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COMPARISON OF METHODS OF MATHEMATICAL MODELLING OF THE WAVE FIELD IN SHALLOW SEISMICS

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Abstract: The paper deals with utilizing mathematical modelling in shallow seismic reflection surveying. In view of the specific properties of the medium near the surface, the model based on the method of finite differences is compared with the ray solution of the problem of seismic wave propagation. An experimental calculation for the Brandýs locality is also given.

Key words: shallow seismic surveying; mathematical modelling of the wave field

1. INTRODUCTION

With the advent of digital instruments and development of computer processing, shallow seismic surveying has become an effective method used in solving problems of engineering geology, geotechnics and the geology of building raw materials. Kinematic interpretation of refraction measurements is being used on a routine basis. The result is the determination of the parameters of a stratified model of the geological medium, or of a velocity section.

To deal with the structure of more complicated media, e.g., with velocity inversion, with small velocity contrast, or to increase the resolution, shallow reflection has been introduced. This is a method which, by using relatively high frequencies (Hill and Ali (1988) mention frequencies in excess of 200 Hz), achieves a resolution of as much as 0.5 m (Jogerius and Helbig, 1988). A number of authors (Knapp and Steeples, 1986b; Hill and Ali, 1988; and others) have pointed out that the disadvantage of the method is its dependence on actual conditions which affect the interpretation. In particular circumstances, this dependence, for example, makes it impossible to monitor the reflections from all boundaries, or makes it necessary to use considerably simpler methods of processing (Hunter *et al.*, 1984). The choice of the optimum scanning window for given reflection, which is closely related to the source frequency and the time factor of the source function, plays an important role. The choice of the trace spacing depends on the necessity to correlate the phases. The seismic apparatus and the wave sources have to display sufficiently high frequencies of the generated and recorded signals. Shallow geological media, frequently characterized by considerable variability in the horizontal as well as vertical directions, and the presence of strong and week reflection boundaries require a record of considerable dynamics (Knapp and Steeples, 1986). The geometry of measurement involving shallow boundaries produce records of rays incident at the surface under a very wide range of angles. This results in phase and amplitude shifts between traces, which may cause problems in correlating the phases (Hunter *et al.*, 1984).

The complicated nature and complexity of the factors affecting the wave field prompted the idea of using mathematical models of propagation of seismic waves to solve the problem of the conditions under which shallow reflection can be used, or of using the solution of the direct problem for iterative interpretation.

2. MATHEMATICAL METHODS OF MODELLING SEISMIC WAVE PROPAGATION

There are several methods of solving the direct problem related to the propagation of seismic waves. In the method of finite differences the complete wave field is computed in the time and space domain. It is based on solving the equation of motion of the continuum with the appropriate boundary and initial conditions. This involves the integration of a differential equation in which the derivatives are approximated by differences.

A similar method is the method of finite elements. In this method the functions being sought approximated within the scope of elements by relatively simple dependences. The problem is to determine the coefficients of the aproximation functions in the separate elements so that they satisfy the differencial equation involved.

Methods referred to as "reflectivity methods" make use of resolving the initial wave field into a sum of plane waves. The propagation of elementary plane waves through a 1-D structure can be solved by matrix methods. Once the waves have propagated through the structure, they are added up to produce the resultant wave field.

Ray methods *a priori* resolve the wave field into elementary wave types propagating along the rays. In this case their travel times and amplitudes decreasing due to geometric spreading and conversion at the internal boundaries are computed.

Each of the methods mentioned has its advantages and disadvantages. In this paper we have used the finite-difference methods and the ray method. The properties of both methods we have indicated will be briefly described in the following sections.

3. CHARACTERISTIC OF COMPUTING THE WAVE FIELD USING THE FINITE-DIFFERENCE METHOD

For practical reasons the computation is carried out in a grid with a finite number of nodes. As a rule, a homogeneous, isotropic, rectangular grid with two dimensions (x, z) and spacing $\Delta x = \Delta z$ is used, i.e. the area of interest is formed by a rectangle with sides $n\Delta x$ times $m\Delta z$. In the internal nodes of the grid the difference schema is derived from the equation of motion, however, boundary conditions have to be formulated separately for the boundary nodes. It is relatively easy to formulate the boundary conditions for a free surface, because these can be derived from the condition of zero stress components, $\alpha_{zz} = \tau_{zx} = 0$. The situation becomes more complicated if a non-reflecting boundary has to be defined, across which the waves should propagate as whole, and the continuing medium simulated in the given direction. There are several ways of establishing non-reflecting boundaries (Smith, 1974; Reynolds, 1978; Sochacki *et al.*, 1987; Cerjan *et al.*, 1985; Clayton and Engquist, 1977). The general feature of all non-reflecting boundaries is that they do not absorb the incident wave completely, and that it is always necessary to assess the degree of contamination of the wave field by parasitic reflections. Parasitic reflections may be eliminated in an elementary way by separating the boundaries sufficiently; however, this is usually connected with having to deal with a grid of unmanageable size.

If a function $f(x_i)$, say a displacement component, is approximated by discrete values f_i, the derivatives of this function can be approximated by a linear combination of approximate values (e.g. $f'(x_i) = (f_{i+1} - f_{i-1})/2\Delta x$). It can be proved that function f_i, being determined by the finite-difference method, must not vary too quickly, in other words that the value Δx must be sufficiently small. The actual conditions with regard to the magnitude of Δx follow from the order of accuracy of the difference formula. For difference schemes of second-order accuracy the shortest wave length $\lambda_{\min}$ must be equal to or larger than $10\Delta x$, for difference schemes of fourth-order accuracy $\lambda_{\min} \geq 5\Delta x$ (Zahradník and Hron, 1991). Difference schemes of an order higher than the fourth have proved to be impractical. Similar limitations apply to the step in the time domain, Δt. For example, in the P-SV problem with a difference scheme of second-order accuracy $\Delta t \leq \Delta x/(\alpha^2 + \beta^2)^{1/2}$ (Aki and Richards, 1980). If these conditions for Δx and Δt are not satisfied, the computation fails completely and becomes quite unstable. It can thus be proved that the finite-difference method is suitable namely for computing the low-frequency components of the wave field, the limiting case being problems of static strain. Since the condition for Δt must hold within the model as whole, the computation for a medium with large velocity contrasts is time-consuming. The presence of regions with high values of α and/or β requires small values of Δt to be used and, consequently, also a large number of time levels to be computed.

In spite of all the restrictions mentioned, the finite-difference method is used frequently, however, as a rule for solving problems of seismic wave propagation in structures whose dimension is at least of the order of 100 m. The advantage of the finite-difference method is namely that it produces the wave field as a whole and no wave selection is required as in the ray method. The wave field thus contains all wave types, including all reflected and surface waves. In principle, this provides the easiest way of comparison with real data.

4. CHARACTERISTIC OF COMPUTING THE WAVE FIELD USING THE RAY METHOD

Ray method are convenient namely for studying wave types of interest. They provide data on the travel times and amplitudes of wave types selected in advance.

In order to construct a synthetic seismogram, it is, therefore, necessary to superimpose all energetically significant waves, and the correct wave field, strictly speaking, can then only be obtained in the limiting case of an infinite summation.

The source of seismic waves has a given time function and radiation diagram. The shape of the rays is determined by solving the eikonal equation which depends on the distribution of propagation velocities. By integrating the time increments along the ray, the travel time is determined. The amplitudes decrease as a result of geometric spreading, and as a result of reflection and refraction at internal boundaries. The geometric spreading values depend on the curvature of the wave surfaces. i.e. conversely on the shapes of the rays. The coefficients of reflection and refraction can be determined using matrix methods, which assume a plane wave incident at a planar boundary.

It can be proved that ray methods provide sufficiently accurate solutions only if the signal frequency is sufficiently high. They cannot be used to solve problems connected with ray refraction and diffraction on inhomogeneities of small size. The simpler versions of ray methods do not, as a rule, yield characteristics of interference (e.g. surface) and inhomogeneous (e.g. head) waves.

Ray method computations are relatively fast and, as compared to the finite-difference method, a larger number of problems can be dealt with.

5. FIELD MEASUREMENTS

Experimental measurements aimed at monitoring reflections from shallow boundaries were carried out at the locality of Brandýs nad Labem. This is a locality with a simple geological structure, located in the Bohemian Cretaceous. The geological layers are deposited horizontally to subhorizontally without being affected tectonically to a larger extent. The geological profile is shown in Fig. 1 and depicts the wall of an abandoned stone quarry.

Fig. 1. Locality Brandýs nad Labem. The quarry wall discloses a geological profile in a plane parallel with the seismic profile. Table I gives the seismological division of the profile.

The seismic measurements were carried out along a profile situated in the wooded terrain above the upper edge of the quarry. The perpendicular distance from the quarry wall was about 150 m to avoid lateral reflections. The 24-channel Terraloc MK-II apparatus was used for the measurements. The seismic energy was generated by a vertical impact, and the vertical component of ground motion velocity was recorded. The recorded data could be partly interpreted using methods of classical refractions seismology. The parameters of the adopted geophysical model are summarized in Table I.

Table I. Parameters of the adopted geophysical model

Layer No.	Thickness m	m/s	m/s	Lithology
1	6.5+	300+	150$	loam
2	6.0++	2200+	1200$	argillite
3	—	3500$	2000$	sandstone

\+ values derived from refraction seismology
++ measured on the quarry wall
$ estimate

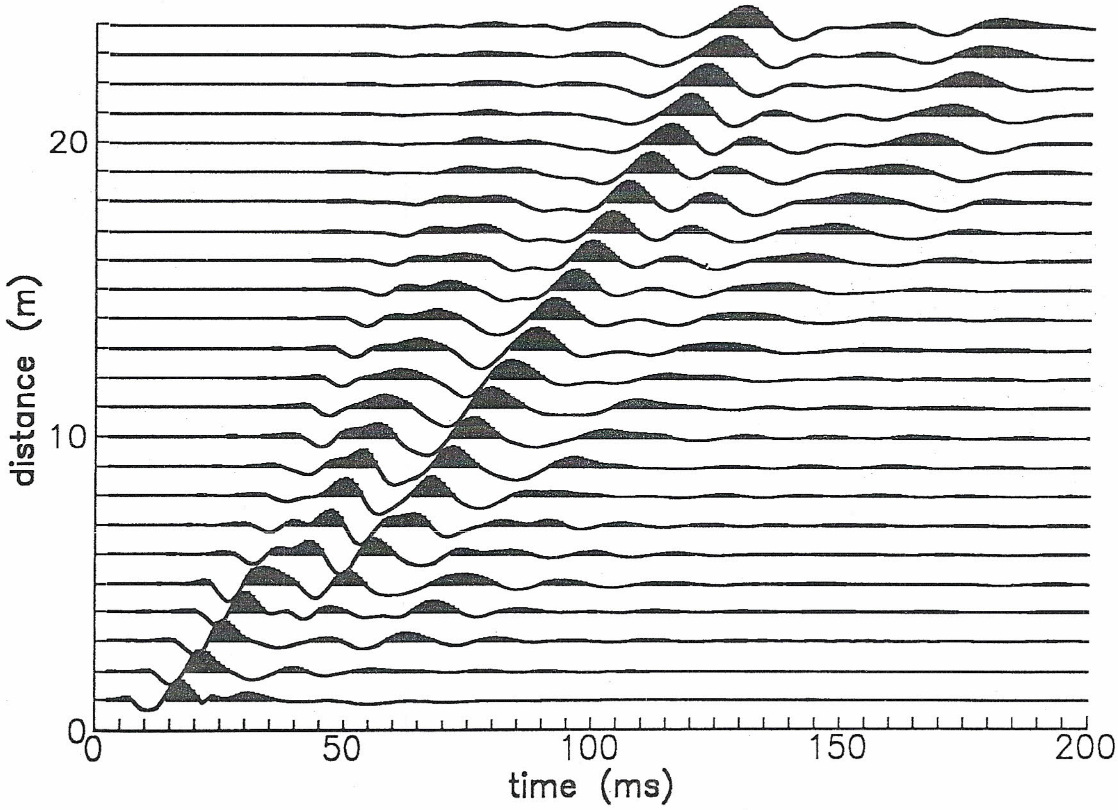

Fig. 2. Example of measured data. Distance of geophones 1 m, offset of point of impact 1 m. Amplitudes normalized within the separate traces to one common level.

An example of the observed data is shown in Figs. 2 and 3. Both figures represent the same record, but in different mapping mode. The amplitudes along each trace were normalized in Fig. 2, and the AGC mode was applied in Fig. 3.

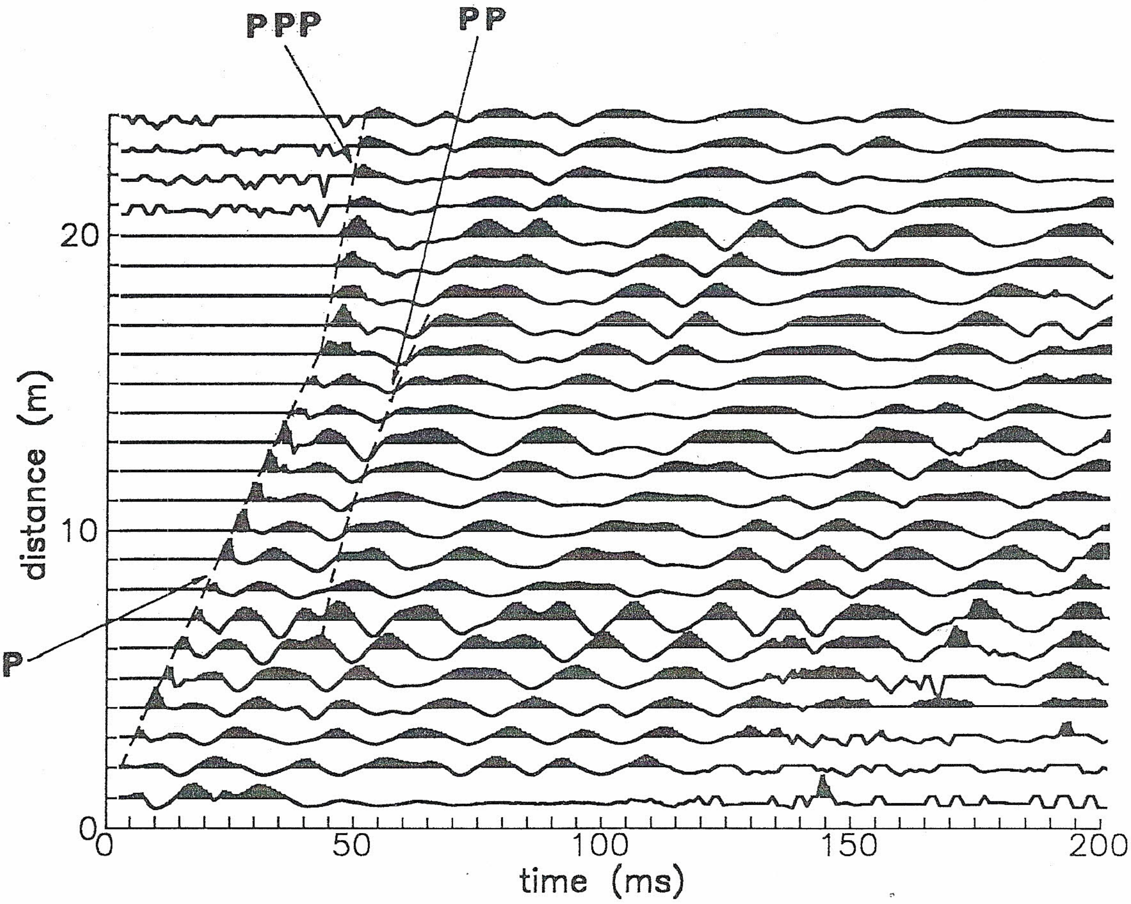

Fig. 3. Record in Fig. 2 made in the AGC mode (automatic gain control normalization along each trace carried out first with a moving window).

6. MATHEMATICAL MODELLING

A parallel theoretical computation using the finite-difference method was carried out for a 1-D model with the parameters from Table I. The whole problem was formulated with a view to maximum similarity with the experiment. The problem was solved using a program implemented on the basis of difference schemes of fourth-order accuracy in the space domain and second-order accuracy in the time domain. The computation simulated a free surface at the upper boundary of the region of computation, at the other boundaries Reynolds boundary conditions, simulating an absorbing boundary, were introduced. The source was modelled by a force acting vertically. The program did not account for the attentuation of the wave field. The size of the grid was taken to be $\Delta x = 0.25$ m. On the whole, a total of 6150 time levels had to be computed in a grid of 120×80 points. The computed seismograms are shown in Figs. 4 and 5. Once again it is the same record with normalized amplitudes (Fig. 4) and a representation in the AGC mode (Fig. 5).

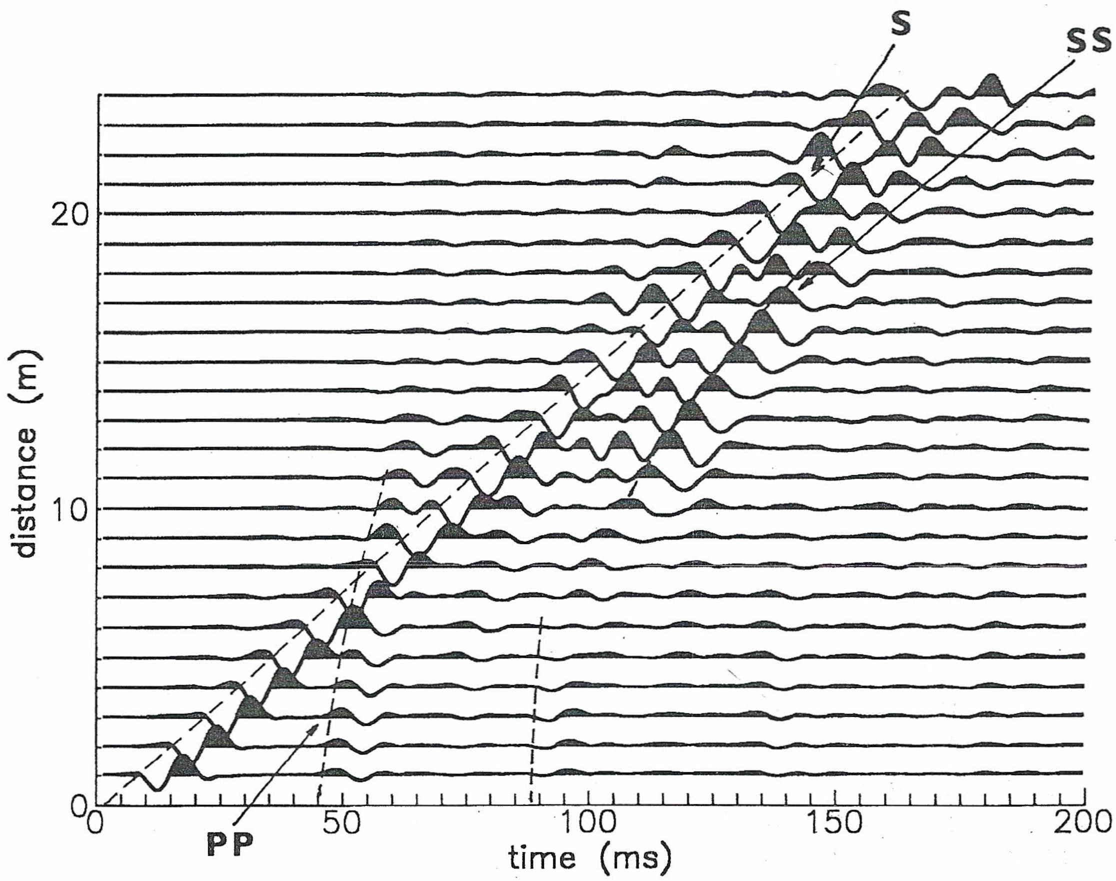

Fig. 4. Model seismogram computed using the finite difference method. The set-up corresponds to Fig. 2. Amplitude is normalized along traces.

7. DISCUSSION

Qualitatively speaking, the observed and computed wave fields have relatively complex seismograms in common: In both cases they contain a number of different waves with a wide interval of mutual amplitude relations. This can be seen by comparing the normalized-amplitude figures (Figs. 2 and 4) the AGC-mode figures (Figs. 3 and 5). In spite of expectations, the computed seismograms appear to be more complicated than the observed.

The observed and computed wave fields contain several mutually corresponding phases which were identified by the ray method. Correspondence between the observed and computed fields can be observed with the P- (direct wave), PPP- (head wave) and partly PP-phases (simple reflection from the first boundary).

Apart from travel-time curves, also the amplitude curves of the separate waves, computed by means of the ray method, proved to be useful in identifying the phases. The ray method, like the finite-difference method, took into account the free surface, i.e. the conversion effect at points where the transducers were located on the surface, was considered. The amplitude curves are depicted in Fig. 6 (PP, PPP), Fig. 7 (SS, SSS), and Fig. 8 (PS, SP). In spite of the restrictions discussed,the ray methods indicate that, in principle, the separate phase can be distinguished on the seismograms. According to Fig. 6, e.g., boundary because its amplitude is about one order of magnitude weaker than the PP-reflection from the first boundary. On the contrary, according to Fig. 7, one may expect a relatively strong SS-reflection from the first boundary at larger epicentral distances, and, according to Fig. 8, also relatively strong converted PS- and SP-types.

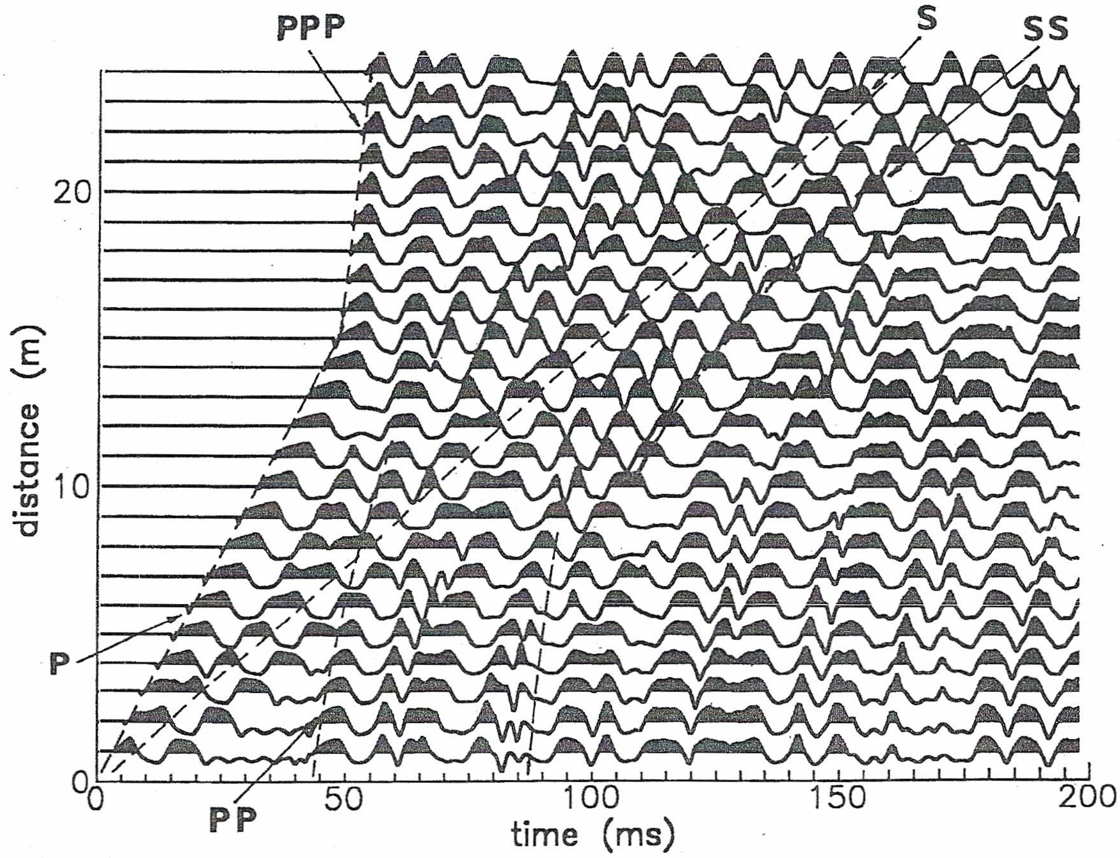

Fig. 5. Model from Fig. 4 made in the AGC mode.

In the computed wave field, several other phases which, however, have no counterpart in the observed data, could be identified in this manner. A simple SS-reflection and the direct S-wave are involved. The direct S-wave is the dominating phenomenon of the theoretical seismogram; however, it is lacking in the field data.

The nature of the agreement of the observed and theoretical seismograms (rel-

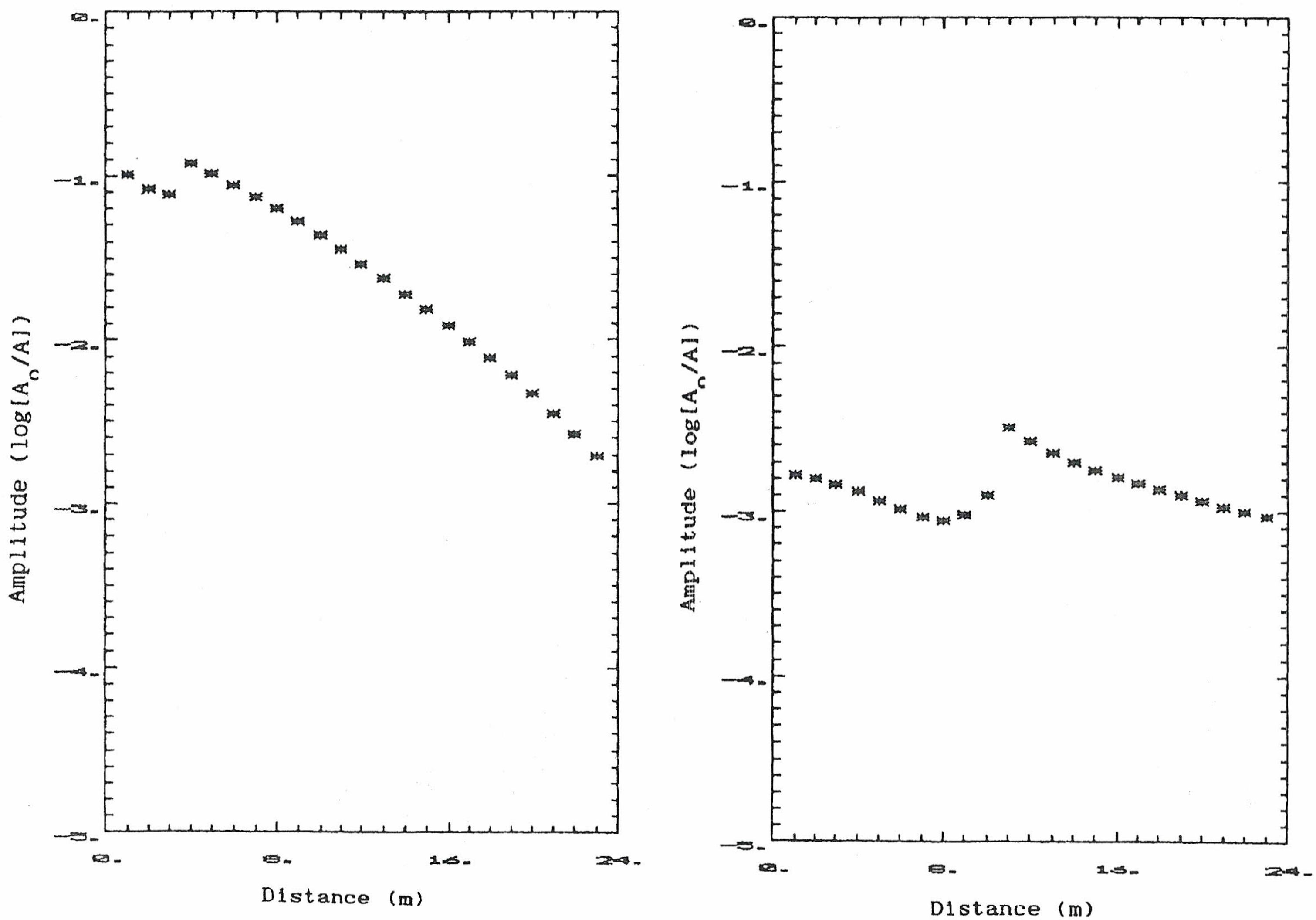

Fig. 6. Amplitude as function of distance for *PP*- (simple reflection from the first boundary) and *PPP*-waves (simple reflection from the second boundary).

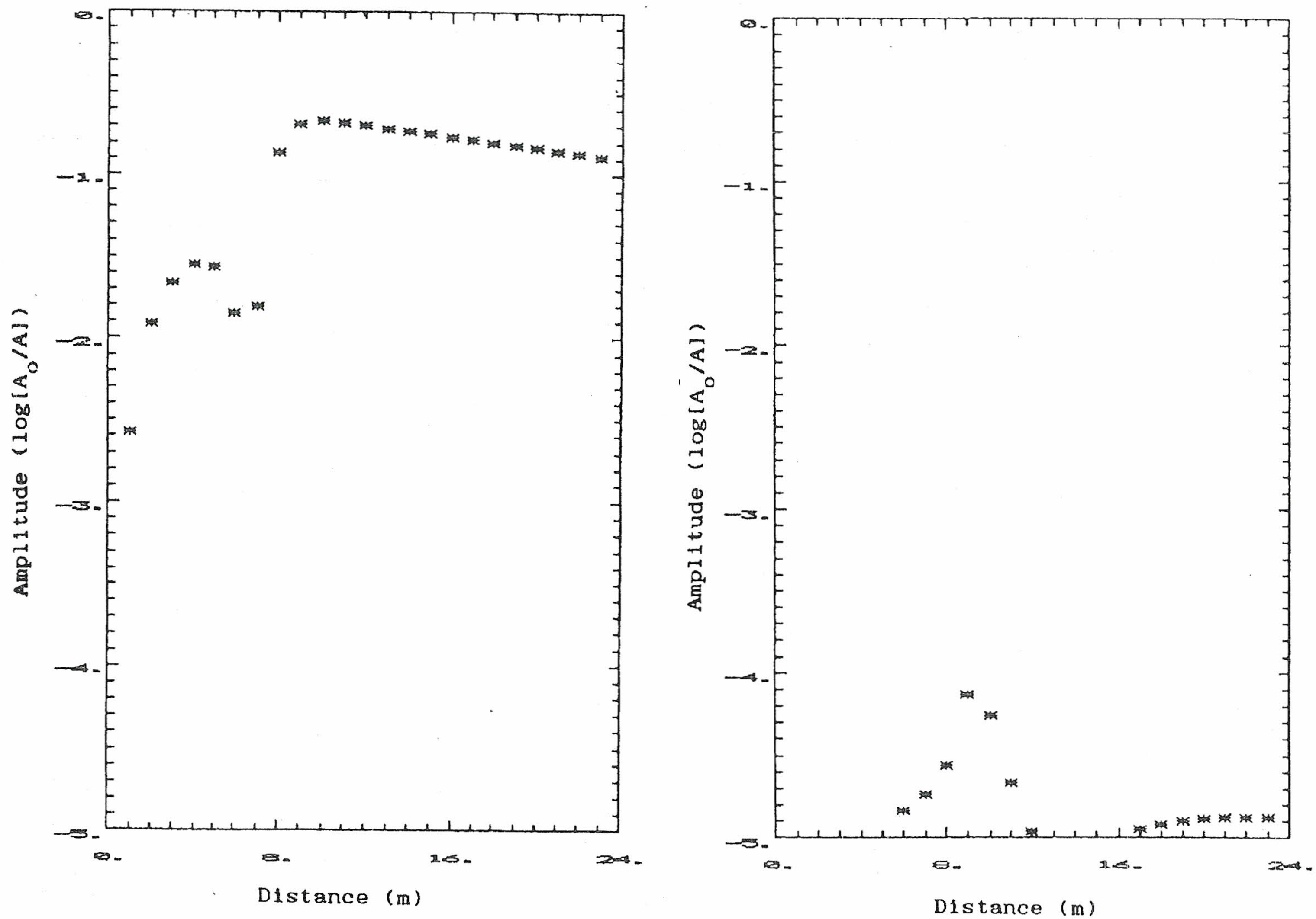

Fig. 7. Amplitude as function of distance for *SS*- (simple reflection at the first boundary) and *SSS*-waves (simple reflection at the second boundary).

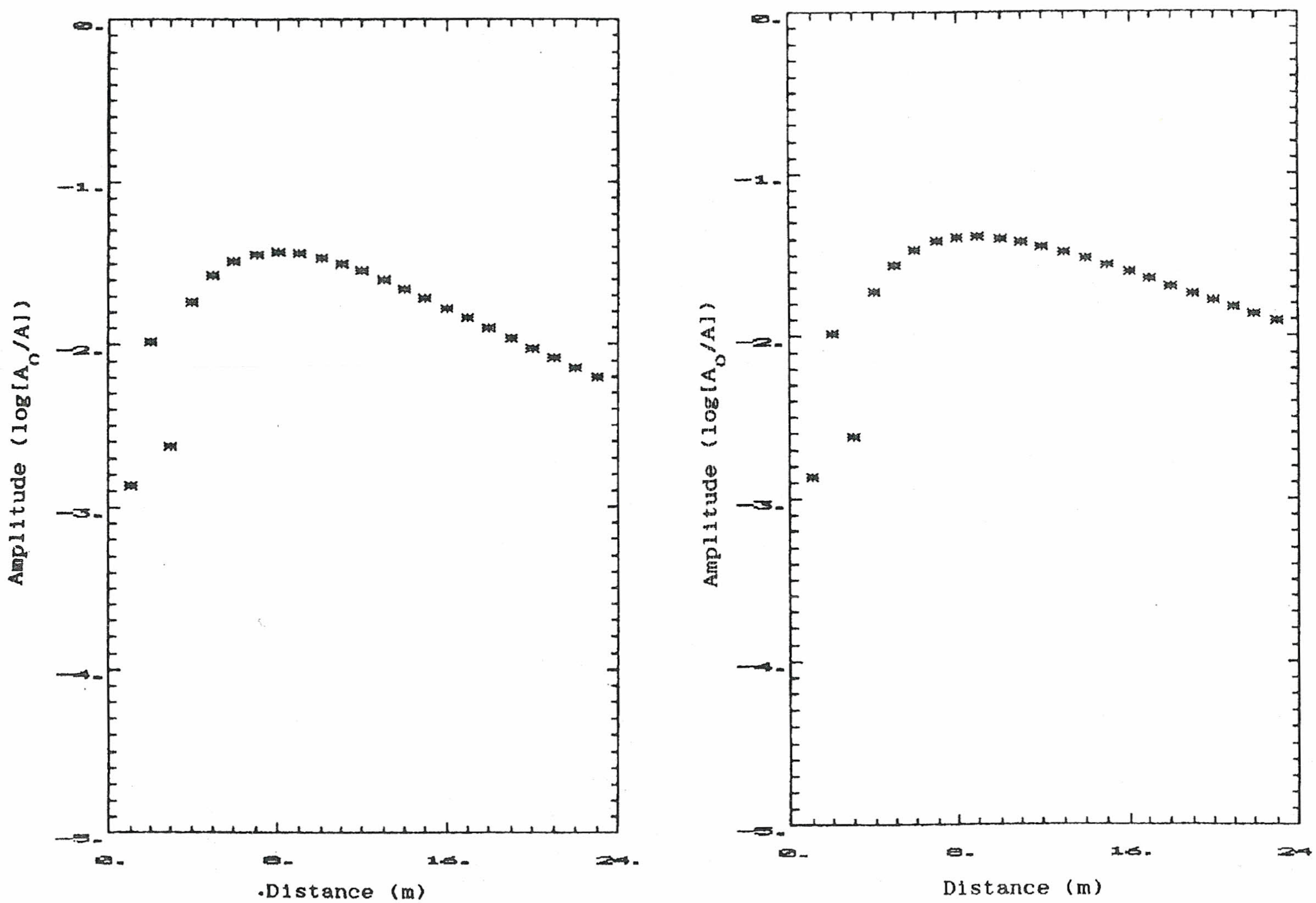

Fig. 8. Amplitude as function of distance for *PS*- and *SP*-waves (simple converted reflections from the first boundary).

ative agreement of the P-phases, relative disagreement of the S-phases) indicates one of the possible causes of this phenomenon: Whereas the propagation velocities of the P-waves were derived at least partly from the experiment, the propagation velocities of the S-waves were only estimated. Using ray methods, it is possible to prove that namely the amplitude relation are sensitive to the wave propagation velocities. To determine the cause of disagreement of the observed and computed seismograms it would be necessary, among other things, to determine the actual propagation velocities of the S-waves; under favourable circumstances, one may, in principle, consider reverse modelling to determine the velocity pattern.

Another possible cause of the disagreement may be neglecting attenuation in applying the finite-difference method. In the weathered subsurface parts of the geological profile one may admit to a substantial affect of attenuation on the wave field pattern. However, to be able estimate the actual magnitude of attenuation from field data, a more profound and independent analysis would be required.

The amplitude curves in Figs. 6–8 all display abrupt changes of values in dependence on the horizontal coordinate, sudden jumps are usually associated with phase changes. The form of the signal changes in intervals where the phase relation changes, and this can explain a loss of correlation between the traces.

8. CONCLUSION

The following conclusions can be briefly formulated based on the measured and theoretical computations we have carried out:

(1) Parallel computations of theoretical seismograms using the finite-difference method, as well as of the travel-time and amplitude curves using the ray method supplement one another conveniently.

(2) Partial agreement was achieved between the theoretical and observed seismograms as regards P-type phases, however, no agreement was achieved as regards the S-type phases.

(3) It appears that the disagreements are due to inadequate estimate of S-wave propagation velocities and general neglect of attenuation.

(4) Under conditions of shallow seismics (large velocity contrasts) it would be technically more suitable to use the finite-difference method with a variable step, or to adopt the method of finite elements altogether.

(5) Shallow contrasting boundaries do not create suitable conditions for monitoring reflections because of the complicated interferences, and abrupt amplitude and phase changes of reflected waves in combination with a relatively low signal frequency.

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REFERENCES

Aki, P. and P.G. Richards (1980). *Qualitative Seismology: Theory and Methods*, W.H. Freeman, San Francisco.

Cerjan, Ch., D. Kosloff, R. Kosloff, and M. Reshef (1985). A nonreflecting boundary condition for discrete acoustic and elastic wave equations, *Geophysics* **50**, 705–708.

Clayton, R. and B. Engquist (1977). Absorbing boundary conditions for the numerical simulation of wave equations, *Bull. Seism. Soc. Am.* **67**, 661–665.

Emerman, S.H. and R. Stephen (1983). Comment on "Absorbing boundary conditions for acoustic and elastic wave equations", by R. Clayton and B. Engquist, *Bull. Seism. Soc. Am.* **67**, 1529–1540.

Hill, I.A. and J.W. Ali (1988). Application of shallow seismic reflection methods to site investigation, *Trans. Inst. Min. Metall. (Sect. B: Appl. Earth Sci.)* **97**, 29–35.

Hunter, J.A., S.E. Pullan, R.A. Burns, R.M. Gagne, and R.L. Good (1984). Shallow seismic reflection mapping of the overburden-bedrock interface with the engineering seismograph. Some simple techniques, *Geophysics* **49**, 1381–1385.

Jongerius, P. and K. Helbig (1988). Onshore high-resolution seismic profiling applied to sedimentology, *Geophysics* **53**, 1276–1283.

Knapp, R.W. and D.W. Steeples (1986a). High-resolution seismic profiling applied to sedimentology, *Geophysics* **53**, 1276–1283.

Knapp, R.W. and D.W. Steeples (1986b). High-resolution common-depth-point reflection profiling: Field acquisition parameter design. *Geophysics* **51**, 283–294.

Reynolds, A. (1978). Boundary conditions for the numerical solution of wave propagation problems, *Geophysics* **43**, 1099–1110.

Smith, W. (1974). A nonreflecting plane boundary for wave propagation problems, *J. of Computational Physics* **15**, 492–503.

Sochacki, J., R. Kubichek, J. George, W.R. Fletcher, and S. Smithson (1987). Absorbing boundary conditions and surface waves, *Geophysics* **52**, 60–71.

Zahradník, J. and F. Hron (1991). Robust finite-difference scheme for elastic waves on coarse grids, *Studia Geodaetica et Geophysica* (in press).

SROVNÁNÍ METHOD MATEMATICKÉHO MODELOVÁNÍ VLNOVÉHO POLE V MĚLKÉ SEISMICE

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Článek se zabývá využitím matematického modelování v mělkém reflexněseismickém průzkumu. S ohledem na specifiku připovrchového prostředí je srovnáván model využívající metodu konečných diferencí s paprskovým řešením problému šíření seismických vln. Uveden je pokusný výpočet na lokalitě Brandýs.

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