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SEISMIC MOMENT TENSOR AND ITS APPLICATION IN MINING SEISMICITY STUDIES: A REVIEW

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Abstract

There is growing evidence that alternative earthquake mechanisms other than shear failure are possible. The most prominent cases of anomalous focal mechanisms are reported from mine seismicity studies. The moment tensor approach is in this respect the most general one, since moment tensors describe completely the equivalent forces of a seismic point source. A moment tensor can be decomposed into an isotropic part, a compensated linear vector dipole and a double couple. Such a decomposition seems to be the most interesting one for source studies of seismic events induced by mining. The isotropic component of the source mechanism corresponds to a volumetric change, the compensated linear vector dipole corresponds to a uniaxial compression, and the double couple, of course, corresponds to a shear failure.

There are various methods of inversion for moment tensor components. The inversion can be done in the time domain or in the frequency domain, and different data can be used. The main difficulty in the inversion is a proper calculation of Green's functions for geologically complex media. Moment tensor inversions on a global scale have been routinely performed for several years, but the application of this technique to local events is a relatively recent innovation. A few works only have been published that are related to the use of moment tensor inversion in studies of seismic events induced by mining. This approach is mostly used in South Africa and Poland, and to a limited extend in Japan and Canada.

1. Introduction

Recent results from earthquake focal mechanism studies indicate growing evidence that alternative mechanisms other than shear failure are possible. The most prominent cases of what appear to be anomalous focal mechanisms are reported from mine seismicity studies. The results from studies mostly based on first-motion polarity and radiation patterns were reviewed by Gibowicz (1990). Here the results from moment tensor inversion are considered in some detail.

Moment tensors describe completely, in a first order approximation, the equivalent forces of general seismic point sources, the double-couple source being just one of them. In many applications a point source approximation may be quite satisfactory, provided that the source dimensions are small in comparison to the observed wavelenghts of seismic waves. The concentrated force couples and the corresponding formal introduction of the moment tensor and its decomposition into various components are discussed at the beginning of this review. Then various methods of inversion for the moment tensor elements are briefly considered. Although moment tensor inversions on a teleseismic scale have been routinely performed for several years, the application of this technique to local seismic events is a relatively recent innovation. The results of the application of moment tensor inversion technique to study local seismicity in general and seismicity induced by mining in particular are described at the end of this presentation.

2. Concentrated Force Couples and Moment Tensor

It is well known that the displacement field $u_i(\mathbf{x}, t)$ generated by a single body force is equal to the convolution of this point force $F(t)$ by the Green's function G_{ij}

$$u_i\ (\mathbf{x}, t) = F(t) * G_{ij}(\mathbf{x}, t), \tag{1}$$

where the asterisk is a commonly used abbreviation for the convolution and the Green's function is simply the medium response to the delta function. If the body force is concentrated but its orientation is arbitrary $\mathbf{f}(\mathbf{x}, t)$ =

$\mathbf{F}(t)\delta(\mathbf{x}-\xi)$, where $\mathbf{F}(t)=(F_1,F_2,F_3)$, then the total displacement is equal to the sum of the displacements generated by the forces F_1, F_2, and F_3 directed along the x_1, x_2, and x_3 directions (e.g., Pujol and Herrmann, 1990)

$$u_i = F_1 * G_{i1} + F_2 * G_{i2} + F_3 * G_{i3} = F_j * G_{ij} \; . \tag{2}$$

Although the single force is one of the simplest models of a seismic source, the supposed external application of a force is unlikely to occur in natural earthquakes. It is more probable that the force action is of self-balancing type, such as a pair of opposite forces acting simultaneously on two adjacent parts of the medium with the resultant force equal to zero.

Let us consider a pair of forces of equal magnitude acting along the positive and negative x_3 directions at a small distance ϵ apart in the x_2 direction. The two forces are $(0,0,F_3)$ acting at point $(\xi+\epsilon e_2/2)$ and $(0,0,-F_3)$ acting at $(\xi-\epsilon e_2/2)$, where e_2 is a unit vector in the x_2 direction. The total displacement u_i caused by the two forces is the sum of the displacements caused by each force (e.g., Pujol and Herrmann, 1990)

$$u_i = \epsilon F_3 * [G_{i3}(\xi+\epsilon\, e_2/2) - G_{i3}(\xi-\epsilon\, e_2/2)]/\epsilon \; . \tag{3}$$

Taking the limit of u_i as F_3 tends to infinity and ϵ tends to zero, in such a way that the product ϵF_3 remains finite, the following relation is obtained

$$u_i = M_{32} * \frac{\partial G_{i3}}{\partial \xi_2} \; , \tag{4}$$

where $M_{32} = \epsilon F_3$. This pair of forces is known in classical mechanics as a couple and the quantity M_{32} as the moment of the couple, which has dimension of force by length and my be a function of time.

There are nine possible combinations of force and arm directions, shown in Fig. 1, represented by the moment M_{ij} of couple with forces in the x_i direction and arm in the x_j direction. When x_i and x_j are the same, the couple is known as a vector dipol or a couple without moment. All the other couples have nonzero moment, equivalent to torque. If a general body force representing a seismic source can be expressed as a linear combination of couples with moments M_{ij}, then the displacement caused by this force is the sum of the displacements caused by individual couples

$$u_k = M_{ij} * \frac{\partial G_{ki}}{\partial \xi_j} = M_{ij} * G_{ki,j} . \tag{5}$$

The set of nine terms M_{ij} is known as the moment tensor of the source, represented by a matrix **M** with elements M_{ij}. The full expression for u_k in (5) can be found, for example, in the book of Aki and Richards (1980).

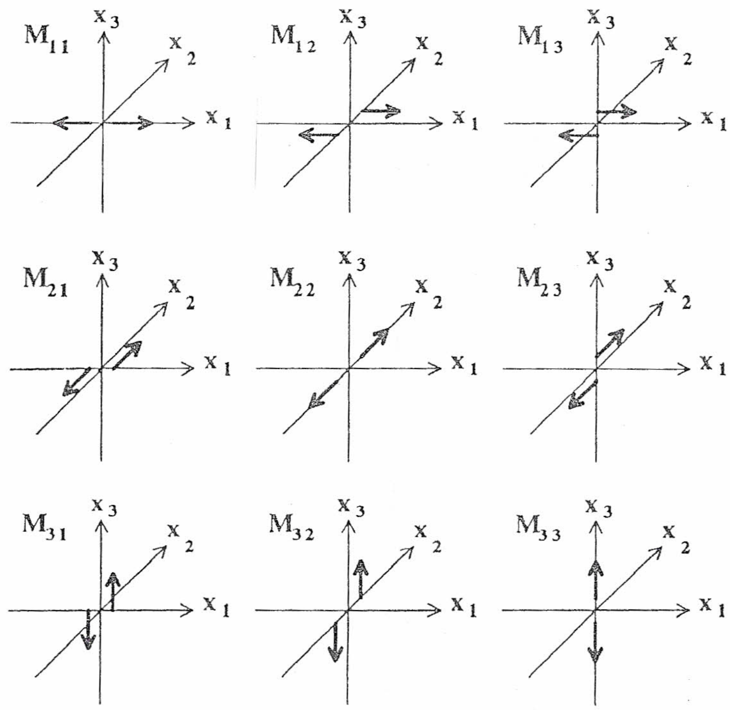


Fig. 1. Representation of the nine possible couples M_{ij}. The subindexes i and j denote the directions of the force and the arm of the couple, respectively. (From Aki and Richards, 1980.)

The seismic moment tensor was introduced by Gilbert (1970) to calculate the displacement which can be expressed as a sum of moment tensor elements times the corresponding Green's function. The linearity between the moment tensor and Green's function elements was first used by Gilbert (1973) to calculate moment tensor elements from observations, known as moment tensor inversion.

The concept of seismic moment tensors was further extended by Backus and Mulcahy (1976) and Backus (1977a, b).

A convenient description of seismic sources is provided by considering them as arising from deviations from the elastically predicted stress state. The stress tensor is divided into purely elastic part and inelastic part, which Backus and Mulcahy (1976) called the stress glut. The physical source region is characterized by the existence of the equivalent forces, arising as the result of differences between the model stress and the actual physical stress. Outside the source region, the stress glut and the equivalent forces vanish.

The displacement u_k generated at a point $\mathbf{x}$ at the time t by a distribution of equivalent body force densities f_i within a source volume V is given by (e.g., Aki and Richards, 1980; Kennett, 1988; Jost and Herrmann, 1989)

$$u_k(\mathbf{x}, t) = \int_{-\infty}^{\infty} \int_V G_{ki}(\mathbf{x}, t; \mathbf{r}, t') \, f_i(\mathbf{r}, t') \, dV \, dt', \tag{6}$$

where $G_{ki}(\mathbf{x}, t; \mathbf{r}, t')$ are the Green's functions containing the propagation effects between the source $(\mathbf{r}, t')$ and the receiver $(\mathbf{x}, t)$. The Green's functions may be expanded in a Taylor series (Stump and Johnson, 1977) about the point $\mathbf{r} = \xi$ as follows

$$G_{ki}(\mathbf{x}, t; \mathbf{r}, t') = \sum_{n=0}^{\infty} \frac{1}{n!} (r_{j_1} - \xi_{j_1}) \ldots (r_{j_n} - \xi_{j_n}) \, G_{ki, j_1 \ldots j_n}(\mathbf{x}, t; \xi, t'), \tag{7}$$

where the comma between indices describes partial derivatives with respect to the coordinates after the comma. The location of the reference point would normally be the hypocenter for small sources, whereas for extended faulting an improved representation is obtained by considering the centroid of the source. Defining the time dependent force moment tensor as

$$M_{ij_1 \ldots j_n}(\xi, t') = \int_V (r_{j_1} - \xi_{j_1}) \ldots (r_{j_n} - \xi_{j_n}) \, f_i(\mathbf{r}, t') \, dV, \tag{8}$$

the displacement can be written as a sum of terms which resolve additional details of the source, known as multiple expansion (e.g., Backus and Mulcahy, 1976; Stump and Johnson, 1977; Aki and Richards, 1980; Jost and Herrmann, 1989)

$$u_k(\mathbf{x}, t) = \sum_{n=1} \frac{1}{n!} G_{ki,j_1 \ldots j_n}(\mathbf{x}, t; \xi, 0) * M_{ij_1 \ldots j_n}(\xi, t), \tag{9}$$

where * denotes the temporal convolution.

In many applications a point source approximation may be quite satisfactory. Finite sources, on the other hand, may be generated by direct superposition of simple point sources. If the source dimensions are small in comparison to the observed wavelengths of seismic waves, only the first term in relation (9) needs to be considered, and then the displacement can be written as

$$u_k(\mathbf{x}, t) = G_{ki,j}(\mathbf{x}, t; 0, 0) * M_{ij}(0, t) \tag{10}$$

for $\xi = 0$. Assuming that all components of the time dependent seismic moment tensor have the same time dependence $s(t)$, the case known as synchronous source (Silver and Jordan, 1982), the displacement can be expressed as

$$u_k(\mathbf{x}, t) = M_{ij} \, [G_{ki,j} * s(t)], \tag{11}$$

where $s(t)$ is often called the source time function. Thus the displacement u_k is a linear function of the moment tensor elements and the terms in the square brackets. If the source time function is a delta function, the only term left in the square brackets is $G_{ki,j}$ describing nine generalized couples (e.g., Jost and Herrmann, 1989), shown in Fig. 1.

In general, the source moment tensor $\mathbf{M}$ of second order, describing a general dipolar source, has nine components M_{ij} and is represented as a 3x3 matrix in a given reference frame. It can be written as (Ben-Menahem and Singh, 1981)

$$\mathbf{M} = M_{ij}\mathbf{e}_i\mathbf{e}_j = \frac{1}{3}(M_{11} + M_{22} + M_{33})(\mathbf{e}_1\mathbf{e}_1 + \mathbf{e}_2\mathbf{e}_2 + \mathbf{e}_3\mathbf{e}_3)$$

$$+\frac{1}{3}(2M_{11}-M_{22}-M_{33})e_1e_1$$

$$+\frac{1}{3}(2M_{22}-M_{33}-M_{11})e_2e_2+\frac{1}{3}(2M_{33}-M_{11}-M_{22})e_3e_3 \tag{12}$$

$$+\frac{1}{2}(M_{32}+M_{23})(e_3e_2+e_2e_3)+\frac{1}{2}(M_{32}-M_{23})(e_3e_2-e_2e_3)$$

$$+\frac{1}{2}(M_{13}+M_{31})(e_1e_3+e_3e_1)+\frac{1}{2}(M_{13}-M_{31})(e_1e_3-e_3e_1)$$

$$+\frac{1}{2}(M_{21}+M_{12})(e_2e_1+e_1e_2)+\frac{1}{2}(M_{21}-M_{12})(e_2e_1-e_1e_2),$$

where e_i and e_j are the unit vectors along the x_i and x_j directions. The first term on the right hand side of this equation describes a center of compression and the successive terms describe three dipoles along the coordinate axes, three double couples and three torques about the coordinate axes, respectively. This is known as the decomposition theorem. The center of compression comes from the isotropic part of the moment tensor, corresponding to a volume change in the source. The remaining nine sources form the deviatoric part of the moment tensor. This deviatoric part can be further decomposed; a multitude of different decompositions are possible.

The conservation of angular momentum for the equivalent forces leads to the symmetry of the seismic moment tensor (Gilbert, 1970). If the moment tensor is symmetric, then $M_{ij} = M_{ji}$ and the torques in equation (12) vanish. The eigenvalues m_1, m_2, and m_3 of a symmetrical second-order tensor are all real and its eigenvectors a_1, a_2, and a_3 are mutually orthogonal. Then from equation (12) it follows that a moment tensor can be decomposed into an isotropic part and three vector dipoles (Ben-Menahem and Singh, 1981; Jost and Herrmann, 1989)

$$M=\frac{1}{3}(m_1+m_2+m_3)\,I+\frac{1}{3}(2m_1-m_2-m_3)a_1a_1$$

$$+\frac{1}{3}(2m_2-m_3-m_1)a_2a_2+\frac{1}{3}(2m_3-m_1-m_2)a_3a_3\,, \tag{13}$$

where $I = \delta_{ij}$ is the identity matrix. The isotropic component of the source mechanism corresponds to a volumetric change which is often thought to be associated with a phase transition expected to occur in the source of deep-focus earthquakes. Most recent results show, however, that a sudden implosive phase change can rather be ruled out as the primary physical mechanism for deep earthquakes (Kawakatsu, 1991). The source process of a shallow earthquake, on the other hand, which occurred on May 14, 1985 off the northern Mozambique, has been considered as a combination of a normal fault and a subsequent isotropic source (Honda and Yomogida, 1991).

Equation (13) may also be written in the form

$$M = \frac{1}{3}(m_1 + m_2 + m_3)\, I + \frac{1}{3} m_1 (2a_1a_1 - a_2a_2 - a_3a_3)$$

$$+ \frac{1}{3} m_2 (2a_2a_2 - a_3a_3 - a_1a_1) + \frac{1}{3} m_3 (2a_3a_3 - a_1a_1 - a_2a_2), \qquad (14)$$

where $2a_1a_1 - a_2a_2 - a_3a_3$ represents a compressional dipole of strength 2 in the direction of the eigenvector a_1 and two dilatational dipoles each of unit strength along the a_2 and a_3 axes. This type of source is known as a compensated linear vector dipole (CLVD). A general dipolar source with a symmetric moment tensor, therefore, is equivalent to a center of compression and three mutually orthogonal compensated linear vector dipoles. A compensated linear vector dipole is equivalent to two double couples since

$$2a_1a_1 - a_2a_2 - a_3a_3 = (a_1a_1 - a_2a_2) + (a_1a_1 - a_3a_3); \qquad (15)$$

and a double couple is given, for example, by $a_1a_1 - a_2a_2$.

Alternatively, a symmetric moment tensor can be decomposed into an isotropic part and three double couples. Equation (13) can be written as

$$M = \frac{1}{3}(m_1 + m_2 + m_3)\, I + \frac{1}{3}(m_1 - m_2)(a_1a_1 - a_2a_2)$$

$$+\frac{1}{3}(m_2 - m_3)(a_2a_2 - a_3a_3) + \frac{1}{3}(m_3 - m_1)(a_3a_3 - a_1a_1), \tag{16}$$

which represents a center of compression and three double couples. Another decomposition of a moment tensor is into an isotropic component and a major and minor double couple introduced by Kanamori and Given (1981). The major couple seems to be the best approximation of a general seismic source by a double couple, since the direction of the principal axes of the moment tensor remain unchanged (Jost and Herrmann, 1989). To construct the major double couple, the eigenvector of the smallest eigenvalue (in the absolute sense) is taken as the null axis, and it is assumed that the purely deviatoric eigenvalues m_i^d of the moment tensor

$$m_i^d = m_i - \frac{m_1 + m_2 + m_3}{3} \tag{17}$$

are such that $|m_3^d| \geq |m_2^d| \geq |m_1^d|$. Then the complete decomposition can be written as (Jost and Herrmann, 1989)

$$M = \frac{1}{3}(m_1 + m_2 + m_3)\ I$$

$$+ m_3^d\ (a_3a_3 - a_2a_2) + m_1^d\ (a_1a_1 - a_2a_2), \tag{18}$$

in which the second term represents the major double couple and the third term represents the minor couple. A best double couple can be constructed in a similar way by replacing m_3^d by the average of the largest (in the absolute sense) two eigenvalues (Giardini, 1984).

Following Knopoff and Randall (1970) and Fitch *et al.* (1980), a moment tensor can be decomposed into an isotropic part, a compensated linear vector dipole and a double couple. Assuming again that $|m_3^d| \geq |m_2^d| \geq |m_1^d|$ in relation (17) and that the same principal stresses produce the CLVD and the double couple radiation, the following decomposition is obtained (Jost and Herrmann, 1989)

$$M = \frac{1}{3}(m_1 + m_2 + m_3)\,I + m_3^d\,F\,(2a_3a_3 - a_2a_2 - a_1a_1)$$

$$+ m_3^d\,(1 - 2F)(a_3a_3 - a_2a_2), \qquad (19)$$

where $F = -\,m_1^d\,/\,m_3^d$.

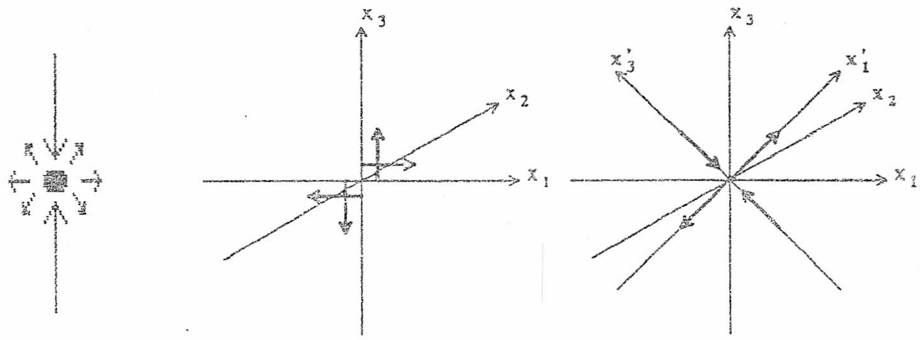


Fig. 2. The compensated linear vector dipole (CLVD), corresponding to a uniaxial compression, and the double couple and the corresponding double dipole system that gives the same radiation pattern as the double couple.

Such a decomposition seems to be the most interesting one for source studies of seismic events induced by mining. The compensated linear vector dipole, corresponding to a uniaxial compression, and the double couple and the corresponding double dipole system that gives the same radiation pattern as the double couple are shown in Fig. 2. The CLVD source was considered as a model for sudden phase transitions in deep earthquakes (Knopoff and Randall, 1970), tensile failure of rock in the presence of high-pressure fluids (Julian and Sipkin, 1985; Foulger, 1988), and source complexity (e.g., Frohlich *et al.*, 1989). The CLVD source corresponding to a uniaxial compression could possibly explain one of the mechanisms of pillar-associated seismic events, observed in situ in deep hard rock mines in South Africa and reported by Lenhardt and Hagan (1990). The other reported mechanism could be explained by a source composed of the CLVD and double couple components. The four possible mechanisms of pillar-associated seismic events at the Western Deep Levels gold mine in South Africa are shown in Fig. 3, reproduced from Lenhardt and Hagan (1990).

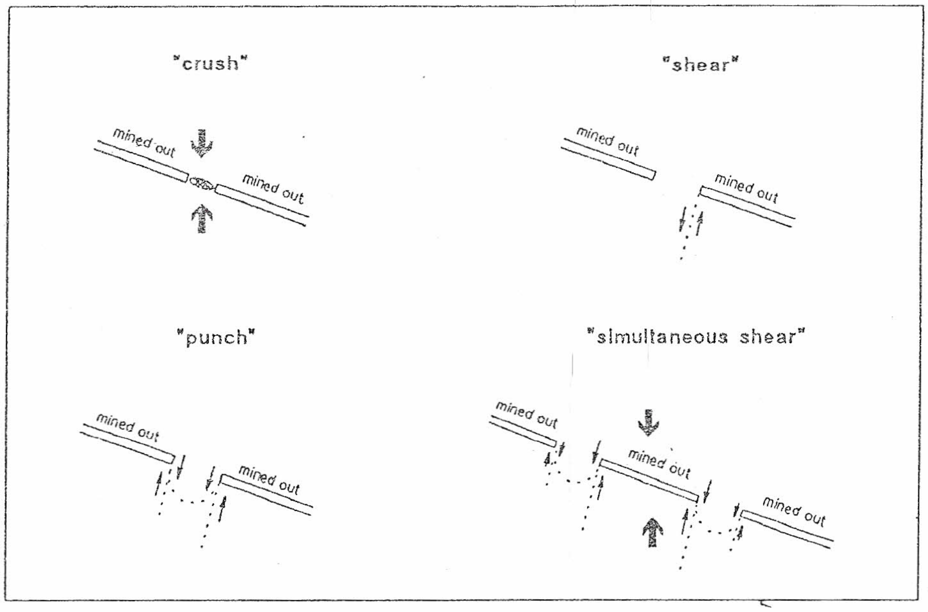


Fig. 3. Four possible mechanisms of pillar-associated seismic events at the Western Deep Levels gold mine, South Africa. (From Lenhardt and Hagan, 1990.)

3. Moment Tensor Inversion

There are various methods of inversion for moment tensor elements. The inversion can be done in the time or frequency domain, and different data can be used separately or in combination. The moment tensor inversion in the time domain can be based on the formulation described by equation (11) (e.g., Gilbert, 1970; Stump and Johnson, 1977; Strelitz, 1978; Fitch *et al.*, 1980). If the source time function is not known or cannot be assessed or the assumption of a synchronous source is not upheld, the frequency domain approach is chosen (e.g., Gilbert, 1973; Stump and Johnson, 1977; Kanamori and Given, 1981). The displacement in the frequency domain, corresponding to the formulation in (11), can be written as

$$u_k(\mathbf{x}, t) = M_{ij}(f)\ G_{ki,j}(f) \tag{20}$$

for each frequency f. Both approaches (11) and (20) lead to linear inversions in the time or frequency domain, respectively, for which a number of fast computational algorithms are available (e.g., Lawson and Hanson, 1974; Press *et al.*, 1990).

Both equations either (11) or (20) can be written in a matrix form

$$\mathbf{u} = \mathbf{G}\ \mathbf{m}. \tag{21}$$

In the time domain, the vector **u** consists of n sampled values of the observed ground displacement at various stations, G is a $n \times 6$ matrix containing the Green's functions calculated using an appropriate algorithm and earth model, and $\mathbf{m} = (M_{11}, M_{12}, M_{22}, M_{13}, M_{23}, M_{33})$ is a vector containing the six moment tensor elements to be determined. In the frequency domain, equations (21) are written separately for each frequency. The vector **u** consists of real and imaginary parts of the displacement spectra, the matrix G and the vector m contain real and imaginary parts as well, and **m** contains also the transform of the source time function of each moment tensor element. The details of solving equations (21) for **m** are given by Aki and Richards (1980). A detailed description of the procedure for regional and local seismograms is given by Oncescu (1986). The application of moment-tensor inversion to microseismic events is described by O'Connell and Johnson (1988).

The first step in a moment tensor inversion is the data acquisition and preprocessing. Data with good signal to noise ratio and that have a good coverage of the focal sphere (see Satake, 1985) are essential. High frequency noise in the data is removed by low-pass filtering.

In the second step synthetic Green's functions are calculated, taking into account geological structure between the source and receiver, the location of the point source, and the position of the receiver. The source time function in equations (11) is often assumed to be a step function (Knopoff and Gilbert, 1959) or a ramp function (Haskell, 1964) and used in the inversion (e.g., Gilbert, 1970; Stump and Johnson, 1977; Dziewonski *et al.*, 1981; Kanamori and Given, 1981). Powerful waveform inversion methods are available when source time functions need to be recovered (e.g., Burdick and Mellman, 1976; Wallace *et al.*, 1981).

The third step is the proper inversion - the solution of equations (21). The inversion is usually formulated in terms of least squares problems (norm L_2; Gilbert and Buland, 1976; Mendiguren, 1977; Stump and Johnson, 1977), though other norms (e.g., norm L_1) can have advantages when less sensitivity to gross errors is required (e.g., Claerbout and Muir, 1973).

The main difficulty in the moment tensor inversion is a proper calculation of Green's functions for geologically complex media. The Green's function is in general different for different displacement components and takes different values for particular stations. The simplest approach in the time domain is to use directly the source radiation formulation for *P*, *SV* or *SH* waves. This approach was used by Fitch *et al.* (1980) and De Natale *et al.* (1987) and others. A more rigorous approach is based on the method of matrix propagators of Haskell-Thomson (Haskell, 1953), modified by Knopoff (1964) and Dunkin (1965), and used in the frequency domain. Another method of evaluation of Green's functions, especially valuable for highly complex structures including possible lateral inhomogeneities, is empirical one using Green's functions determined from observations and a known source. This relative moment tensor determination was first proposed by Strelitz (1980) in a study of subevents of complex deep-focus earthquakes. The method was extended by Oncescu (1986) to individual small events recorded at a few stations.

If the focal depth is not known with sufficient precision, then a linear inversion can be done for a number of trial depths. The most probable depth will minimize the corresponding error between observed and theoretical waveforms (e.g., Mendiguren, 1977; Patton and Aki, 1979; Sipkin, 1982).

It is convenient to characterize the moment tensor by its eigenvalues. This can be done by a rotation of the moment tensor from geographical coordinates into its principal axes. Then the moment tensor can be written in the diagonal form

$$M_{ij} = m_i \delta_{ij} , \tag{22}$$

where m_i are the eigenvalues of **M** and the Kronecker delta $\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ for $i \neq j$. For a general moment tensor all eigenvalues m_i are different. Seismic sources with no volume change can be obtained by constraining the moment tensor to have zero trace

$$\mathrm{tr}\ \mathbf{M} = m_1 + m_2 + m_3 = 0, \tag{23}$$

or in a more general form

$$\mathrm{tr}\ \mathbf{M} = M_{11} + M_{22} + M_{33} = 0. \tag{24}$$

The sum of the diagonal elements of the moment tensor divided by 3 is a measure of the volume change associated with the source.

It can be readily shown that for the moment tensor of the double-couple source, one principal value of $\mathbf{M}$ must vanish which means that the determinant det $\mathbf{M}$ must also vanish

$$\det \mathbf{M} = m_1 m_2 m_3 = 0, \tag{25}$$

or in a general form

$$\det \mathbf{M} = M_{11}M_{22}M_{33} + 2M_{12}M_{23}M_{13} - M_{11}M_{23}^2 - M_{22}M_{13}^2 - M_{33}M_{12}^2 = 0. \tag{26}$$

The vanishing of det $\mathbf{M}$ and tr $\mathbf{M}$ are therefore necessary and sufficient conditions for a double-couple source.

In the more general case, the eigenvalues of the moment tensor $\mathbf{M}$ can be readily found, following for example Kennett (1988). Any isotropic component is removed by bringing the trace to zero, simply by modifying the diagonal elements of the original moment tensor

$$M'_{ij} = M_{ij} - \frac{1}{3} \operatorname{tr} \mathbf{M}\, \delta_{ij} . \tag{27}$$

The diagonalization of this new tensor $\mathbf{M}'$ results in a cubic equation for its eigenvalues m

$$m^3 - Km - \det \mathbf{M}' = 0, \tag{28}$$

where

$$K = M_{12}^2 + M_{13}^2 + M_{23}^2 - M_{11}M_{22} - M_{22}M_{33} - M_{11}M_{33}.$$

If det $\mathbf{M}' = 0$, the solutions of equation (28) are

$$m = 0, \quad K^{1/2}, \quad -K^{-1/2}; \tag{29}$$

if det $\mathbf{M}' \neq 0$, the solutions are

$$m = q\cos\theta, \qquad q\cos(\theta + 2\pi/3), \qquad q\cos(\theta + 4\pi/3), \tag{30}$$

where

$$q = 2(K/3)^{1/2}, \qquad \theta = \frac{1}{3}\cos^{-1}\left(\frac{3\det \mathbf{M}'}{Kq}\right).$$

The principal directions may be found by solving the following equations for the components of the eigenvector a

$$(M'_{ij} - m\,\delta_{ij})a_j = 0. \tag{31}$$

If the trace of the original tensor $\mathbf{M}$ is non-zero, then (1/3) tr $\mathbf{M}$ has to be added to each eigenvalue.

The non-isotropic constraint of zero trace on the moment tensor is linear, whereas for double-couple sources the constraint of zero determinant on the moment tensor is nonlinear. To solve the linear system of equations (21) under these constraints, the method of Lagrange multipliers is used (Strelitz, 1980; Oncescu, 1986). The system must be solved iteratively until the determinant det $\mathbf{M}$ and the trace tr $\mathbf{M}$ converge to zero. The scalar seismic moment M_o can be determined from a given moment tensor, corresponding to a double-couple source, by

$$M_o = \frac{1}{2}\left(|m_1| + |m_2| \right), \tag{32}$$

where m_1 and m_2 are the largest eigenvalues in the absolute sense. The seismic moment can equivalently be estimated by the following relations (Silver and Jordan, 1982)

$$M_o = \left[\frac{\sum M_{ij}^2}{2}\right]^{1/2} = \left[\frac{\sum m_i^2}{2}\right]^{1/2}. \tag{33}$$

After the recovery of moment tensor, the deviation of the solution from the pure double-couple model can be evaluated from the ratio (Dziewonski *et al.*, 1981)

$$\epsilon = \frac{|m^d|_{min}}{|m^d|_{max}} , \tag{34}$$

where $|m^d|_{min}$ is the smallest and $|m^d|_{max}$ is the largest deviatoric eigenvalue in the absolute sense. The values of this ratio can range from 0 for a pure double-couple source to 0.5 for a pure compensated linear vector dipole. Alternatively, the ratio ϵ can be expressed in percentages of CLVD by multiplying ϵ by 200. The percentage of a pure double couple is $100(1 - 2\epsilon)$. The variation of ϵ against seismic moment and earthquake space distribution was studied by Dziewonski and Woodhouse (1983) and Giardini (1984).

Silver and Jordan (1982) have developed a method for the estimation of the isotropic and deviatoric components of the moment tensor, introducing the isotropic, deviatoric and total scalar seismic moments. Graphical methods have been recently suggested for identifying non-double-couple moment-tensor components (Pearce *et al.*, 1988; Hudson *et al.*, 1989; Riedesal and Jordan, 1989). A method for the exact mapping of error bounds on seismic waveforms into bounds on certain moment-tensor properties was presented by Vasco (1990). The properties are the three invariants of the moment tensor: the trace, the determinant, and the sum of the determinants of the diagonal minors. Finding upper and lower bounds on these unique coordinate-free invariants allows to determine if significant volume change is associated with the source or if a non-double-couple mechanism is needed to satisfy the data. Furthermore, the range of models that fit the data is an indication of how well constrained the source properties are.

In general, moment tensor inversions involve two major assumptions. First, it is assumed that the earthquake may be treated as a point source for a given frequency of seismic waves; second, that the effect of the earth structure on the seismic waves is properly modeled. If the earthquake cannot be represented as a point source or the assumed model of structure is incorrect, the apparent moment tensor may contain a large non-double-couple component, even if the source mechanism is a double couple (Strelitz, 1978; Barker and Langston, 1982). Increasing the complexity of the source structure model, improving the azimuth coverage, and leaving the time function free to compensate for the deficiences of the Greeen's functions decrease the size of the non-double-couple component (Johnston and Langston, 1984).

4. Investigations of Local Seismic Events

Moment tensor inversions have been routinely performed for several years by the U. S. Geological Survey. Centroid-moment tensor solutions (simultaneous inversion of the waveform data for the hypocentral parameters of the best point source and for the six independent elements of the moment tensor) are regularly published by the Harvard University group for all larger earthquakes recorded at teleseismic distances.

The application of a moment tensor inversion technique, however, to local events is a relatively recent innovation. Saikia and Herrmann (1985, 1986) used this technique for the interpretation of the observed body wave amplitudes at local distances for two aftershocks of the 1982 Miramichi, Canada, earthquake and three 1982 Arkansas, USA, swarm earthquakes. Oncescu (1986) used a simple and efficient method for relative moment tensor determination of 95 intermediate depth small earthquakes from the Vrancea region, Romania, recorded by a local seismic network. A moment tensor inversion was performed by De Natale *et al.* (1987) for ten small volcanic events from the Campi Flegrei in Italy. O'Connell and Johnson (1988) made moment tensor inversions in the frequency domain for three microearthquakes from The Geysers geothermal field in California.

Most recently, Ebel and Bonjer (1990) performed moment tensor inversion of small earthquakes, with magnitudes from 0.5 to 2.2, in southwestern Germany and confirmed that direct *P*- and *S*-wave amplitudes can be inverted for the source focal mechanism. Koch (1991a) examined two methods for moment tensor inversion of waveform data for applicability to high-frequency near-source data. An algorithm was developed for near-source data, in which a stabilization procedure was introduced. Both methods, one in the time domain and one in the frequency domain, allow the retrieval of the complete time-dependent moment tensor. The technique was applied to eight aftershocks of the May 1980 Mammoth Lakes earthquakes (Koch, 1991b). Ohtsu (1991) applied moment tensor analysis to acoustic emission recorded during an in-situ hydrofracturing test. He used moment tensor components to classify crack types and to determine crack orientations.

5. Investigations of Seismicity Induced by Mining

A few works only have been published that are related to the use of moment tensor inversion in studies of the source mechanism of seismic events induced by mining. Spottiswoode (1984) has studied the focal mechanism of 11 mine tremors at Blyvooruitzicht gold mine, South Africa, in the frequency domain, and he found that the data were consistent with zero volume change in the seismic source area and were then interpreted as shear failures on plane striking parallel to the advancing face or to either of two dikes cutting across the face.

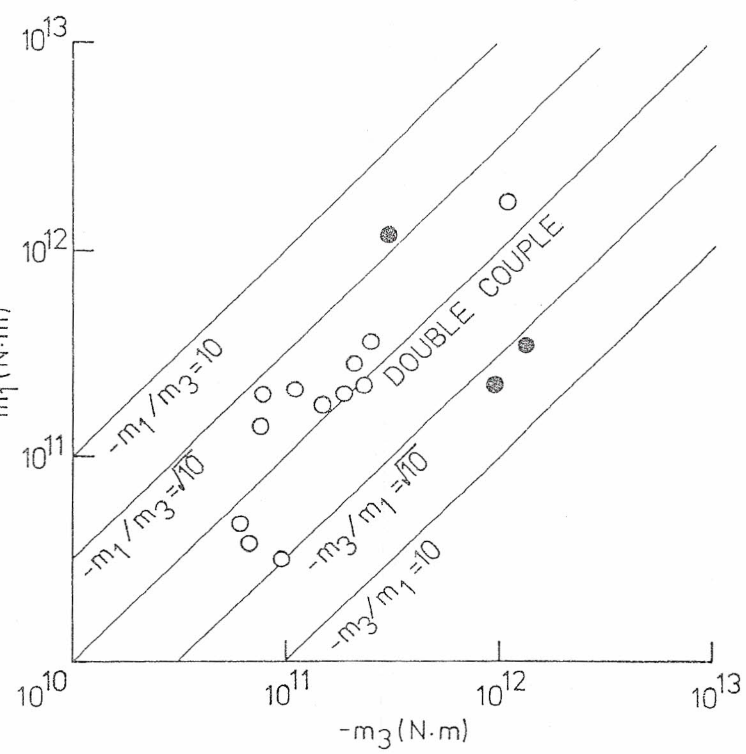


Fig. 4. Relation between the major and minor principal values of the moment tensor for 15 seismic events that occurred on January 29, 1986 during a large outburst at the Sunagawa coal mine, Japan. Solid circles indicate non-double-couple events. (From Sato and Fujii, 1989.)

Sato and Fujii (1989) have studied the source mechanism of a large-scale gas outburst at Sunagawa coal mine in Japan, which occurred in January 1986. They used a new method to evaluate the moment tensor in the frequency domain and applied it to 15 seismic events recorded by the mine underground seismic network. The procedure consists of two steps. In the first step an iterative least squares method was used to determine the quality factor Q (representing attenuation and scattering effects) and the apparent seismic moment for each record from the P-wave displacement spectrum in the frequency range from 4.9 to 48.8 Hz. In the second step, once the apparent seismic moment had been calculated for each station, the moment tensor was determined from simple relations between the apparent seismic moment and the moment tensor, taking into account the geometrical spreading, free-surface effect, and direction cosines. Out of 15 studied tremors associated with the outburst, 12 seismic events could be interpreted in terms of a double-couple focal mechanism (see Fig. 4). In contrast to these results, the moment tensor inversion performed on the observations from two small seismic events at Horonai coal mine in Japan has shown that they are non-double-couple events (Fujii and Sato, 1990). The tremors were associated with longwall mining and were located in the vicinity of the longwall face.

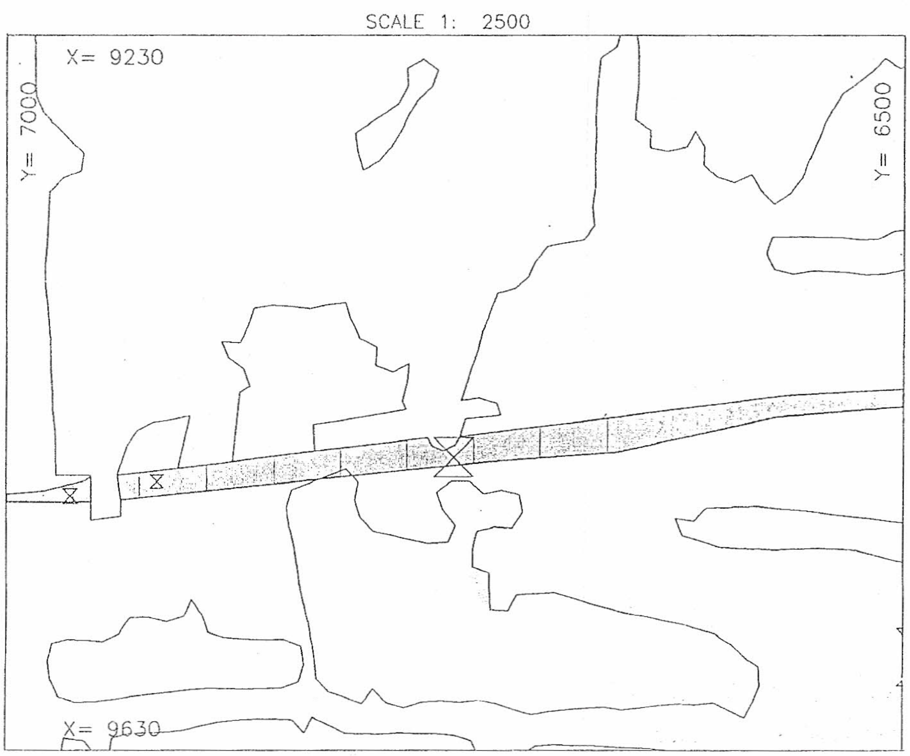


Fig. 5. Location of the main seismic event (marked by a large double triangle) and two aftershocks (small double triangles) in a vertical dyke (shaded area), which occurred on April 9, 1991 at the Western Deep Levels gold mine, South Africa. Their source is characterized by large isotropic components.

The Integrated Seismic System (ISS) recently introduced in the Welkom gold mining district in South Africa (Mendecki, 1990; Mendecki *et al.*, 1990) includes a software package which calculates all components of the higher order moment tensor. Calculations are done in the frequency domain using maximum entropy method for the inversion (Brawn, 1989). The use of higher than second order moment tensors permits to assess a number of source properties, besides those provided by the inversion of a standard second order moment tensor, such as the direction of the rupture propagation, the rupture velocity, duration and size, and the source geometry, in terms of the orientation of the plane of rupture and overall shape of the source. Although a few examples of the higher moment tensor inversions for mine tremors were given by Mendecki (1990), no systematic studies of this complex problem have been published so far.

Inversions of a standard second order moment tensor are routinely performed within the ISS Welkom system at Western Deep Levels gold mine in South Africa, though the results are not readily available in the professional literature. Moment tensor inversions for three seismic events with volume sources, which occurred within a dyke on April 9,1991, provide good examples to illustrate the kind of information becoming available from moment tensor analyses. The main event with moment magnitude M = 2.1 occurred at 07^h09^m and two aftershocks with the same moment magnitude M = 1.8 occurred at 09^h24^m and 12^h27^m, respectively. Their location in a dyke is shown in Fig. 5, elaborated by H. Aswegen from Western Deep Levels mine. The dyke is vertical and strikes EW. The reef being mined is planar and dips 16 degrees easterly. The main event created intense but highly localized damage, and the dyke "exploded". The routine moment tensor inversion provided the following results (in a diagonalized form in coordinate system defined by eigenvectors):

MOMENT TENSOR ISOTROPIC CLVD DC

$$\begin{bmatrix} 0.14 & 0.00 & 0.00 \\ 0.00 & -0.06 & 0.00 \\ 0.00 & 0.00 & 1.41 \end{bmatrix} = 0.494 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 0.355 \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} + 0.202 \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

with the largest isotropic component corresponding to extension and the smallest double-couple (DC) component.

Two aftershocks occurred also in the dyke. Damage during the first aftershock was more wide-spread than that during the main event, but it was less intense. The moment tensor inversion shows more shear than volume change corresponding this time to contraction:

$$\begin{array}{cccc} \text{MOMENT TENSOR} & \text{ISOTROPIC} & \text{CLVD} & \text{DC} \end{array}$$

$$\begin{bmatrix} -0.44 & 0.00 & 0.00 \\ 0.00 & -1.13 & 0.00 \\ 0.00 & 0.00 & 0.73 \end{bmatrix} = -0.281 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 0.162 \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} + 0.685 \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The third event caused no apparent damage and is characterized by even larger double-couple component and smaller volume change corresponding to extension:

$$\begin{array}{cccc} \text{MOMENT TENSOR} & \text{ISOTROPIC} & \text{CLVD} & \text{DC} \end{array}$$

$$\begin{bmatrix} 0.15 & 0.00 & 0.00 \\ 0.00 & -0.66 & 0.00 \\ 0.00 & 0.00 & 1.21 \end{bmatrix} = 0.241 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 0.095 \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} + 0.809 \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Wiejacz (1991) has studied the source mechanism of 60 small seismic events (in the seismic moment range 10^{11} - 10^{12} N·m) at Rudna copper mine in the Lubin mining district in Poland, which occurred in 1990 and 1991. He performed the moment tensor inversion in the time domain using the first motion amplitudes and signs of *P* and the amplitudes of *SV* waves recorded by the mine underground network composed of over 20 vertical seismometers.

Wiejacz (1991), before performing the inversion on real observations, carried out a number of numerical tests to check his algorithm. These tests are highly informative and I have selected a few examples to illustrate the problems involved in focal mechanism studies. Three solutions – general six-free-component moment tensor, constrained moment tensor with no volume changes, and constrained moment tensor corresponding to double couple – were sought for a fixed network composed of 24 stations with exact synthetic observations, for the same network with observations disturbed by 10% noise generated by random numbers, and for the network with randomly distributed stations and 10% random noise in the observations. The results of a numerical test for a purely explosional source are shown in Fig. 6. They are rather obvious and only last two solutions are of some interest, as they show that a poor focal sphere coverage may lead even to a not-too-bad-looking double-couple solution of the explosional source.

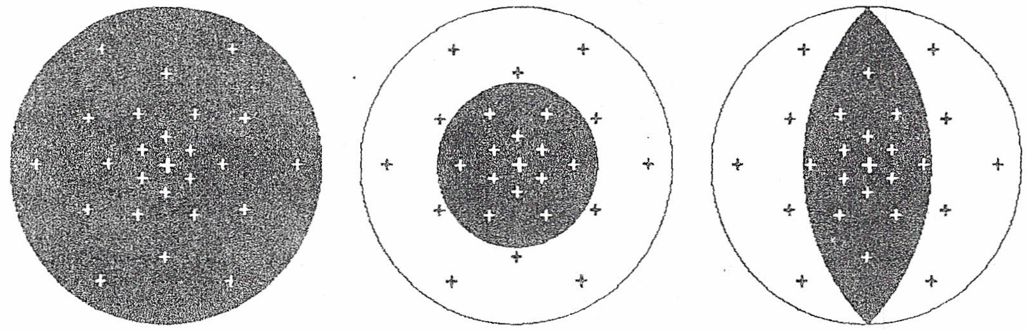
The nine solutions from the numerical test for a CLVD source, corresponding to vertical compression, are shown in Fig. 7. Once again, the double-couple solutions corresponding to normal faulting look rather good, with a few inconsistent observations only, but they are entirely unstable showing a wide range of possible distributions of nodal planes for the three considered cases. All the nine solutions for a shear source corresponding to dip-slip reverse faulting, shown in Fig. 8, are practically identical. The isotropic and CLVD components contribute no more than about 1% to the total solution. The nine solutions from the moment tensor inversion for a seismic source composed of the double couple (dip-slip reverse fault), 10% CLVD and 10% explosional components are shown in Fig. 9. The unconstrained solutions show 8% of explosional components. The solutions for a general deviatoric source have from 5 to 9% of CLVD components, whereas the double-couple solutions are highly stable and well constrained. This test shows how difficult it is to detect the non-double-couple components when they are relatively small and the double-couple sources are dominant.

The horizontal distribution of selected seismic events and seismic stations at Rudna copper mine, selected by Wiejacz (1991) for the moment tensor inversion, is shown in Fig. 10. The time-independent solutions were obtained for a general six-free-component moment tensor, constrained solutions corresponding to sources without volume changes, and constrained solutions corresponding to double-couple sources. Examples of such solutions for 10 selected events are shown in Fig. 11. In general, the solutions that are well constrained by observations (good coverage of the focal sphere) have dominant shear components, though occasionally the isotropic component (showing both extension and contraction) could be as large as 25 percent of the mechanism. The CLVD component corresponds to uniaxial compression in all cases and is usually larger than the isotropic component.

Fig. 6. Numerical test of the moment tensor inversion for an expolosional source; a lower hemisphere equal-area projection is used. Shaded areas represent the regions of up motion for *P* waves (individual observations marked by +) and unfilled areas represent the regions of down motion (observations marked by -). Three solutions were sought for a fixed network composed of 24 stations with exact synthetic observations, for the same network with observations disturbed by 10% noise generated by random numbers, and for the network with randomly distributed stations and 10% random noise in the observations (solutions are shown in three horizontal rows). These three solutions are: for a general six-free-component moment tensor, for a constrained moment tensor corresponding to sources with no volume changes, and for a constrained moment tensor corresponding to double couple sources; they are shown in three vertical columns. (From Wiejacz, 1991.)

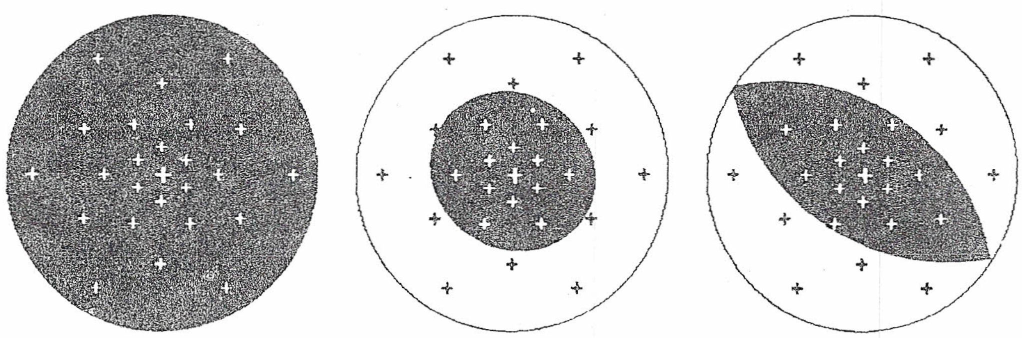
FULL	TRACE 0	TRACE 0, DET 0

EXACT



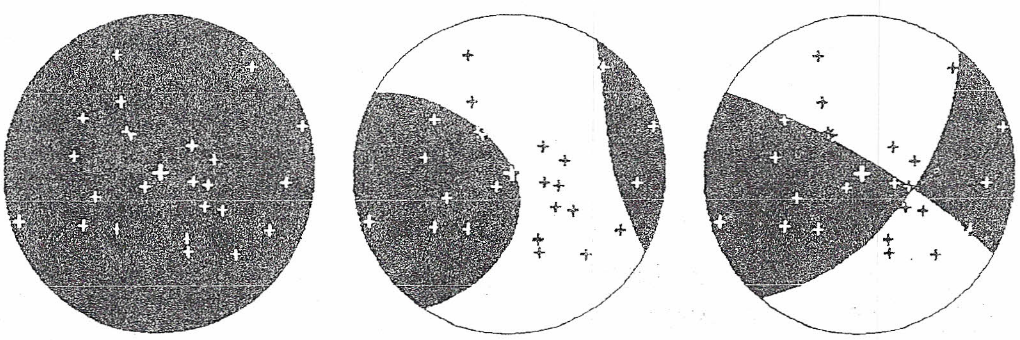
FULL	TRACE 0	TRACE 0, DET 0
.816E+12 -.468E+04 -.316E+04	-.243E+12 -.436E+04 -.890E+04	-.422E+08 -.422E+10 -.545E+04
-.468E+04 .815E+12 -.199E+05	-.436E+04 -.244E+12 -.306E+05	-.422E+10 -.422E+12 -.355E+05
-.316E+04 -.199E+05 .815E+12	-.890E+04 -.306E+05 .487E+12	-.545E+04 -.355E+05 .422E+12
MO= .999E+12 ERR= .0%	MO= .422E+12 ERR= 90.7%	MO= .422E+12 ERR= 91.9%
EXPL= 100.0% CLVD= .0%	CLVD= 99.8%	

DISTURBED



FULL	TRACE 0	TRACE 0, DET 0
.818E+12 -.403E+11 .233E+11	-.252E+12 -.403E+11 .233E+11	-.287E+12 -.189E+12 .222E+11
-.403E+11 .851E+12 .479E+09	-.403E+11 -.220E+12 .479E+09	-.189E+12 -.124E+12 .526E+10
.233E+11 .479E+09 .803E+12	.233E+11 .479E+09 .472E+12	.222E+11 .526E+10 .411E+12
MO= .101E+13 ERR= 4.9%	MO= .411E+12 ERR= 91.0%	MO= .411E+12 ERR= 92.2%
EXPL= 93.6% CLVD= 1.8%	CLVD= 81.6%	

RND STN



FULL	TRACE 0	TRACE 0, DET 0
.795E+12 .958E+10 .473E+10	-.302E+12 .149E+12 -.144E+12	-.406E+12 .133E+12 -.195E+12
.958E+10 .820E+12 .103E+11	.149E+12 .266E+12 -.320E+12	.133E+12 .291E+12 -.200E+12
.473E+10 .103E+11 .829E+12	-.144E+12 -.320E+12 .355E+11	-.195E+12 -.200E+12 .115E+12
MO= .998E+12 ERR= 1.3%	MO= .477E+12 ERR= 94.0%	MO= .477E+12 ERR= 93.4%
EXPL= 97.3% CLVD= .1%	CLVD= 67.8%	

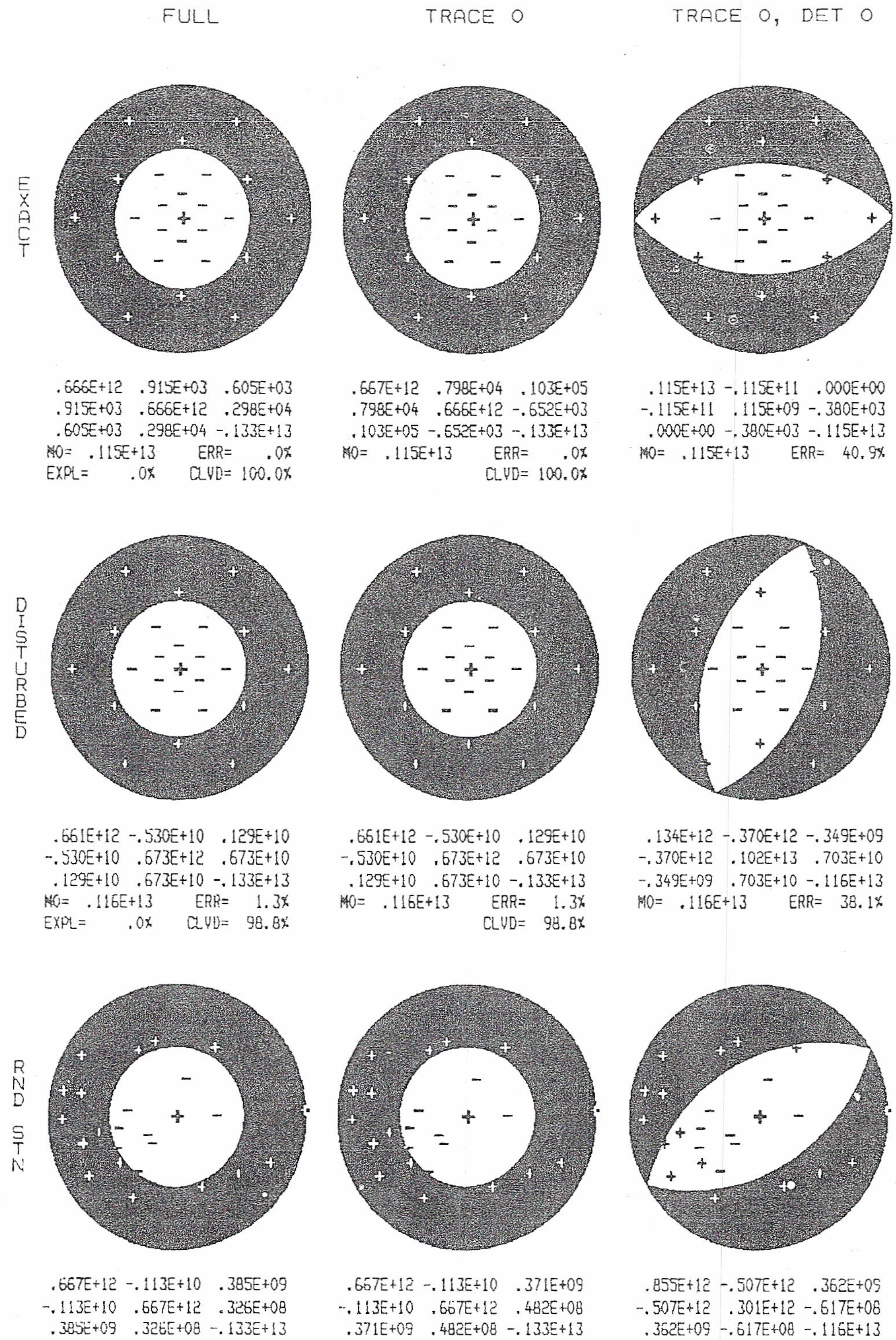


Fig. 7. Numerical test of the moment tensor inversion for a compensated linear vector dipole (CLVD) source corresponding to vertical compression. The nine solutions are described in Fig. 6. (From Wiejacz, 1991.)

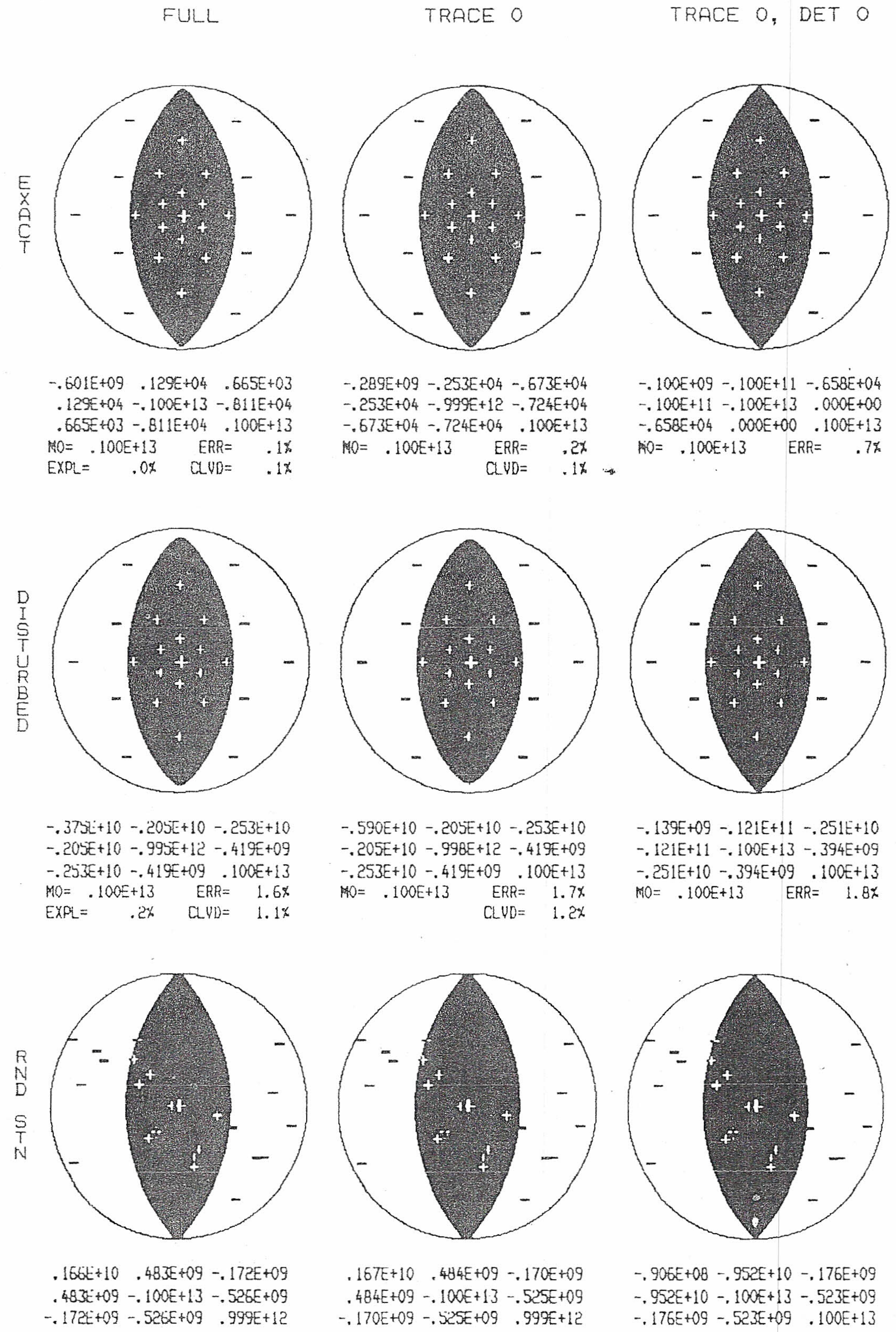


Fig. 8. Numerical test of the moment tensor inversion for a shear (double-couple) source corresponding to dip-slip reverse fault. The solutions are described in Fig. 6. (From Wiejacz, 1991.)

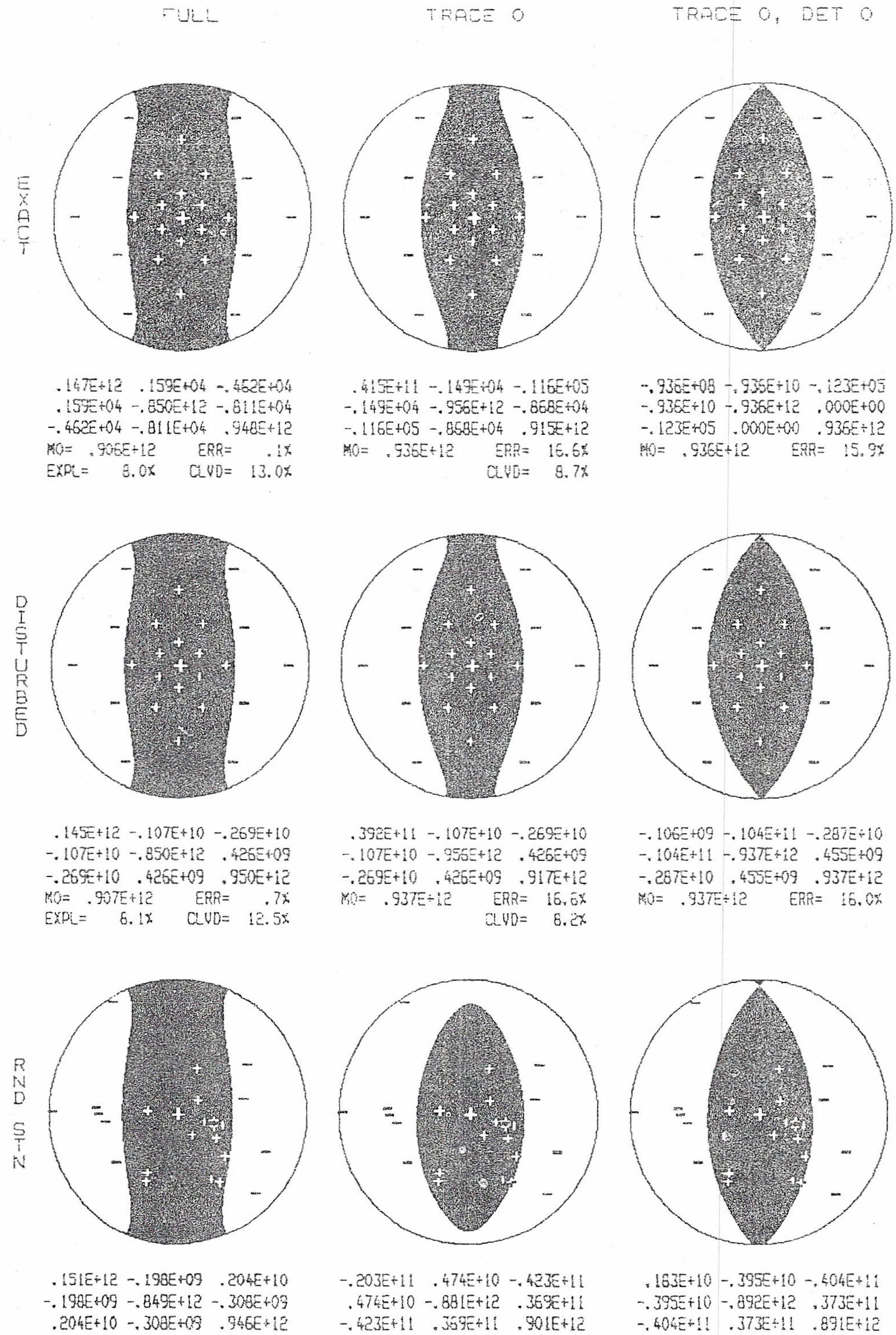


Fig. 9. Numerical test of the moment tensor inversion for a seismic source composed of the double-couple (dip-slip reverse fault), 10% CLVD and 10% explosional components. The solutiona are described in Fig. 6. (From Wiejacz, 1991.)

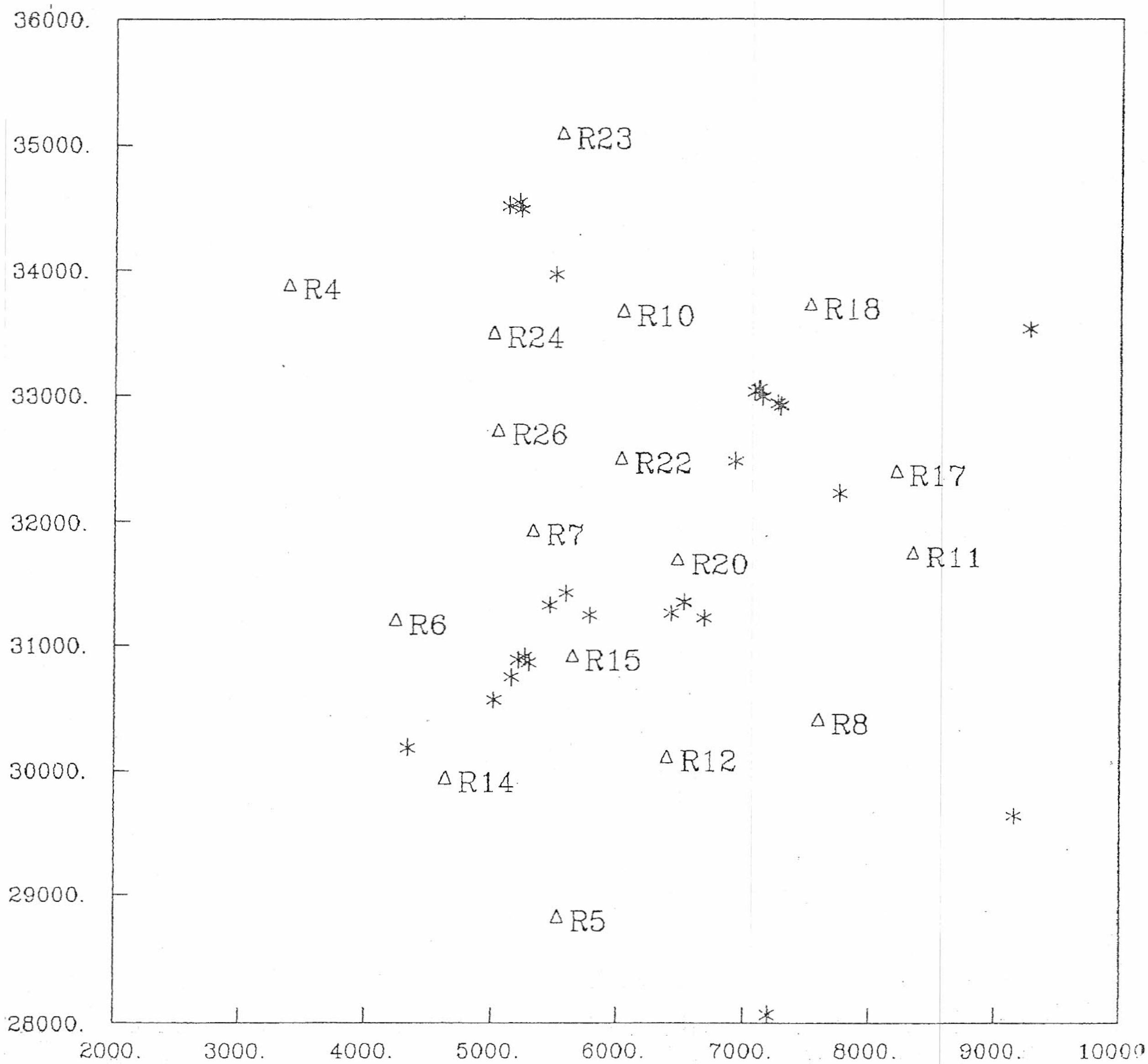


Fig. 10. Horizontal distribution of selected seismic events which occurred in 1991 (marked by stars) and seismic stations used for the moment tensor inversion (open triangles) at Rudna copper mine, Poland. (From Wiejacz, 1991.)

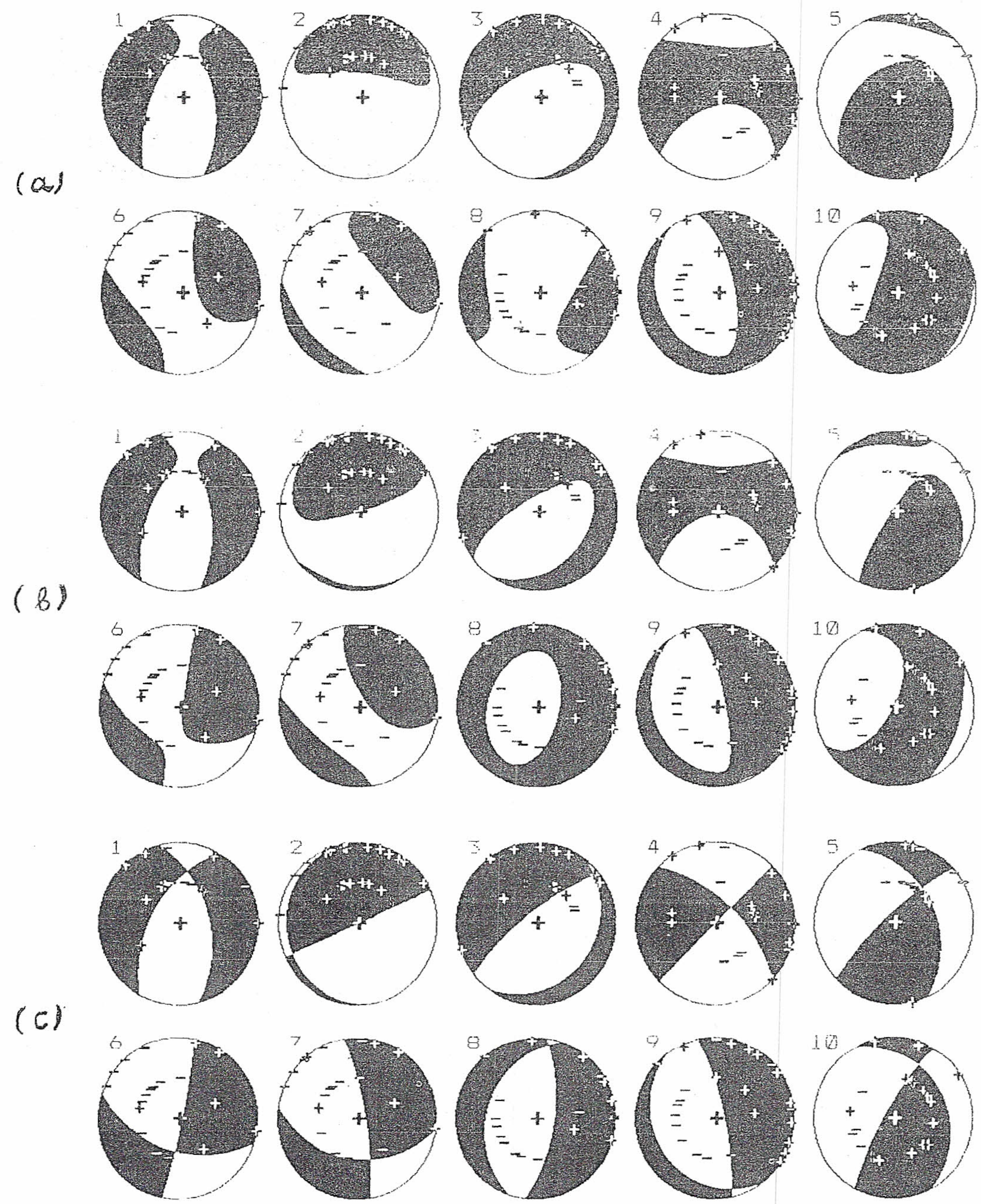


Fig. 11. Nodal lines deduced from moment tensor inversion for 10 selected seismic events which occurred in 1991 at Rudna copper mine, Poland. Three solutions are shown for each event: (a) for a general six-free-component moment tensor, (b) for a constrained moment tensor corresponding to sources with no volume changes, and (c) for a constrained moment tensor corresponding to double-couple sources. (From Wiejacz, 1991.)

Another case of the application of moment tensor inversion to study the mode of failure of small seismic events induced by mining was recently reported by Feignier and Young (1992). They studied 33 microevents (in the $-4<M<-2$ moment magnitude range) induced by drilling a tunnel at 420 m depth in the Canadian shield granite at the Underground Research Laboratory in Pinawa, Manitoba. From the moment tensor inversion they obtained the ratio of isotropic to deviatoric components and they found a number of explosional and implosional sources. Furthermore, the location of events displaying extensional components corresponded to a breakout observed in the roof of the tunnel. Although they also found a number of purely deviatoric sources, no attempt was made to decompose the deviatoric moment tensors into, for example, the CLVD and double-couple components. Thus the presence of purely shear seismic events could not be detected.

Most recently, McGarr (1992) reported three tremors in the magnitude range from 2.1 to 3.4, recorded in early 1988 on the surface and at an underground station in one of the major gold mines in South Africa, with seismic moment tensors having substantial implosional components. In the mines generating these tremors the subhorizontal tabular ore bodies are offset (typically by several hundred meters) by major faults. Mining in the vicinity of the faults stimulates seismicity resulting in renewed fault slip as well as excavation closure, which manifests as a volumetric contribution to seismic moment tensors.

7. Conclusions

Moment tensor inversion, as long as the seismic source can be considered as a point source, is probably the best approach to study the mode of failure of seismic events induced by mining. The immediate proximity of openings in underground mines creates favorable conditions for generation of non-shearing seismic events, especially in the stope area. Decomposition of a moment tensor into an isotropic part corresponding to volumetric changes, a compensated linear vector dipole corresponding to a uniaxial compression, and a double couple corresponding to shearing seems to be the most interesting one for source studies in mines, especially where pillar-associated seismic events are observed. It should be noted, however, that relatively small non-shearing components of the source mechanism are difficult to detect; the dominant shear component leading to well-constrained classical double-couple solutions.

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