# HOW MUCH CAN WE TRUST SOME MOMENT TENSORS OR AN ATTEMPT OF SEISMIC MOMENT ERROR ESTIMATION – 2 DATA REINTERPRETATION, METHODOLOGY IMPROVEMENT

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ABSTRACT
Non-liner inversion of ratio of (relative) P to S waves amplitudes of 3 selected KTB events was performed in Kolář (2007a) to determine moment tensors (MTs). Here further development of the method is presented: *(i)* re-calibration of the seismograms, *(ii)* re-interpretation of the S wave maximal amplitudes readings, *(iii)* a linear inversion of the (re-calibrated) amplitudes including MTs errors determination, and *(iv)* four methods of transformation of MT errors into errors of their decomposed parts. Generally, the new results confirm previous ones, however remain some open questions about MT errors transformation. New methodology is more accurate and data processing more user-friendly.

KEYWORDS: seismic moment tensor inversion, error estimation, seismic moment tensor decomposition

*In commemoration of work of Tycho Brahe (died 1605 in Prague), who measured so precisely, more precisely than had ever been measured before him, that his measurement would provided a basis to falsify his own hypothesis about the Solar system and the Universe.*
*(adopted after Horský and Plavec, 1962 and Horský, 1980)*

## INTRODUCTION

The events under interest come from an active experiment: fluid injection into the KTB borehole (for details about the experiment see Baish et al., 2002). The seismograms were recorded by a network consisting of about 40 stations, there were available records of events with magnitude ranging from -1.2 to 1.1. The analysis was restricted only to data from surface stations and to the set of about 150 events with depth about 5 km (events were also located around depth 9 km, but they were not subject of this analysis). In the previous study (Kolář, 2007a) we determined full seismic moment tensors (MT) for 3 events from the set ($M_w$ = 1.02, 0.55, 0.22).

The data sets – seismograms – are partly distorted and as the most distorting it has appeared the lack of calibrations constants due to which the amplitudes could not be determined absolutely. Therefore ratios of relative P to S waves amplitudes were used as inversion input data. We processed maximal displacement of P and S waves amplitudes. Remember, that while inversion of absolute P and/or S amplitudes is a simple linear problem, inversion of P to S ratios is then generally non-linear.[1]

Determined MTs were finally decomposed into their volumetric (VOL), compensated linear vector (CLVD) and double couple (DC) parts and their orientation (strike, dip, rake), which is nowadays probably most often used way of MT decomposition. Errors of MT are then also "decomposed" into errors of DC, non-DC parts and geometrical orientation.

Further development of several points of the previous work is presented in this paper: *(i)* re-calibration of the seismograms, *(ii)* re-interpretation of the S wave amplitudes, *(iii)* a linear inversion of the (re-calibrated) amplitudes is performed, and *(iv)* four methods of transformation of MT errors into errors of their decomposed parts are design, applied and compared.

[1] Even if division of two linear equations lead generally in non-linear problem, Julian and Foulger (1996) developed a method which enable linear inversion of P to S waves ratio.

## SEISMOGRAM RE-CALIBRATION

Once MTs are determined (Kolář, 2007a) it is also possible to evaluate forward problem, i.e. using well know equation

$$U = G * MT , \qquad (1)$$

where $U$ is displacement, $G$ is (derivative of) Green's function and $MT$ is seismic moment tensor (see e.g. Jost and Herrmann, 1989), "theoretical" amplitudes can be calculated. It is known, that the stations of KTB network were equipped with three-component seismometer Mark L4-3c and with recording units PDAS-100 (Baish et al., 2002). This instruments posses a pre-amplifier which gain can be changed by orders. Let suppose now, that the seismogram absolute amplitudes ambiguity is consequence only of usage wrong values of this $10^n$ gain in raw data restitution and that values were not changed during the experiment.[2] Those pre-gains for each station are searched in form of

$$A_{abs_i} = 10^{n_i} A_{relat_i} \quad \text{i=1- number of stations} \qquad (2)$$

where $A_{abs}$ is absolute (displacement) amplitude and $A_{relat}$ relative amplitude for each station and we try to set individual $n_i$ from their mutual comparison to reach best fit - trial and error method was used.[3] The possible influence of noise, which is eliminated when P to S ratio is used, is (partly) reduced by svismogram filtration (band pass filter 2.5 – 40.0 Hz) and using amplitudes determined as $A_{max}$ - $A_{onset}$ (for details see Kolář, 2007a). The effect of the correction is demonstrated in Figure 1; the values of correction are given in Table 1. Note, that by this method the calibration constants can be determined but for a multiplicative constant (common for all the stations).

**Table 1** Found values of re-calibration constants (Data_truth = Correction * Data_orig ).

| Station | Correction |
|---|---|
| 303 | 0.01 |
| 322 | 0.10 |
| 323 | 0.10 |
| 341 | 10.00 |
| 347 | 0.10 |
| all others | 1.00 |

## S WAVES AMPLITUDE RE-INTERPRETATION

During S waves picking in the seismograms under the interest, it was often difficult to identify maximal S wave amplitude and particle motion diagrams were used systematically to distinguish them – see Kolář (2007a). Also time shift between S waves maxima on different components were often observed. We decided to re-interpret S waves picking with different seismogram viewer (Kolář, 2006, 2007b) and to test different methodology of S waves interpretation. We adopted methodology proposed Silver and Chan (1991) which is based on time and spatial correlated seismograms.[4] In this method we search for new components F and S given by transformation of original component R and T (in ZRT components system). The transformation consists of rotation of (horizontal components) by angle *dFi* and their mutual time shift by *dN*. These values are chosen in such a way (following Silver and Chan, 1991), that determinant D

$$D = \begin{bmatrix} C_{FF} & C_{FS} \\ C_{FS} & C_{SS} \end{bmatrix} , \qquad (3)$$

has minimal value. Coefficients C represent corresponding correlation of new components F and S. In such a way the shape of S waves is simplified - it has usually one principal maximum which can be easily identified. To obtain SH and/or SV amplitudes we rotated (time shifted) components F and S backward by angle *–dFi* while the (new) time position is kept unchanged.

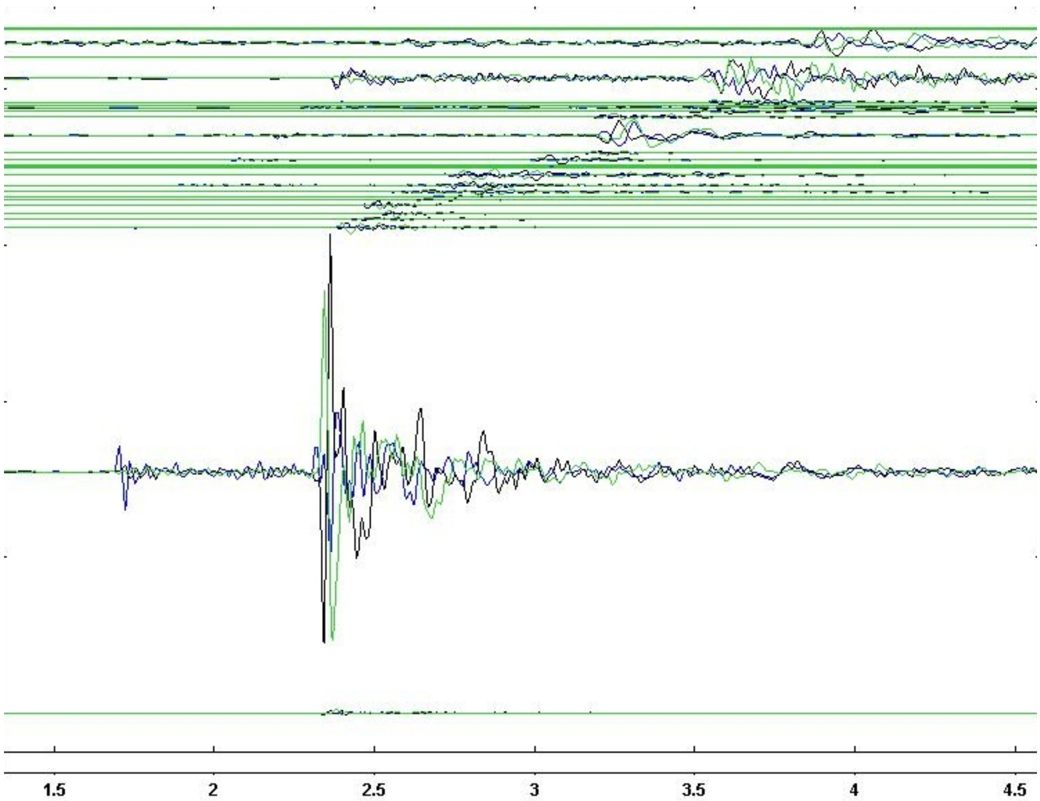
It has appeared, that maximal S amplitudes can be more easily identify in new transformed components and particle motion diagram was used only in a few cases, mainly to distinguish between two similar maxims of different orientation. The reinterpreted data were used then for (linear) inversion; P waves amplitudes were not re-interpreted and originally interpreted values (Kolář, 2007a) were used.

Two slightly different ways of S waves processing were designed. First only horizontal components of seismograms are correlated with no regard to vertical one (method refereed here after as SLVR1). In the later, vertical Z and radial R component are rotated (keeping transversal T component constant) so that the vertical component is minimal before correlation calculation. This approach

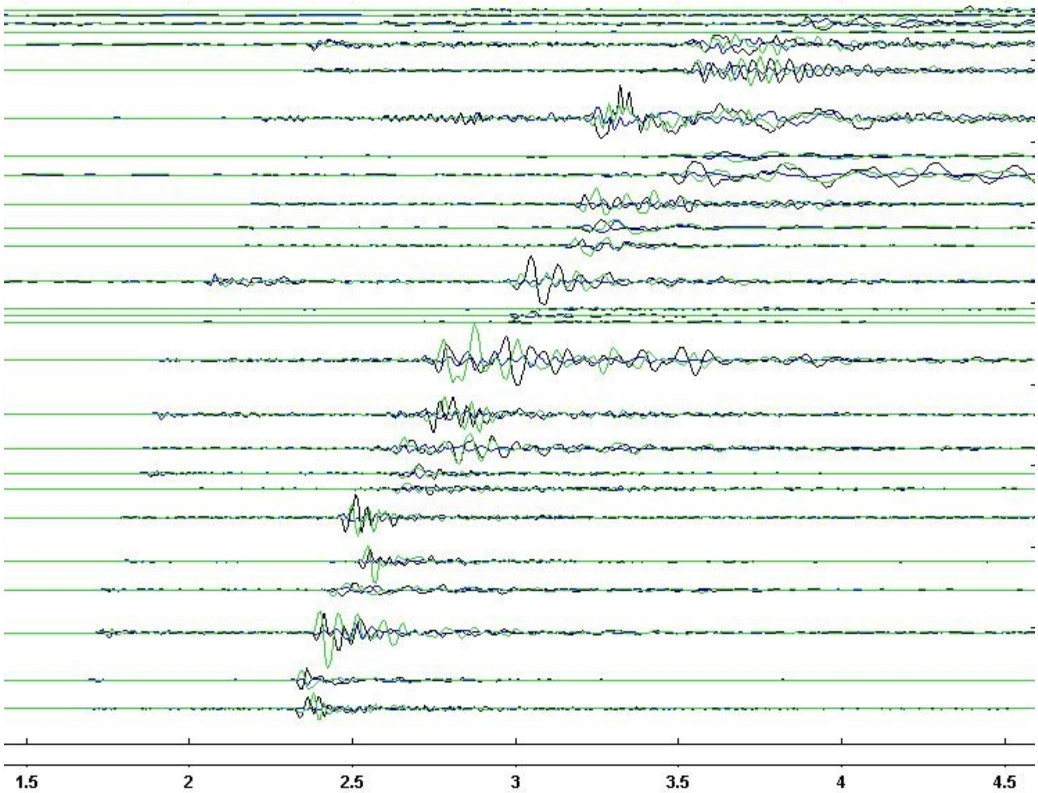
[2] Value of calibration constants are likely lost for ever (if they ever had been known correctly). However on the base of discussion with people operating seismometers (namely J. Horálek – personal communication) and on the structure of data decoding program, where various constants in form of $10^n$ for particular stations are applied, we consider our assumption to be at least probable, if not more.

[3] This idea was lanced by colleagues Z. Jechumtálová, V. Vavryčuk and T. Fischer – personal communication.

[4] This method was originally developed for processing of waves affected by medium with anisotropy. Here we do not study origin of the effect (shift of maxima can be caused not only by anisotropy but e.g. also by free surface effect).




**a**




**b**

**Fig. 1** An example of one event data set (event 048290, seismograms are sorted according epicentral distance, all 3 components are plotted). There are raw original data in Figure 1a and the same seismograms after re-calibration in Figure 1b. The improvement is obvious, the remaining amplitude variation is consider to be the effect of radiation pattern, propagation, surface effects, etc.

is a sort of determination of dynamic incident angle. The angle of rotation was allowed to range in interval ($0^o$ ; $45^o$). Then new horizontal components were again correlated (method refereed here after as SLVR2). A byproduct of this operation is a set of values of *dN*, *dFi* and dynamic incident angles which should be the subject of the future investigation.

**LINEAR INVERSION OF RE-CALIBRATED DATA**

Absolute P and/or S amplitudes can be put into (1), i.e. in a system of over-determined linear equations in our case. Matrix's coefficients – Green functions – are the same as in previous work (Kolář, 2007a): a homogeneous half space model with $v_P$=5.6 km/s and $v_P/v_S$=1.8 was used. Remember that this simple model yielded better fit of the data then other considered model – a 400m low velocity layer over a half space gradient model (Málek et al., 2000).[5] MTs were then determined as a solution of system of over determined linear equations (i.e. a matrix inversion is performed - see e.g. Menke, 1989). MTs are computed from re-calibrated P wave amplitudes and P+S wave amplitudes. Remember that MT cannot be inverted only from SH waves amplitudes which are used as S waves representation. Remember, that we restricted on SH waves to avoid problem with uncertainty or even singularities with free surface conversion coefficient for SV waves – for more detailed discussion see Kolář (2007a).

Errors of these linear solutions can be determined by a standard way: independency of the parameters (MT components $m_{ij}$) is supposed and the errors are determined as diagonal members of a covariance matrix when estimation of data standard error *s* is given as (following e.g. Rektorys et al., 1995)

$$s = \sqrt{\frac{1}{N-k-1}\sum_{i=1}^{N}\left(x_{0i}-x_i\right)^2} \quad , \qquad (4)$$

where $x_0$ are predicted values and $x$ are observed data, $N$ is number of observations, $k$ number of parameters ($k = 6$ in our case). It is supposed, that into this estimation of the data standard error it is also projected error of inadequate medium modeling which is indeed not a homogeneous half space in reality. Note, that these "linear" errors are fully deterministic to the contrary to the previous "non-linear" errors determined in Kolář (2007a). Those "non-linear" errors were an estimation and even if they were found to be realistic and consistent, they inevitably posses a stochastic part and may vary for different realizations of non-linear inversions.

It can be expected that P and S waves amplitudes are determined with different reliability (S waves amplitudes are expected to posses bigger error as they can be disturbed by noise generated by P waves, they are often of more complicated shape and therefore more difficult to be interpreted, etc.). Therefore weighting of S wave data is optionally used: the S waves amplitudes are scaled with factor *w* which is estimated (following e.g. Bartsch, 2000) as

$$\mathrm{w} = {s_P^2}\Big/{s_S^2} \quad , \qquad (5)$$

where $s_P$ is error estimation from only P waves data solution evaluated using (4). Error estimation $s_S$ cannot be calculated directly as it is not possible to invert only for SH waves amplitudes (see above). Therefore we used following approach: we evaluate theoretical $S_{theorP}$ waves amplitudes from $MT_P$ – moment tensor determined from P waves. Error $s_S$ is then calculated as difference between those theoretical $S_{theorP}$ values and S waves data.

To be able to compare reliability of various solutions we introduce a single number: an average of normalized errors of $m_{ij}$ components. The averages of normalized error of MTs of different considered data sets are given in Table 2. It follows from the table that lower or even the lowers error is produced for linear inversion of P + S weighted data, ZRT coordinates system and SLVR1 data reading method. In addition this method enable also user-friendly way of data processing. The results of this (linear) inversion are given in Table 3 (MTs), Table 4 (MTs decomposition) and plotted in Figures 2 and 3. It follows from the Table 4 that MT obtained by non-linear and linear inversion are consistent when the system is sufficiently over determined (events 048290 and 058266). For event 065259 the situation is different. We suppose that it is consequence low number of observations (data only from 9 stations are applicable) and also (slightly) different data set is used for linear and non-linear inversion: results of non-liner inversion of only P to S amplitudes ratios were unstable and had to be stabilized by adding signs of P waves onsets, which modify data set form statistical point of view. Note that similar approach for inversion stabilization was used also by Vavryčuk et al. (2008).

From the re-calibrated data it was also possible to determine value of scalar seismic moment $M_0$ and consequently moment magnitude $M_{M0}$ by using formula

$$M_{M0} = \frac{2}{3}\left(\log_{10} M_0 - 9.1\right) \quad , \qquad (6)$$

defined by Hanks and Kanamori (1975) – see Table 3. Determined $M_{M0}$ values are higher than original values $M_{orig}$ , however serious discussion of this effect would need enlarge the processed event set.

**MT ERRORS DECOMPOSITION**

In addition to the knowledge of MT we determined also errors of MT components. In this

[5] No other testing of medium models was performed in the present work.

**Table 2** Average of MTs error of linear inversion of different data sets are given (inversion of P, P+S, P+Sweighted and corresponding weight). Two systems of components were used (ZNE and ZRT) and three methods of maximal amplitude finding: particle motion diagram (PM) and two methods of time-spatial correlation SLVR1, SLVR2 for data interpretation – see the text. The combination of ZRT components system, SLVR1 method of interpretation and weighted S amplitudes data yields generally low or the lowest error value (bold); the inversion of non weighted P + S waves data set seems to be the worst possibility, but for 06259 event. We assign this effect to the low number of observations in this particular case.
On the base of given result with regards to user-friendly way of method SLVR1 data processing, we chose the combination of ZRT coordinates system and SLVR1 interpretation method as suitable for further processing.

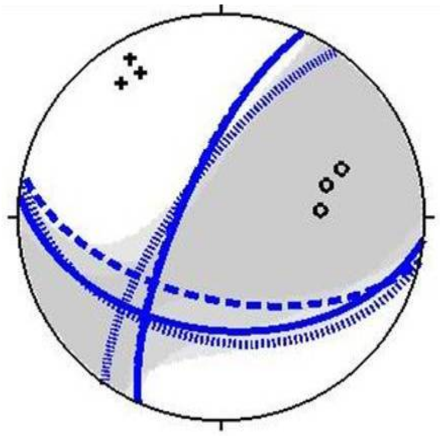
| Event | Components / method | P | P+S | P+S weighted | P/S weight |
|---|---|---|---|---|---|
| 046290 | ZNE PM | 0.18 | 0.19 | 0.13 | 35 |
| | ZRT PM | 0.18 | 0.36 | 0.13 | 34 |
| | ZRT SLVR1 | 0.18 | 0.35 | **0.14** | 43 |
| | ZRT SLVR2 | 0.18 | 0.42 | 0.15 | 128 |
| 058266 | ZNE PM | 0.33 | 0.93 | 0.26 | 7 |
| | ZRT PM | 0.22 | 0.24 | 0.16 | 13 |
| | ZRT SLVR1 | 0.22 | 0.42 | **0.11** | 99 |
| | ZRT SLVR2 | 0.22 | 0.50 | 0.11 | 149 |
| 065259 | ZNE PM | 1.27 | 0.16 | 0.37 | 40 |
| | ZRT PM | 0.39 | 0.48 | 0.19 | 115 |
| | ZRT SLVR1 | 0.39 | 1.07 | **0.20** | 221 |
| | ZRT SLVR2 | 0.39 | 1.13 | 0.20 | 192 |

**Table 3** Moment tensor determined as linear inversion of P and weighted S waves amplitudes. There are given number of event, number of used stations, original magnitude $M_{orig}$, magnitude determined from scalar seismic moment $M_{M0}$, scalar seismic moment M0, normalized components $m_{ij}$ of MT, their errors and average of errors of $m_{ij}$ .

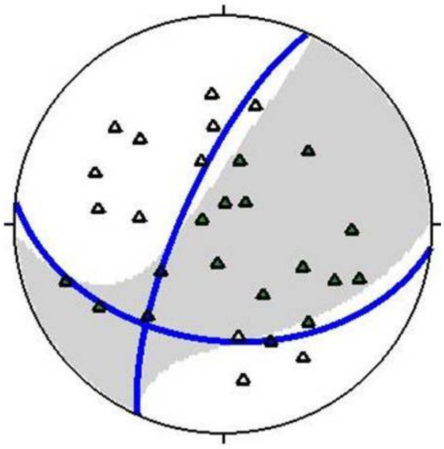
| event | stats. | $M_{orig}$ | $M_{M0}$ | M0 x$10^9$ [Nm] | $m_{11}$ | $m_{12}$ | $m_{13}$ | $m_{22}$ | $m_{23}$ | $m_{33}$ | averr. error |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 048290 | 29 | 1.02 | 1.33 | 123 | -0.63 | 0.66 | -0.24 | -0.04 | 0.52 | 0.29 | |
| | | | +/- 0.04 | +/- 16 | +/- 0.23 | +/- 0.16 | +/- 0.07 | +/- 0.23 | +/- 0.07 | +/- 0.07 | 0.14 |
| 058266 | 31 | 0.55 | 1.01 | 41.2 | -0.47 | 0.72 | -0.48 | 0.41 | 0.17 | -0.25 | |
| | | | +/- 0.01 | +/- 0.9 | +/- 0.19 | +/- 0.14 | +/- 0.06 | +/- 0.18 | +/- 0.07 | +/- 0.06 | 0.12 |
| 065259 | 9 | 0.22 | 0.42 | 5.3 | 0.78 | 0.18 | -0.80 | 0.01 | 0.13 | 0.15 | |
| | | | +/- 0.71 | +/- 1.3 | +/- 0.48 | +/- 0.23 | +/- 0.22 | +/- 0.14 | +/- 0.06 | +/- 0.08 | 0.20 |

paragraph we investigate how these errors can be transformed during MT decomposition. We named this transformation as MT errors decomposition.

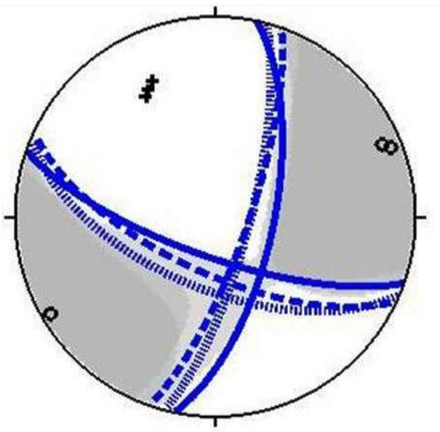
In Kolář (2007a) errors of MTs were determined by mapping the parameter space around the (non-linear) solution and they were transformed into errors of source geometrical orientation (strike, dip, rake) and errors of VOL, CLVD and DC components. It was done it in a following way: all possible combinations of MT's values and their errors were used as input data for MT's decomposition; the maximum or minimum of the decomposed values respectively were then taken as an error interval of MT's decomposition results. Two versions of method were used: either all the possible combination of $m_{ij}$ +/- $m_{ijerror}$ were consider – i.e. $2^6 = 64$ combinations (hereafter referred as MAP64 method) or all $m_{ij}$ were kept constant but one, which were vary in the same way as before – i.e. $2*6 = 12$ combinations were considered (hereafter referred as MAP12 method). These methods are simple and easy to evaluate. Method MAP64 is very conservative error estimation, method MAP12 is rather errors lower limit than their estimation. Moreover these method are not fully robust (namely MAP64) for errors of periodical values (strike, dip, rake) sometimes cannot be



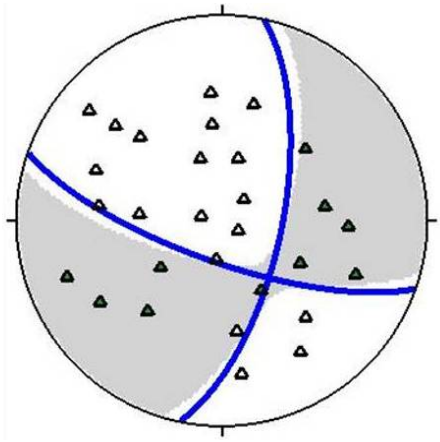
**2a**



**3a**



**2b**



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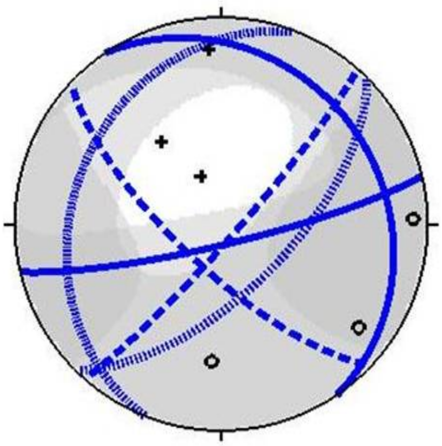
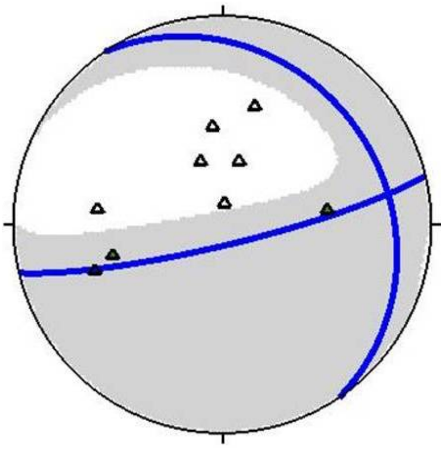


**2c**



**3c**

**Fig. 2** Comparisons of different MT solutions; for each event there are plotted non-liner solutions (dotted), linear solutions from P+S waves (dashed), and P+S weighted waves (full line). Note that solutions for P and P+S weighted waves do not differ significantly (but for 065259 event) and are not plotted. There are also plotted P ('+') and T ('o') axes, the mechanisms are plotted by MEPL2 program (Kolář, 2007c). Event 04290 is in 3a, 058266 in 3b, 069259 in 3c.

**Fig. 3** The best solutions of linear inversion of P+S weighted waves including stations used for the inversion. Event 04290 is in 4a, 058266 in 4b, 069259 in 4c.

**Table 4** Decomposition of MTs and errors. For all 3 processed events they are given non-linear (Kolář 2007a) and linear solutions. For each solution we give also errors determined by four methods: MAP64, MAP12, Gauss and Chol – see the text (Chol errors is not given for non-linear solution as necessary covariance matrix is not know). Note, that the non-liner errors may slightly vary from the values given in Kolář (2007a) as they were recalculated by slightly improved algorithm. Values marked with '*' were determined only from one side error interval, 'NaN' stands for non-determined value. The results are discussed in the text.

| Event | Inversion / Error | ISO [%] | CLVD [%] | DC [%] | Strike1 [dgr] | Dip1 [dgr] | Rake1 [dgr] | Strike2 [dgr] | Dip2 [dgr] | Rake2 [dgr] |
|---|---|---|---|---|---|---|---|---|---|---|
| 048290 | Non-Lin | 2 | -38 | 60 | 215 | 73 | 55 | 102 | 39 | 152 |
| | MAP64 | +/- 10 | +/- 29 | +/- 28 | +/- 10 | +/- 7 | +/- 14 | +/- 10 | +/- 10 | +/- 16 |
| | MAP12 | +/- 4 | +/- 11 | +/- 11 | +/- 4 | +/- 2 | +/- 4 | +/- 4 | +/- 3 | +/- 5 |
| | Gauss | +/- 5 | +/- 15 | +/- 16 | +/- 5 | +/- 3 | +/- 6 | +/- 5 | +/- 5 | +/- 7 |
| | Lin | -10 | -41 | 49 | 204 | 74 | 49 | 96 | 43 | 156 |
| | MAP64 | +/- 15 | +/- 34 | +/- 36 | +/- 19 | +/- 16 | +/- 27 | +/- 16 | +/- 13 | +/- 21 |
| | MAP12 | +/- 6 | +/- 11 | +/- 10 | +/- 8 | +/- 6 | +/- 9 | +/- 6 | +/- 4 | +/- 7 |
| | Gauss | +/- 8 | +/- 16 | +/- 16 | +/- 3 | +/- 7 | +/- 12 | +/- 11 | +/- 7 | +/- 11 |
| | Chol. | +/- 9 | +/- 18 | +/- 16 | +/- 11 | +/- 9 | +/- 17 | +/- 16 | +/- 6 | +/- 16 |
| 058266 | Non-lin | -11 | -5 | 84 | 113 | 60 | -163 | 15 | 75 | -31 |
| | MAP64 | +/- 14 | +/- 34 | +/- 24 | +/- 14 | +/- 17 | +/- 13 | +/- 12 | +/- 9 | +/- 18 |
| | MAP12 | +/- 4 | +/- 13 | +/- 10 | +/- 10 | +/- 9 | +/- 4 | +/- 7 | +/- 3 | +/- 10 |
| | Gauss | +/- 6 | +/- 16 | +/- 16 | +/- 7 | +/- 4 | +/- 11 | +/- 10 | +/- 10 | +/- 6 |
| | Lin | -9 | -21 | 70 | 109 | 74 | -155 | 12 | 66 | -18 |
| | MAP64 | +/- 13 | +/- 34 | +/- 34 | +/- 9 | +/- 10 | +/- 11 | +/- 8 | +/- 10 | +/- 11 |
| | MAP12 | +/- 5 | +/- 8 | +/- 11 | +/- 4 | +/- 3 | +/- 3 | +/- 4 | +/- 3 | +/- 3 |
| | Gauss | +/- 7 | +/- 14 | +/- 19 | +/- 5 | +/- 4 | +/- 5 | +/- 5 | +/- 5 | +/- 4 |
| | Chol | +/- 7 | +/- 15 | +/- 17 | +/- 14 | +/- 5 | +/- 10 | +/- 12 | +/- 5 | +/- 12 |
| 065259 | Non-lin | 34 | 12 | 54 | 46 | 63 | -79 | 201 | 29 | -111 |
| | MAP64 | +/- 15 | +/- 45 | +/- 33 | +/- 19 | +/- 10 | +/- 28 | +/- 41 | +/- 12 | +/- 45 |
| | MAP12 | +/- 7 | +/- 20 | +/- 12 | +/- 8 | +/- 3 | +/- 12 | +/- 26 | +/- 4 | +/- 21 |
| | Gauss | +/- 8 | +/- 27 | +/- 22 | +/- 10 | +/- 5 | +/- 15 | +/- 29 | +/- 5 | +/- 45 |
| | Lin | 24 | 33 | 43 | 77 | 82 | -110 | 326 | 22 | -22 |
| | MAP64 | +/- 15 | +/- 47 | +/- 41 | +/- 26* | +/- 8* | +/- NaN | +/- 31* | +/- 20* | +/- NaN |
| | MAP12 | +/- 7 | +/- 15 | +/- 21 | +/- 4 | +/- 6 | +/- 13 | +/- 29 | +/- 10 | +/- 29 |
| | Gauss | +/- 8 | +/- 22 | +/- 26 | +/- 7 | +/- 7 | +/- 17 | +/- 22 | +/- 15 | +/- 25 |
| | Chol. | +/- 10 | +/- 25 | +/- 21 | +/- 52 | +/- 18 | +/- 56 | +/- 35 | +/- 20 | +/- 49 |

determined uniquely as errors intervals can sometime over crossed for complementary solutions. Therefore a standard Gaussian method of error propagation based on evaluation of function differential (A1) was applied (method is hereafter referred as Gauss). The method however had to be adapted to our particular case, the technical details are given in Appendix A. This method is rather time consuming from point of view of evaluation, but the results are robust to the contrary to MAP64/12 methods. All the above mentioned methods deal only with individual errors of $m_{ij}$. From linear inversion we have not only $m_{ij}$ components errors but full covariance matrix is known. Therefore we can try to estimate also influence of its non-diagonal members. The algorithm is described in Appendix B, the method is referred as Chol (according to Cholesky matrix factorization which is used in the calculation). We consider Chol method as the most apposite to our problem (decomposition MT errors). Under this method the independency of MT components $m_{ij}$ need not be assumed. The disadvantage is the required knowledge of full covariance matrix, which cannot be know when e.g. non-linear inversion is performed.

All four methods (MAP64, MAP12, Gauss and Chol) were applied and the results are given, together with MTs decompositions, in Table 4.

It follows from the table that determined errors are generally consistent. Similarity between MAP64 and Chol and between MAP12 and Gauss can be observed. The MAP64 method is generally most conservative however Chol method, which we consider to be the most apposite, yields sometimes bigger errors of geometrical orientation especially for bigger $m_{ij}$ errors (event 065259). This could be influence of significant values of non-diagonal members of covariance matrix.

It follows also from the table, that reliability of MT of 065259 event, especially from point of view of MAP64 and/or Chol method, is very low (its accuracy is of order of tens of percent or degrees respectively) and further possible exploitation of results of such uncertainty seems to be rather problematic.

**CONCLUSIONS**

It can be concluded, that amplitudes of processed KTB seismograms were probably successfully re-calibrated. Results of performed linear inversion of re-calibrated amplitudes are in acceptable agreement with previous results from non-linear inversion of relative P to S amplitudes ratios (see Figure 2 and Table 4). The discrepancies are within the limit of determined errors or can be explained as solution instability caused by low number of observations. We designed four methods of MT errors decomposition. The results are discussed, however the appropriate usage of the method remains still open, namely for MT with relatively bigger errors.

From general point of view, within the frame of performed work:

- we design way of recovery of lost seismograms calibration constants. Hopefully, such situation is an exception and the method would not have to be used in future.
- it has been developed, or at leas implemented, a new method of amplitudes reading – time-spatial correlation of horizontal components were used instead of particle motion diagrams. The data interpreted by the method seem to yield slightly smaller errors than old one, the new method is definitely faster and more user-friendly.
- we performed MT linear inversion, this is a standard operation, there is no innovation in this point.[6]
- we design four methods of MT errors decomposition and discuss their features. Accuracy of find MT solutions and their decomposition is an important problem, however it remains open to the future investigation, namely role of non-diagonal components of covariance matrix.

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**Appendix A**

*In the* Appendix A *we describe a new symbolical-numeric method developed for evaluation of partial differential of values of MT decomposition, the method could be however used for other application also.*

Error $\Delta f$ of function f = f(x) can be expressed by function's differential in form of

$$\Delta f(x^0) = \sqrt{\sum_i \left( \frac{\partial f(x^0)}{\partial x_i} \Delta x_i^0 \right)^2} \quad , \qquad \text{(A1)}$$

where function $f$ is MT's decomposition in our case, $x^0$ is a solution and $\Delta x$ is its error (both values are known from inversion, see above);. Even if it is given analytically or in form of computer program code respectively, the formula is too complicate to express its partial derivatives analytically. Therefore we used a tool for symbolic calculus in MATLAB. Even on this platform the formula A1 cannot be simply implemented. Formulas for MT decomposition contain such elementary functions as *absolute_value(x)*, *minimum(xi)*, *maximum($x_i$)*, *relation operations*, etc. and such functions cannot be directly treated under Symbolic MATLAB Toolbox formalism. However, in our case we do not need to know full analytical form of function f derivatives, it is enough to know the derivatives in a particular point - in a solution. For this particular case a Symbolical-Numeric Process (here after SNP) was developed: one more dimension of size 2 was added to all variables in program code for MT decomposition (in used MATALB platform such extension can be done very easily). In the first part of an extended variable the treated value is stored as symbolic type, in the second as numerical (double) type with value of a particular point. Any time any symbolically generally non-defined function is used, its actual numerical value is seen and its symbolical value is evaluated in concordance. Whole principle is demonstrated in following (simplified) example of MATLAB's function for absolute value:

```
function ret=absSNP(val)
%
% evaluate abs value in SNP Method
% val=[val_symbolic, val_numeric];
% ret=[new_val_symbolic,...
% new_val_numeric];
%
  ret=val;
% numerical value is tested
   if val(2) < 0
   ret(1:2)=val(1:2) * -1;                (A2)
  end
return
```

[6] Even if from practical point of view linear MT inversion would be most common task, real live data sometime brings need of non-linear inversion as it was in our case (Kolář, 2007a). In addition the same data set was already used as test of newly developed non-liner inversion method to supplement the synthetic testing problems (system of polynomial equations) – Málek et al. (2007).

In such a way all required functions, namely: `abs, min, max, atan2, eig` and `relation operations` were modified.

Once having symbolical expression of strike, dip, rake, VOL, DC, CLVD (in a particular point – a particular solution), their partial derivatives and differential can be expressed. The numerical part of variable serves, in addition, to checking of the result (the value must be the same as for direct MT decomposition).

Notice that the method is rather computer time consuming. While the other methods (MAP64/12, Chol – see the text) are evaluated almost immediately, the SNP method is fairly exigent - it requires tens of minutes on relatively powerful computer and the memory allocation must be maintained carefully.[7]

Developed SNP method can be useful also in similar situations, when we need a partial derivative of complicated functions which cannot be treated neither analytically nor directly symbolically. The method was also described in Kolář (2007d).

**Appendix B**

I*n the* Appendix B *we describe method of MT errors decomposition with use of full covariance matrix.*

The MT errors decomposition is based on Aster et al. (2005), Example B.10:

Find the Cholesky factorization **L** of (covariance) matrix **C**

$$C = LL^T \ , \tag{B1}$$

Let **Z** be a vector of *n* independent N(0,1) random numbers (i.e. normal distribution). Let

$$X = m + LZ \ , \tag{B2}$$

where **m** is a MT (a solution) in our case. The decomposed MT errors are then determined as a standard deviation of decomposed **X** values.

Note that the results obtained by this method posse a random part, however it can be effectively eliminated by choosing *n* sufficiently big. We used *n* = 1000, however even with *n* = 100 the results were almost identical and stable.

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[7] Notice that described computations under new MATLAB 2007b version appeared to be 2-3 times faster (in comparison to version 2006b) and the results seem to be more stable – especially those concerning geometrical orientation which evaluation include goniometrical functions.