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SPECTRAL ANALYSIS OF SHORT SEISMIC SIGNALS

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Abstract

One of the input conditions of correct determination of harmonic components by means of a discrete Fourier transformation is that the length of duration of the signal analysed must be an integer product of period. In general this condition cannot be fulfilled for seismic signals. In this research task a method of determining harmonic components with a non-integer number of periods over the duration of the analysed time series (DHC) has been applied for the analysis of such signals (short recording, unknown period of the signal). The DHC method starts the signal analysis by determining periods of individual components

$$y(t) = \sum_{i=1}^{m} [S_i \sin \omega_i t + C_i \cos \omega_i t]$$

where m is number of harmonic components. Then the remaining parameters of components are determined by a current method.

The above-mentioned method was applied for processing of several rock bursts in Ostrava-Karvina Coal Basin. The spectra of longer seismic signal intervals obtained either by FFT method or by DHC method are very similar. The results indicate further that the DHC method enables a more accurate determination of short signal spectral parameters. The consequence is a better interpretation of physical parameters of rock burst focal zones.

Key words : determination of harmonic components, spectral analysis, Fourier transformation

1. INTRODUCTION

The seismic pulse is as a rule a short-time waving characterised by its shape, amplitude, complex spectrum and polarisation. On the basic of theoretical routes developed by Fourier the majority of functions, even the seismic pulse (continuous function of time), can be approximated by the sum of waves of sinusoidal and cosinusoidal form [CAMINA et al., 1984]. This function is represented by a frequency spectrum. Study of spectra is of utmost significance for the up-to-date processing of seismic data oven in the field of interpretation of rock bursts.

When investigating the geometrical and physical parameters of the foci of the rock bursts one goes often out from the spectral parameters of a registered signal. To calculate the spectra from the records of rock bursts the discrete Fourier transformation by means of the algorithm of rapid transformation (FFT) is usually applied. Nevertheless, the input conditions are not kept-on here and thus, the spectrum could be rather misrepresentative, especially with analysis of short-time intervals. In the framework of investigation we have used, for analysis of such signals, the method of determination of the harmonic components with a non-integer number of periods (DHC).

2. CONFINED APPLICABILITY OF THE FFT-ALGORITHMS

The analysis of signal by means of the discrete Fourier's transformation, with the help of the FFT-algorithm, leads to correct discrete spectrum, when the following preconditions are fulfilled [ČÍŽEK, 1981]:

1. The sampling period has to meet the sampling theorem
2. The errors due to sampling digitalisation and calculation have to be negligible
3. The original signal should be periodical
4. The duration of the time series should equal to the integrated multiple of the period of original signal.

The consequence of breach of the assumptions of applicability for the spectrum of a harmonic signal is shown in Fig. 1. Nevertheless, with acceptable approximations and modifications the applicability of this algorithm at analysing is well reasoned. For our analyses the long enough intervals of seismic cycle are usually applied, whereby these intervals are represented by continuous spectra.

The problems with calculation of spectrum are emerging with analysis of rock bursts registered in small distances, as the applied non-interfered part of wave is rather short. One of the ways of determination of the spectrum of short signals is transfer of signals into series of harmonic components whose periods are not integral parts of duration of the given sector of signal [MATYÁŠ, 1984]. Determination of the harmonic components of signal has to start in this case by determination of period and/or frequency of the individual components. Then, the residual parameters of components may be determined in common manner.

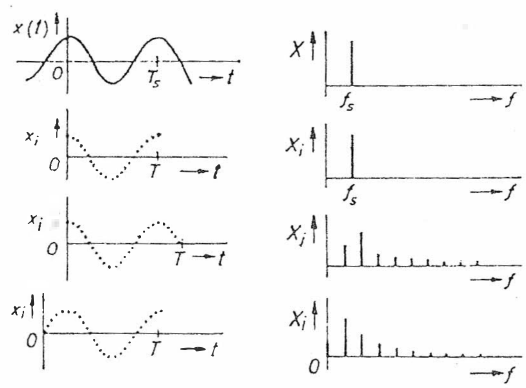

Fig. 1. Example of deformation of spectrum, unless the preconditions for possible utilisation of the FFT-algorithm are kept-on.

a) Harmonic signal with period T_S and its spectrum
b) Time series of length $T = T_S$ and its spectrum
c) Time series of length $T = 1.5 \times T_S$ and its spectrum
d) Time series of length $T = T_S$ taken off half of period sooner and its spectrum

3. THE DHC-ALGORITHM

For determination of the harmonic components there is applied approximation of signal by the trigonometric functions. The amplitude of the individual sine and cosine components will be determined from the course of the signal in question and from the frequency values of the relevant components. This is also the principle of the Fourier's transformation that, as mentioned earlier, is going out from the assumption of equality of the length of analysed signal and the integrated multiple of the signal period. The DHC-methodology can be described as follows [MATYÁŠ, 1984].

The frequency of the harmonic components of the given signal (where the discrete spectrum is assumed) can be determined as follows: The signal *y(t)* can be expressed by a series of harmonic components with frequencies f_l or $\omega_l = 2\pi f_l$. If m is the number of components, it can be written:

$$y(t) = \sum_{l=1}^{m} [S_l \sin \omega_l t + C_l \cos \omega_l t] \tag{1}$$

If the signal is in sector of the length T transferred to the time series with n specimens, then with implementation $o_l = \omega_l T / n$ the samples in the time series can be expressed:

$$y(i) = \sum_{l=1}^{m} [S_l \sin o_l i + C_l \cos o_l i] \tag{2}$$

As is valid

$$\begin{aligned} y(i \pm k \mp 1) &= \sum_{l=1}^{m} [S_l \sin(i \pm k \mp 1) o_l + C_l \cos(i \pm k \mp 1) o_l] = \\ &= \sum_{l=1}^{m} \begin{bmatrix} S_l \sin i o_l \cos(k-1) o_l \pm S_l \cos i o_l \sin(k-1) o_l + \\ + C_l \cos i o_l \cos(k-1) o_l \mp C_l \sin i o_l \sin(k-1) o_l \end{bmatrix} \end{aligned} \tag{3}$$

There can be written:

$$y(i + k - 1) + y(i - k + 1) = 2 \sum_{l=1}^{m} \cos(k-1) o_l [S_l \sin i o_l + C_l \cos i o_l] \tag{4}$$

Then also is valid

$$\begin{aligned} &\sum_{k=1}^{m} V_k [y(i + k - 1) + y(i - k + 1)] = \\ &= 2 \sum_{k=1}^{m} V_k \sum_{l=1}^{m} \cos(k-1) o_l [S_l \sin i o_l + C_l \cos i o_l] \end{aligned} \tag{5}$$

If the weight coefficients V_k are selected so that would be valid

$$\sum_{k=1}^{m} V_k \cos(k-1)\,\omega_l = \cos m\omega_l \tag{6}$$

then is obtained

$$\begin{aligned}&\sum_{k=1}^{m} V_k\left[y\,(i+k-1)+y\,(i-k+1)\right]= \\ &\quad =2\sum_{l=1}^{m}\cos m\omega_l\left[S_l\sin i\omega_l+C_l\cos i\omega_l\right]=y\,(i+m)+y\,(i-m)\end{aligned} \tag{7}$$

This inscription can be simplified by introduction

$z_{ik} = y\,(i+k-1)+y\,(i-k+1)$

in the form

$$\sum_{k=1}^{m} V_k z_{ik} = z_{i,m+1} \tag{8}$$

For $i \in \langle m+1, n-m \rangle$ we have in total $n-2m$ equations for m weight coefficients V_k. By solving this system of equations one gets the weight coefficients V_1 to V_m. For such weight coefficients the frequency figures of the harmonic components are given by Eq(6). This can be solved by transferring into a not-linear equation

$$\sum_{k=0}^{m} W_k \cos^k \omega = 0 \tag{9}$$

The real roots of Eq(9) provide $\cos\omega_1$ up to $\cos\omega_m$, which results into ω_1 to ω_m . As soon as the frequency figures of the harmonic components are known, the amplitudes of the sine and cosine components can be calculated from Eq(2) for $n \succ 2m+1$ by using the least-square method [MATYÁŠ, 1984]. In this respect the Gauss eliminating method with partial pivoting was used for solution of the system of linear equations, while the Newton-Horner method was applied at solution of the roots of non-linear Eq(9). Application of the Newton-Horner method reduces the number of frequency figures to $m = 12$ max.

The software for analysing the seismic signal by the method in question was elaborated by the specialists of the laboratory of mine geophysics of the Mining Institute CSAS GRUNTORÁD, 1991. The software enables to calculate by means of the DHC-method for a given time interval the corresponding harmonic components and to illustrate them in spectrum obtained by the FFT-method. At calculation one can select start and end of the analysed signal. Then, the number of frequency figures should be chosen so as to reach the maximum. This number can be estimated as one third the number of samples of the analysed signal. This method of illustration enables to make use simultaneously of two methods at interpretation.

For the sake of testing the compatibility of the two methods of calculation there have been applied signals we have acquired as a sum of the sinusoidal functions with known frequency figures and amplitudes. As follow even from Fig. 2 the DHC-method provides a higher accuracy of determination of the harmonic components of signal. There is to underline here that the price for this precision is the discrepancy in the size of amplitudes of spectra of the two methods. Accordingly, the present method cannot be applied for more precise evaluation of the plateau -values of spectra.

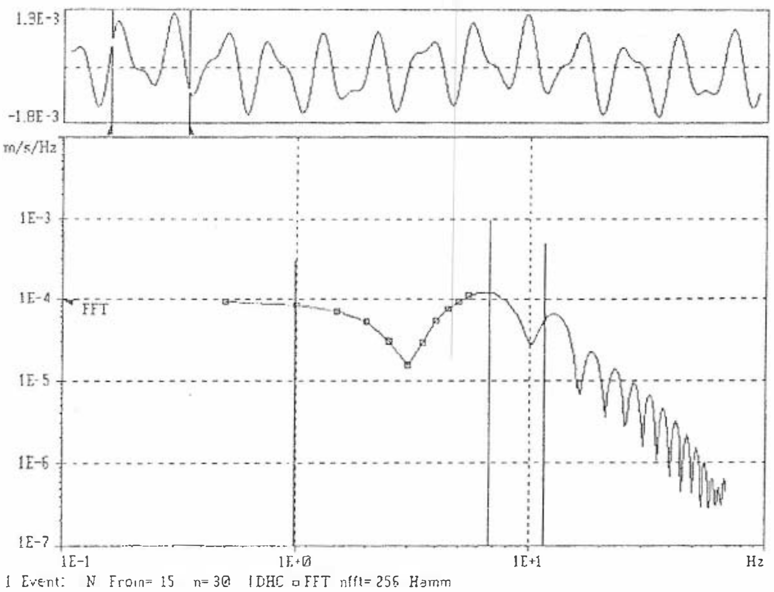

Fig. 2. Example of calculated signal:

$$y\ (t\ ) = 0{,}0003\ \sin(2\pi t\ ) + 0{,}0010\ \sin(2\times 6{,}8\pi t\ ) + 0{,}0005\ \sin(2\times 11{,}7\pi t\ ).$$

4. SPECTRA OF THE ROCK BURSTS

From the comparison of the results of the two methods there is obvious that both the FFT-method and the DHC-method (Fig. 3) provide similar results for a signal lasting for a longer time. The two methods were used for processing the same sectors of signal; the DHC-method for an interval of 30 samples, the FFT-method for the same interval modified by the Hamming weighing function and completed by zero from the right-hand side to almost 64 samples. The individual significant spectral DHC-lines in Fig. 3 refer to the maximum of the FFT-spectrum, however, the amplitudes of the harmonic components of the DHC-method are higher if compared with the FFT-method. Fig. 4 shows interpretation of a not-interfered sector of signal processed by the FFT- or the DHC-method. The spectrum obtained by the DHC-method has a very sharp maximum and the frequency of this maximum is about by 1 Hz smaller than the frequency of the FFT-maximum. The difference of the spectra of short sectors of signal is much more distinct in Fig. 6 whereby the spectra of the further intervals of a seismic signal in Fig. 5 are very similar.

The wrong calculation of spectrum from short sector of seismic signal by the FFT-method brings errors into estimation of the parameters of source. In such cases the method of determination of the harmonic components of a non-integer of number of the periods should be applied here. In our experience this method should be more suitable for automatic destination of parameters of the spectral magnitudes. The DHC-method can be applied at determination of the harmonic components of the interfering waves.

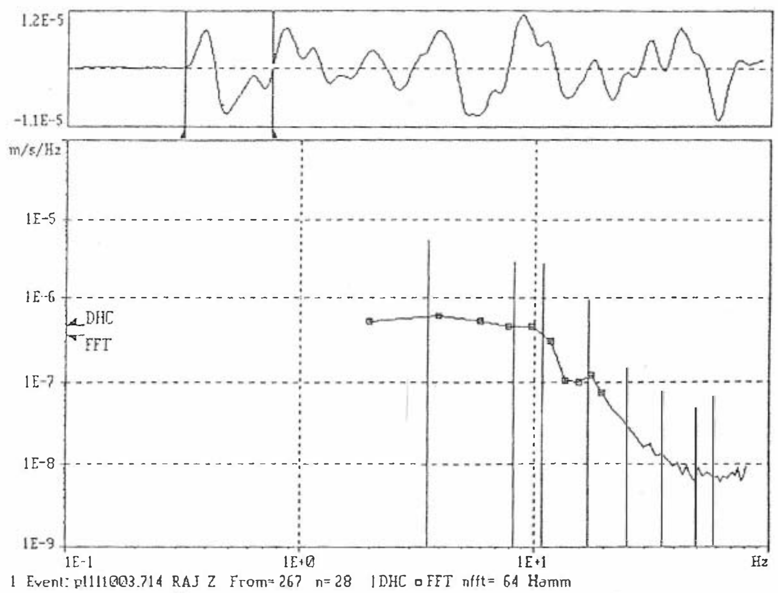

Fig. 3. Comparison of spectra FFT method a DHC method for longer interval of signal.

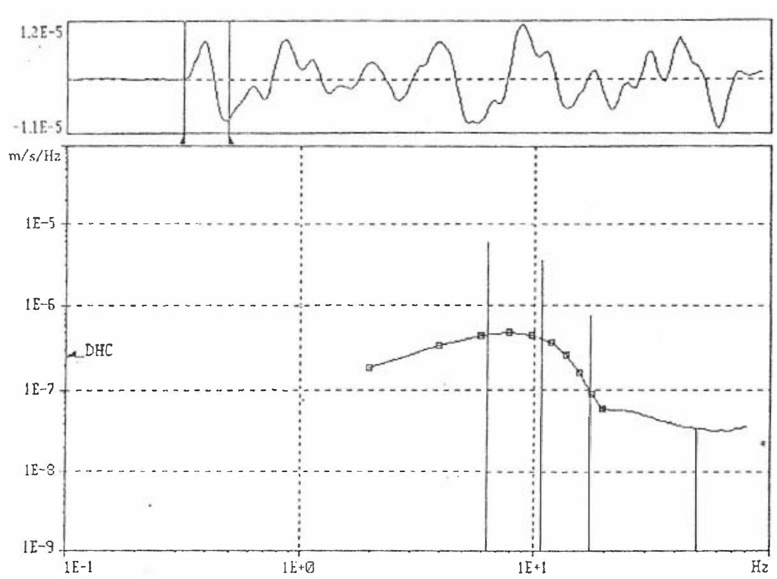

Fig. 4. FFT spectrum and DHC spectrum of not-interfered sector of signal.

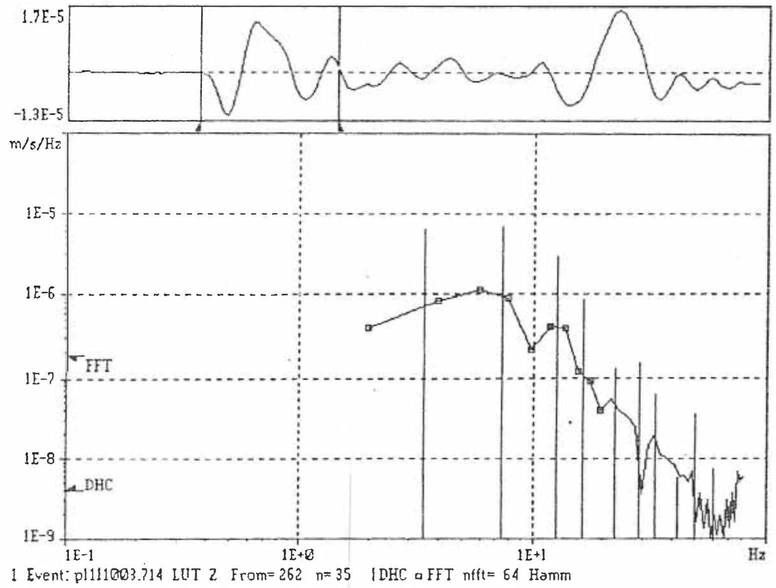

Fig. 5. FFT and DHC spectra of longer interval of signal.

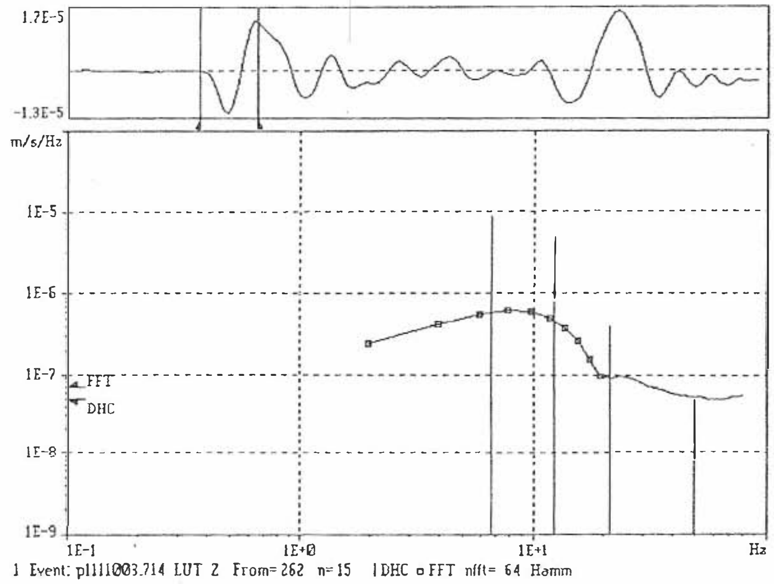

Fig. 6. Spectrum of FFT and DHC methods for short sector of seismic signal.

5. CONCLUSION

The DHC-method cannot be considered for a substitution of the Fourier's analysis of signal. Its task consists above all in determination of the harmonic components of signal whose period is compatible with duration of the analysed time series being a non-integer number of the periods of components. When analysing the rock bursts from the Ostrava-Karviná Coal Field, the record of analysed signal i.e. the non-interfered part of wave, has a length of about 30 to 40 samples with sampling frequency of 125 Hz.

From a visual comparison of the results follow that the two methods provide spectra with similar shape and values of the individual components. The individual significant spectral DHC-lines refer to the maxims of the FFT-spectrum. The largest difference was recorded in the expected zone i.e. at the lowest investigated frequency figures. In addition, this difference is more expressive for spectrum considered from shorter sectors of the analysed signal if compared with spectra from longer intervals.

The incorrect assessment of the parameters of spectrum (from a short sector of a seismic signal) by the FFT-method brings errors into calculation of the parameters of source. In such cases the DHC-method should be utilised for more precise evaluation of the spectrum components.

The present method is permanently investigated, which is associated with numerous still open questions. Further investigation will be focused into both fields - the field of modification of software and the field of method of interpretation of spectra acquired by the DHC-method.

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