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DEFORMATION MEASUREMENT OF ROCK MODELS UNDER THE LOADING STRESS OF CENTRIFUGAL FORCE

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ABSTRACT. In connection with the determination of deformation changes of the surface of rock models, loaded by centrifugal forces in a special centrifuge, an entirely automatized contactless survey method, based on the application of analytical stereophotogrammetry, has been developed. The method enables the spatial displacements of identical survey points between two arbitrary deformation phases, which were documented by photogrammetric measurement always after removal of the rock model from the centrifuge and its location in adequate position for measurements, to be determined. It is not necessary, for the evaluation of such measurements, to define a spatial control point field in the neighbourhood of the object, while preserving a high determination precision of deformation displacements. Digital models of deformation surfaces can be constructed from these discrete data, and these can be further effectively and for the instruction's sake illustrated by means of the author's application software, formerly developed. The application of this method is entirely general.

INTRODUCTION

This surveying problem had to be dealt with in connection with the solution of the slope failure of spoil banks in the Northon- Bohemian coal district. The problem has been investigated by means of rock models, whose loading by gravitational special centrifugal forces, generated in a special centrifuge, originally constructed for the solution of geotechnics problems by means of the so-called frozen photoelastic models (Málek 1977).

1. SCHEME OF THE MODEL EXPERIMENTS.

Fig.1 illustrates the construction diagram of the used centrifuge, its overall dimensions, the size of the "cell", where the rock model was situated, as well the location of this cell at stillstand of the centrifuge. It should be added to these data that the generated centrifugal force can be continuously changed up to its value of 200-multiple of the earth's gravity. The centrifuge does not enable the deformations to be observed visually during the rotation, so that deformation measurements of the model body could not be effectuated before stopping the centrifuge and removal of the steel cell.

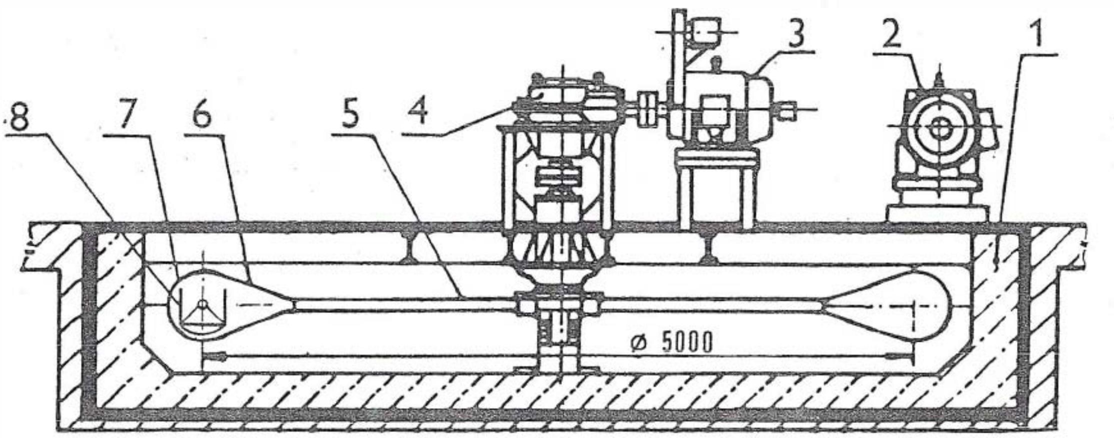

FIG 1. Scheme of centrifuge

1: reinforced-concrete trough, 2: direct current generator, 3: asynchronous motor, 4: gearbox, 5: centrifuge arm, 6: centrifuge cabin, 7:revolving cradle, 8: cell

2. METHOD OF DEFORMATION MEASUREMENTS OF THE ROCK MODEL.

Due to necessity of application of a contactless measurement methods and to the requirement of complex assessment of deformations within the entire model surface range, the so-called normal case of the close-range analytical stereophotogrammetry has been chosen as the surveying method. The layout of such measurement is illustrated in Fig.2. The steel cell with the rock model within it, has been always placed — after removal from the centrifuge — so that it took up approximately the same position at the distance of about 1,2 m from the base of the stereophotogrammetrics system Zeiss: "DOPPELAUFHÄNGUNG" 2× UMK 10/1318. The deformation measurement in a certain phase of the model experiment consisted of providing a classic stereophotogrammetric pair of pictures of the steel cell with the model (Fig.3) at the linear base of 320 mm. The automatized stereocomparator "STECOMETER" was used for the measurement of the picture's coordinates.

3. EVALUATION OF MEASUREMENTS AND DETERMINATION OF DEFORMATIONS.

The deformations of the rock model between two experimental phases were defined as changes of the spatial position of a certain number of "survey points", adequately chosen on the model surface. The selection of these points was aimed at an optimum definition of the deformation course on the entire model surface. This could be made on the basis of stereoscopical observation of pictures, which are documenting both phases of the experiment and appertain always to the same, either the left or the right photogrammetric position. The stereoscopic observation of this couple of pictures — pictures on the so-called time basis — produces, provided the

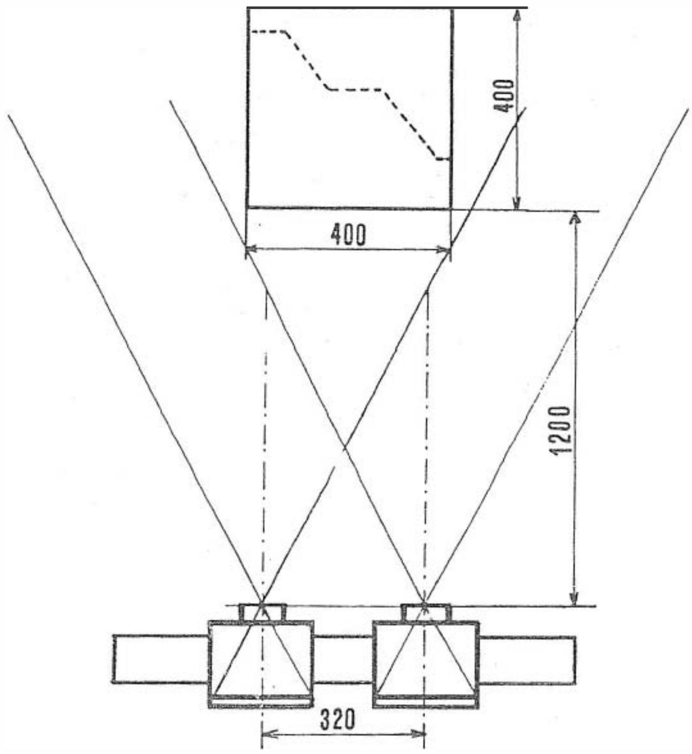

FIG 2. Scheme of photogrammetric measurements

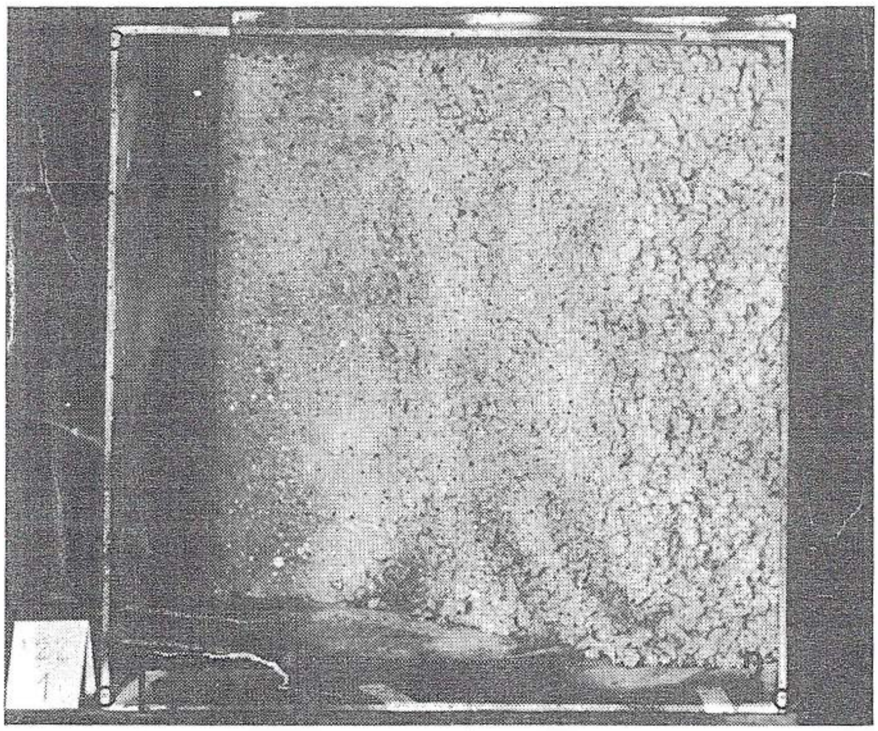

FIG 3. The rock mass model in cell

model surface contains a sufficient quantity of spot object (mass grains, colour spots etc.), a stereoscopic model of the "deformation surface". The shape of this surface illustrates very well the model deformations and their character. The selection of the above-mentioned points is then made so that it would define, in the best way, the morphology of this deformation surface, much like in the case of evaluation of deformation measurements on the planar objects (Vencovský 1989).

The mentioned changes of the spatial positions of survey points are derived as differences of their coordinates, defined within the common rectangular spatial coordinate system XYZ, which remains relatively steady towards the deforming model a which should thus be defined on the structure of the steel cell. Deformations of this cell due to the centrifugal force are not assumed.

Photogrammetric measurements described in the preceding paragraph determine the positions of all survey points within the system of so-called photogrammetric coordinates xyz, which is defined by the position and setting of the stereophotogrammetric equipment and which is relatively steady in relation to the positionally unsteady XYZ system.

The problem of evaluation of photogrammetric measurements within each of the deformation phases of the model experiment consists in finding the so-called transformation key between the systems XYZ and xyz and in transforming, by means of this key, the positions of all survey points from the xyz system into the system XYZ.

This is made by means of the known expressions:

$$\begin{aligned} X &= [(x - x_0).k_1 + (y - y_0).k_2 + (z - z_0).k_3].e_x \\ Y &= [(x - x_0).k_4 + (y - y_0).k_5 + (z - z_0).k_6].e_y \\ Z &= [(x - x_0).k_7 + (y - y_0).k_8 + (z - z_0).k_9].e_z \end{aligned} \tag{1a}$$

which include also the differing scales for e_x, e_y, e_z of transformed coordinates in all the three axes of the system XYZ. The above mentioned formulae can be transcribed into the forms of:

$$\begin{aligned} X &= A + x.K_1 + y.K_2 + z.K_3 \\ Y &= B + x.K_4 + y.K_5 + z.K_6 \\ Z &= C + x.K_7 + y.K_8 + z.K_9 \end{aligned} \tag{1b}$$

where

$$\begin{aligned} K_i &= k_i.e_x \qquad (i = 1,2,3) \\ K_i &= k_i.e_y \qquad (i = 4,5,6) \\ K_i &= k_i.e_z \qquad (i = 7,8,9) \end{aligned} \tag{2}$$

and which become the "mediating" functions for the calculation of 12 constants of the transformation key according to the rules of the adjusting computation.

This computation can only be realized, if coordinates of more than 4 control points marked on the steel cell structure will be known. Coordinates of these points can be determined either by direct measurements by means of mechanical engineering gauges or tools, as well as by the application of a generally known geodetical method. However, this complicates considerably the realization of the whole deformation measurement. In such a case it is quite convenient, from the viewpoint of the required precision, when the XYZ coordinates of control points are derived from their photogrammetric coordinates xyz, obtained during bearing of each deformation phase, by the following procedure:

Three control points are chosen on the structure of the steel cell in that way, that they are situated either in the plane of the bottom or in the plane of the upper wall (Fig.3, points 1,3,4). This plane becomes then the XY-plane of the spatial coordinate system XYZ, the beginning of this system being introduced in one of the chosen point (point 1) and the X-axis being directed towards one of the remaining points (point 2). The system XYZ is defined as righthanded. Coordinates of all control points are defined for each measurement phase, which will give us several sets of these coordinates. From these sets, a single one is created, in which the resulting coordinates of control points are derived as arithmetic means.

The orientation of the coordinate system XYZ in relation to the xyz system is defined by direction cosines k_x, k_y, k_z of the normal vector of the XY plane. Values of these cosines will be determined from the three linear equations mentioned below, which are constructed from the photogrammetric coordinates x, y, z of the chosen three control points:

$$\begin{aligned} x_1.K_x + y_1.K_y + z_1.K_z &= -1 \\ x_2.K_x + y_2.K_y + z_2.K_z &= -1 \\ x_3.K_x + y_3.K_y + z_3.K_z &= -1 \end{aligned} \tag{3a}$$

and on the basis of the following relations:

$$\begin{aligned} k_x &= K_x.P \\ k_y &= K_y.P \\ k_z &= K_z.P \\ P &= \frac{1}{\sqrt{K_x^2 + K_y^2 + K_z^2}} \end{aligned} \tag{3b}$$

The spatial coordinates XYZ of the control points are obtained by means of equations (1a), where

$$\begin{aligned} e_x &= e_y = e_z = 1 \\ k_1 &= -k_y.Q \\ k_2 &= k_x.Q \\ k_3 &= 0 \\ k_4 &= -k_x.k_z.Q \\ k_5 &= -k_y.k_z.Q \end{aligned}$$

$$
\begin{aligned}
k_6 &= \frac{1}{Q} \\
k_7 &= k_x \\
k_8 &= k_y \\
k_9 &= k_z \\
Q &= \frac{1}{\sqrt{1-k_z^2}}
\end{aligned} \tag{4}
$$

where x,y,z are photogrammetric coordinates of the control point, x_0,y_0,z_0 are photogrammetric coordinates of the control point, which has been chosen as initial point of the XYZ system.

Determination of the above mentioned transformation key at predetermined conditions, i.e. adjustment of twelve transformation constants A, B, X, K_1 , ... , K_9 by means of mediating functions (1b), is possible in two ways. Each of these ways must be preceded by preliminary, although not perfectly exact adjustment of these constants, which will be used for determination of scale coefficients e_x, e_y, e_z by following equations:

$$
\begin{aligned}
e_x &= \sqrt{K_1^2 + K_2^2 + K_3^2} \\
e_y &= \sqrt{K_4^2 + K_5^2 + K_6^2} \\
e_z &= \sqrt{K_7^2 + K_8^2 + K_9^2}
\end{aligned} \tag{5}
$$

derived on the basis of orthogonal relations between direction cosines k_1, ... ,k_9 and considering the equations (2).

a) Direct adjustment of constants A, B, C, K_1, ... , K_9. Owing to the fact that in such a case all 12 conditions of orthogonality, which hold true for the elements of the matrix

$$
\begin{bmatrix}
k_1 & k_2 & k_3 \\
k_4 & k_5 & k_6 \\
k_7 & k_8 & k_9
\end{bmatrix}
$$

should be incorporated into an exact adjustment, this computation leads to the solution of a system of 24 "normal" equations. These equations are constructed from linear forms of correction equations, derived from the mediating function (1b) for small changes of arguments of direction cosines k_1, ... , k_9. Owing to the complexity of this adjustment procedure, a simpler method, leading to the solution of a system of only 6 normal equations, should be preferred.

b) Adjustment by means of Euler angles. If these angles are denoted S, T, U, the following relations (Nečas 1977) hold true between them and the constants $k_1, \ldots, k_9$.

$$
\begin{aligned}
k_1 &= -\cos S.\cos U - \sin S.\sin U.\cos T \\
k_2 &= \sin S.\cos U \quad - \cos S.\sin U.\cos T \\
k_3 &= -\sin U.\sin T \\
k_4 &= \cos S.\sin U \quad - \sin S.\cos U.\cos T \\
k_5 &= -\sin S.\sin U \quad - \cos S.\cos U.\cos T \\
k_6 &= -\cos U.\sin T \\
k_7 &= -\sin S.\sin T \\
k_8 &= -\cos S.\sin T \\
k_9 &= \cos T
\end{aligned}
\tag{6}
$$

Equations (1b) can then be formulated in the forms

$$
\begin{aligned}
X &= A + \kappa_x(S,T,U,x,y,z).e_x \\
Y &= B + \kappa_y(S,T,U,x,y,z).e_y \\
Z &= C + \kappa_z(S,T,U,x,y,z).e_z
\end{aligned}
\tag{7}
$$

and these further transformed into linear functions of small changes dS, dT, dU of the approximate values of angles S, T, U, computed from preliminarily adjusted constants $K_1, \ldots, K_9$ on the basis of the following equations, obtained from equations (6) and (2)

$$
\begin{aligned}
\operatorname{tg} S &= \frac{K_7}{K_8} \\
\operatorname{tg} U &= \frac{K_3.e_y}{K_6.e_x} \\
\cos T &= \frac{K_9}{e_z}
\end{aligned}
\tag{8}
$$

For each photogrammetric point, which participates in the adjustment, the following correction equations can be constructed

$$
\begin{aligned}
v_x &= A + \frac{\partial \kappa_x}{\partial S} dS + \frac{\partial \kappa_x}{\partial T} dT + \frac{\partial \kappa_x}{\partial U} dU - X \\
v_y &= B + \frac{\partial \kappa_y}{\partial S} dS + \frac{\partial \kappa_y}{\partial T} dT + \frac{\partial \kappa_y}{\partial U} dU - Y \\
v_z &= C + \frac{\partial \kappa_z}{\partial S} dS + \frac{\partial \kappa_z}{\partial T} dT + \frac{\partial \kappa_z}{\partial U} dU - Z
\end{aligned}
\tag{9}
$$

in which

$$
\frac{\partial \kappa_x}{\partial S} = [x(\sin S.\cos U \quad - \cos S.\sin U.\cos T) +
$$

$$+ y(\cos S.\sin U + \sin S.\sin U.\cos T)].e_x$$

$$\frac{\partial \kappa_x}{\partial U} = [x(\cos S.\sin U - \sin S.\sin U.\cos T) -$$
$$- y(\sin S.\sin U + \cos S.\cos U.\cos T) - z.\cos U.\sin T)]e_x$$

$$\frac{\partial \kappa_x}{\partial T} = (x.\sin S.\sin U.\sin T + y.\cos S.\sin U.\sin T - z.\sin U.\cos T).e_x$$

$$\frac{\partial \kappa_y}{\partial S} = [-x(\sin S.\sin U - \cos S.\cos U.\cos T) -$$
$$- y(\cos S.\sin U - \sin S.\cos U.\cos T)].e_y$$

$$\frac{\partial \kappa_y}{\partial U} = [x(\cos S.\cos U + \sin S.\sin U.\cos T) -$$
$$- y(\sin S.\cos U - \cos S.\sin U.\cos T) + z.\cos U.\sin T].e_y$$

$$\frac{\partial \kappa_y}{\partial T} = (x.\sin S.\cos U.\sin T + y.\cos S.\cos U.\sin T - z.\cos U.\cos T).e_y$$

$$\frac{\partial \kappa_z}{\partial S} = (-x.\cos S.\sin T + y.\sin S.\sin T).e_z$$

$$\frac{\partial \kappa_z}{\partial U} = 0$$

$$\frac{\partial \kappa_z}{\partial T} = (-x.\sin S.\cos T - y.\cos S.\cos T - z.\sin T).e_z$$

From the correction equations (9), the system of six "normal" equations is constructed by the normal well-known procedure from the adjustant computation and the solution of this system yields the adjusted unknowns A, B, C, S, T, U. Owing to the fact, that the linear forms (9) are only an approximate expression for mediating functions (7), the values of unknowns obtained from this first iteration should be considered merely their approximate values and the whole computation should be repeated by second, third, or eventually further iterations. For each iteration, the sum of all correction squares (9) is determined within the range of all participating n control points. The definitive adjustment of the unknowns looked for and thus the termination of the iteration computing corresponds to the situation, when the minimum of this sum is attained:

$$\sum_{i=1}^{n} (v_x.v_x + v_y.v_y + v_z.v_z) = \min$$

After obtaining the adjusted angles S, T, U, the constants $K_1, \ldots, K_9$ are computed by using equations (6) and (2), which, together with the definitive unknowns A, B, C, represent the transformation key, which has been looked for.

4. Measurement results and their precision.

The realization of the described procedure of deformation measurement brought fairly acceptable results concerning both the obtained precision and the global and

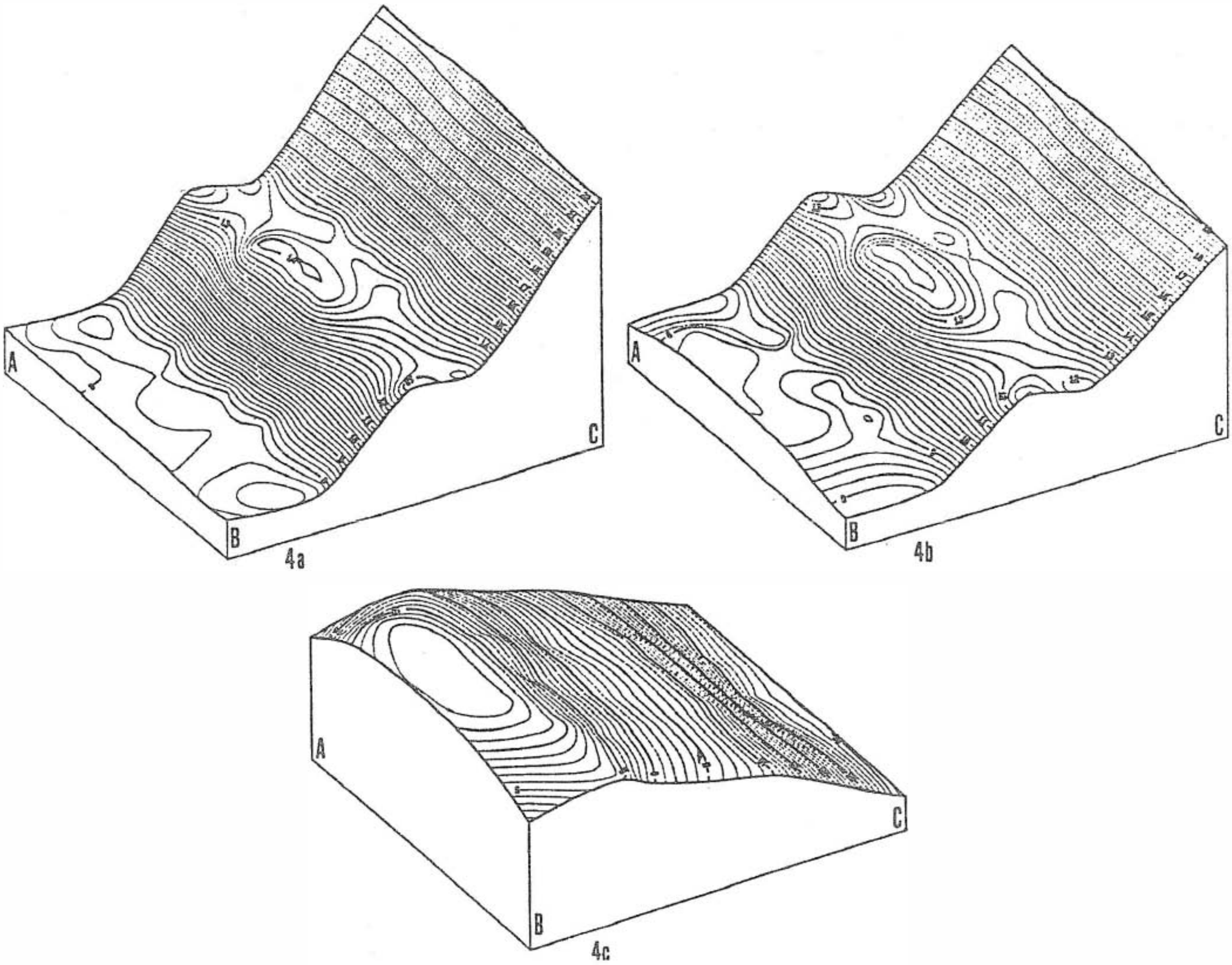

FIG.4: Graphical representation of deformation measurements

detailed determination of deformation strain of the surface of studied rock models. The necessary program for automatic computer treatment of all numerical operations has been designed. The resulting fields of XYZ coordinates of all survey points from all realized measurements of deformation phases of the model experiment have been configurated and put into the external store of the PC, so that a digital model of deformations between two arbitrary deformation states of these experiments could be created and mentioned model further processed by means of some author's program for graphical representation of three-dimensional point fields (Vencovský 1989).

As far as the precision is concerned, the following table indicates both the mean errors m_X, m_Y, m_Z, in the derivation of coordinates XYZ of control points by the above-mentioned method, and also the mean errors m_{DX}, m_{DY}, m_{DZ} of the determination of coordinate components of deformations, i.e. of the differences of coordinates of identical survey points. Mean errors m_X, m_Y, m_Z were obtained from the coordinate sets of control points, derived for 5 deformation phases. Mean errors m_{DX}, m_{DY}, m_{DZ} were then determinated from the dispersion variance of

XYZ coordinates of 36 survey points, assessed by 5-times repeated above-mentioned deformation measurements of a certain physical model, whose surface has been subjected to deformation changes.

$$m_X = 0{,}02\,\text{mm} \qquad m_{DX} = 0{,}05\,\text{mm}$$
$$m_Y = 0{,}02\,\text{mm} \qquad m_{DY} = 0{,}05\,\text{mm}$$
$$m_Z = 0{,}07\,\text{mm} \qquad m_{DZ} = 0{,}12\,\text{mm}$$

Figures 4a,b,c illustrate the graphical form of evaluation of deformation measurement between two phases of the model experiment. Figs.4a,b contain the isoline descriptions of the relief of the investigated physical slope model, in the phase before and after loading, the Fig.4c represents the component of model deformations, which lies in the direction of centrifugal forces.

5. Conclusion.

The described method of deformation measurement proved completely successful in connection with the given survey problem, giving the expected and accurate results. The method is entirely general and has been used lated, with similar success, also for other deformation measurements in mechanical and civil engineering.

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Měření deformací horninových modelů zatěžovaných odstředivými silami

Miloš Vencovský

V souvislosti s určováním deformačních změn povrchu horninových modelů zatěžovaných odstředivými silami ve speciální odstředivce byla vypracována a plně automatizována nekontaktní měřická metoda založená na užití analytické stereofotogrammetrie. Metoda umožňuje určování prostorových posunů identických měřických bodů mezi dvěma libovolnými deformačními fázemi, které byly dokumentovány fotogrammetrickým měřením vždy po vyjmutí horninového modelu z odstředivky a po jeho umístění do polohy umožňující toto měření. K vyhodnocení provedeného měření není třeba definovat v okolí objektu prostorové vlícovací pole bodů, přičemž je zachována vysoká přesnost v určení deformačních posunů. Z těchto diskrétních údajů je možno zkonstruovat digitální modely deformačních ploch a ty dále zobrazovat pomocí autorova softwaru vyvinutého v minulosti. Metoda má zcela obecné použití.