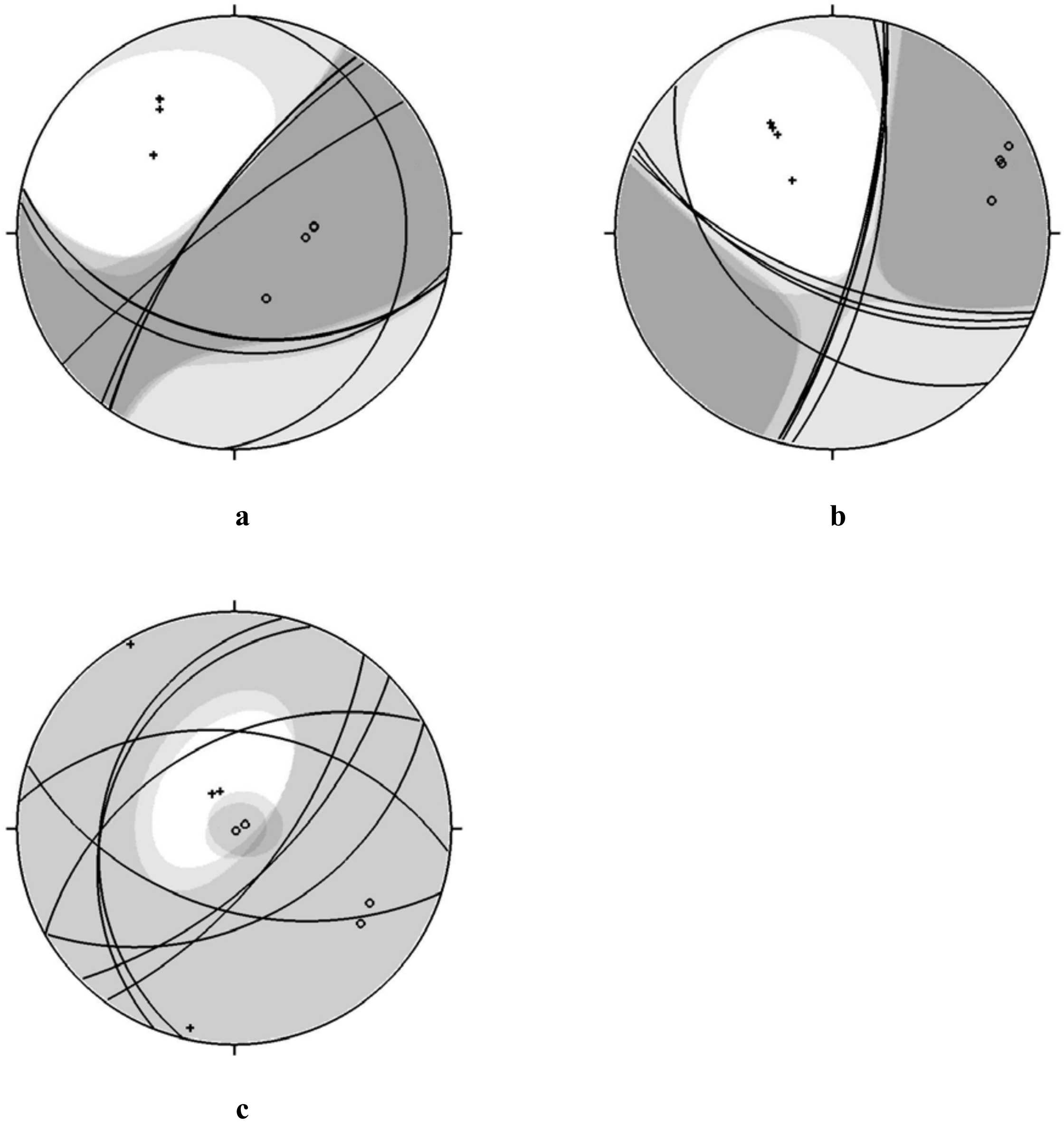

Fig. 3 The same as in Fig. 2 but for Cf given by (4) (solutions which average errors are given in Table 1, lower part). Event 048290 – Fig. 2a, 058266 – Fig. 2b, 065259 – Fig. 2c.

Table 3 Decomposition of the most confident MT (given in Table 2) and their transformed errors. There are given also solutions published by Vavryčuk et al. (2007) - for those solutions the errors were not determined.

event	sts.	Mw	ISO [%]	CLVD [%]	DC [%]	Strike 1 [dgr]	Dip1 [dgr]	Rake1 [dgr]	Strike 2 [dgr]	Dip2 [dgr]	Rake2 [dgr]
048290	29	1.02	2 +/- 10	-38 +/- 28	60 +/- 28	215 +/- 11	73 +/- 6	55 +/- 9	102 +/- 10	39 +/- 9	152 +/- 12
Vavryčuk et al.			-2	-55	43	197	87	34	105	56	177
058266	31	0.55	-11 +/- 14	-5 +/- 34	84 +/- 22	113 +/- 18	60 +/- 15	-163 +/- 11	15 +/- 4	75 +/- 11	-31 +/- 14
Vavryčuk et al.			0	-2	98	110	69	-169	16	80	-22
065259	9	0.22	34 +/- 16	12 +/- 45	54 +/- 35	46 +/- 20	63 +/- 12	-79 +/- 30	201 +/- 45	29 +/- 8	-111 +/- 44
Vavryčuk et al.			31	30	39	37	75	-149	359	60	-18

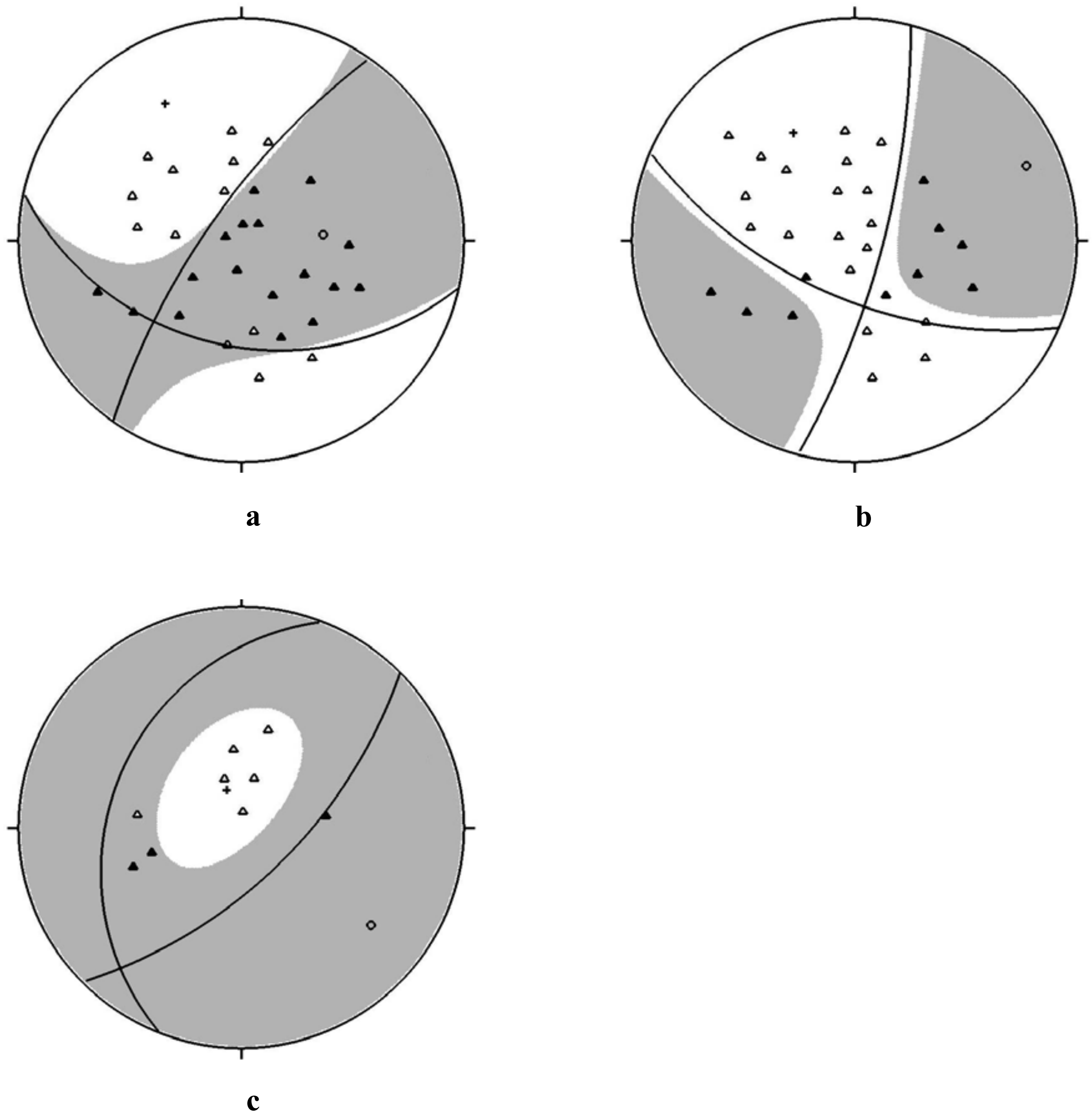

Fig. 4 The most confident solutions (ZRT coordinates system, gradient medium model, Cf given by (4) - Tables 2 and 3). Used stations are marked by triangles; positive first motions are filled, negatives are empty. P axe is marked by '+', T axe by 'o'.

MT are then decomposed (Table 3), the determined errors are transfer (via formula (7)). Results given in Table 3 confirm that geometrical orientation of a source mechanism is usually determined more reliable than amount of its DC and non-DC parts. The biggest relative error of DC part is +/- 35%[8] (event 065259, DC part), the biggest error of geometrical orientation is about +/- 17%[9].

Our results are compare (in Table 3) with results given by Vavryčuk et al. (2007), it can be concluded, that both results are consistent if determined errors taken into account. The discrepancy can be consequence of different way of processing as e.g. different inversion method (Vavryčuk used Genetic Algorithm), different construction of Cost function, etc.

CONCLUSIONS

We determine full seismic MT and its errors for 3 investigated events. It has appeared that stability of

[8] The ISO and CLVD components range in interval <-100%; 100%>, DC in interval <0%; 100%> - see comments to formula (6). To these intervals correspond also calculated relatives errors.
[9] Strike and rake range in interval <0^o; 360^o>, dip in <0^o; 90^o>.

the solution depends on construction of Cost function (we had to emphasize fit of P waves first motion signs). Simultaneously we determined also errors of MT components and transform them during MT decomposition. The magnitude of error generally shows indirect dependency on number of readings, which is then approximately proportional to the event magnitude. The values of relative MT error vary in order from units of percent to first tens of percent. The relative errors of DC and non-DC parts reach first tens of percent; the relative error of DC orientation is smaller and reach a unit of percent (with one exception – see above).

We remember again, that determined MT errors cannot be taken literary: due to used methodology (of the inversion as well as of the errors determination) they unambiguously posses a random part and the results are only sort of an estimation. However, estimated errors seems to be consistent (they increase with decreasing event magnitude) and the relative precision of source orientation is higher than precision of its DC and non-DC parts. Even if (relative) errors of numerical results are not dramatically high, any subsequent geophysical interpretation of the results should not go behind this precision.

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