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POSSIBILITY OF DIMENSION DECREASE IN THE PROBLEM OF SEISMIC EVENTS LOCATION

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ABSTRACT. A classical approach to the location of seismic events consists in identifying the coordinates of epicentre or hypocentre of the tremor and time t_0 – origin time. The time is treated as equivalent to focus coordinates. It is one of the unknowns numerically determined by means of iterative calculations. Since the equations connecting arrival times with seismometer coordinates and unknown focus coordinates are linear, the t_0 parameter can easily be eliminated. As a result, the dimension of the problem with regard to t_0, this parameter decreases and numerical properties of the seismic events location task are improved.

To present the method for decreasing the dimension of iterative task of tremor coordinates calculation, let us assume that the seismic event is localized in homogeneous and isotropic medium, and the velocity of seismic wave is a known parameter of the model. Furthermore, let us assume that the system of nonlinear equations is a system of full order with supernumerary elements ($s > 4$).

Differences between measured arrival times t^{ex} and times calculated from the assumed model, for each seismometric station, are marked by the following symbol:

$$e_j = \phi(\mathbf{r}_j, \mathbf{r}_0) + t_0 - t_j^{\mathrm{ex}} \qquad (j = 1, \ldots, s) \tag{1}$$

Traditional approach to the localization task presented in this way consists most often in the application of Nonlinear Problem of Least Squares with 4 unknowns (x_0, y_0, z_0, t_0).

The traditional method has at least four flaws:

- the solution of system (1) is possible if initial approximation of searched coordinates of the tremor and the moment of its generation are sufficiently close to real location of tremor focus and time t_0 [Fortuna et al. 1982],
- the solution of standard equation of square, symmetrical and positively defined matrix $\mathbf{A}^T\mathbf{A}$ ($\mathbf{A}$ – design matrix) may be improperly conditioned, which results in great iterative fluctuations of calculated corrections of tremor focus parameters. In extreme cases, fluctuations of these corrections may result in the situation, where, instead of coming closer to the searched tremor focus, we may wander away from it [Kornowski 1989],
- in the case of Nonlinear Problem of Least Squares, we have to cope with an Euclidean norm, or, another words, with norm L^2, which means that this method

is sensitive to remote observations, i.e. observations lying outside the main group of observations, and hence, it transfers the solution towards these observations (so called outliers). Norm L^1 is devoid of this flaw [Kijko 1993; Drzęźla 1995],
– the calculation of parameter t_0 is unnecessary since we are only interested in the location of tremor focus.

The last flaw can be easily eliminated. Matrix $\mathbf{A}^T\mathbf{A}$ may be written in the following form:

$$\mathbf{A}^T\mathbf{A} = \begin{bmatrix} \mathbf{W} & \mathbf{c} \\ \mathbf{c}^T & s \end{bmatrix} , \tag{2}$$

where vector $\mathbf{c}^T$ is defined as:

$$\mathbf{c}^T = \left(\sum_{j=1}^{s} \varphi_{j,x}, \sum_{j=1}^{s} \varphi_{j,y}, \sum_{j=1}^{s} \varphi_{j,z} \right) , \tag{3}$$

and superdiagonal terms in the symmetrical matrix $\mathbf{W}$ have the following values:

$$\begin{aligned} \mathbf{W}_{11} &= \sum_{j=1}^{s} \varphi_{j,x}^2 , \quad \mathbf{W}_{12} = \sum_{j=1}^{s} \varphi_{j,x}\varphi_{j,y} , \quad \mathbf{W}_{13} = \sum_{j=1}^{s} \varphi_{j,x}\varphi_{j,z} , \\ \mathbf{W}_{22} &= \sum_{j=1}^{s} \varphi_{j,y}^2 , \quad \mathbf{W}_{23} = \sum_{j=1}^{s} \varphi_{j,y}\varphi_{j,z} , \\ \mathbf{W}_{3,3} &= \sum_{j=1}^{s} \varphi_{j,z}^2 \end{aligned} \tag{4}$$

with

$$\begin{aligned} \varphi_{j,x} &= \frac{\delta(\mathbf{r}_j, \mathbf{r}_0, t_0)}{\delta x_0} , \quad \varphi_{j,y} = \frac{\delta(\mathbf{r}_j, \mathbf{r}_0, t_0)}{\delta y_0} , \\ \varphi_{j,z} &= \frac{\delta(\mathbf{r}_j, \mathbf{r}_0, t_0)}{\delta z_0} , \quad \varphi_{j,t} = \frac{\delta(\mathbf{r}_j, \mathbf{r}_0, t_0)}{\delta t_0} \end{aligned} \tag{5}$$

being the derivatives of function e_j expressed by the equation (1).

After denotation:

$$\begin{aligned} \mathbf{a}^T &= \left(\sum_{j=1}^{s} \varphi_{j,x}\left(\varphi_j - t_j^{\text{ex}}\right), \sum_{j=1}^{s} \varphi_{j,y}\left(\varphi_j - t_j^{\text{ex}}\right), \sum_{j=1}^{s} \varphi_{j,z}\left(\varphi_j - t_j^{\text{ex}}\right) \right) , \\ \alpha &= \sum_{j=1}^{s} \left(\varphi_j - t_j^{\text{ex}}\right) , \\ \varphi_j &= \phi(\mathbf{r}_j, \mathbf{r}_0) \end{aligned} \tag{6}$$

the product $\mathbf{A}^T\mathbf{b}$ may be written as:

$$\mathbf{A}^T\mathbf{b} = \begin{bmatrix} \mathbf{a} \\ \alpha \end{bmatrix} + t_0 \begin{bmatrix} \mathbf{c} \\ s \end{bmatrix} . \tag{7}$$

Taking into consideration formulas (2) and (7) in standard equation of Linear Problem of Least Squares, we obtain:

$$\begin{bmatrix} \mathbf{W} & \mathbf{c} \\ \mathbf{c}^T & s \end{bmatrix} \cdot \begin{bmatrix} \mathbf{h}_x \\ \Delta t \end{bmatrix} = \begin{bmatrix} \mathbf{a} \\ \alpha \end{bmatrix} + t_0 \begin{bmatrix} \mathbf{c} \\ s \end{bmatrix} , \tag{8}$$

where

$\mathbf{h}_x^T = (\Delta x, \Delta y, \Delta z)$ – vector of coordinates corrections of tremor hypocentre in a successive iterative step,

Δt – correction of time t_0 in a successive iterative step.

Traditional solution method of equations system (8) consists in accepting certain initial values of searched parameters and calculating resulting whereof corrections $\Delta x, \Delta y, \Delta z, \Delta t$. The obtained above form of equations system allows to calculate such value of t_0 so as the correction Δt had arbitrarily given value. It is convenient to select t_0 in such a way so as $\Delta t = 0$. The system of equations (8) is separated then into two parts:

$$\begin{aligned} \mathbf{W}\mathbf{h}_x &= \mathbf{a} + t_0\mathbf{c} \\ \mathbf{c}^T\mathbf{H}_x &= \alpha + t_0 s \end{aligned} \tag{9}$$

Calculating t_0 from the second equation of this group of expressions and taking into account the result from the second equation, we obtain a linear system of algebraic equations for the increments of searched coordinates of tremor hypocentre:

$$\left(\mathbf{W} - \frac{1}{s}\mathbf{R}\right)\mathbf{h}_x = \mathbf{a} - \mathbf{c}\frac{\alpha}{s} . \tag{10}$$

Superdiagonal terms of symmetrical matrix $\mathbf{R}$ are as follows:

$$\begin{aligned} &\mathbf{R}_{11} = \Big(\sum_{j=1}^{s} \varphi_{j,x}\Big)^2, \ \mathbf{R}_{12} = \Big(\sum_{j=1}^{s} \varphi_{j,x}\Big)\Big(\sum_{j=1}^{s} \varphi_{j,y}\Big), \ \mathbf{R}_{13} = \Big(\sum_{j=1}^{s} \varphi_{j,x}\Big)\Big(\sum_{j=1}^{s} \varphi_{j,z}\Big), \\ &\mathbf{R}_{22} = \Big(\sum_{j=1}^{s} \varphi_{j,y}\Big)^2, \qquad \mathbf{R}_{23} = \Big(\sum_{j=1}^{s} \varphi_{j,y}\Big)\Big(\sum_{j=1}^{s} \varphi_{j,z}\Big), \\ &\mathbf{R}_{33} = \Big(\sum_{j=1}^{s} \varphi_{j,z}\Big)^2 . \end{aligned} \tag{11}$$

Suggested method decreases the dimension of localization task of tremors and improves numerical conditioning of the problem.

Different method of t_0 elimination results from the condition on the minimum of squared sum of deviations e_j (see (1)) i.e.:

$$\sigma = \sum_{j=1}^{s} e_j^2 . \tag{12}$$

Then, the following is obtained from the equation $\delta\sigma/\delta t_0 = 0$ (one of necessary conditions for the existence of minimum of function (12)):

$$t_0 = -\frac{1}{s} \sum_{j=1}^{s} \left(\varphi_j - t_j^{\text{ex}}\right) . \tag{13}$$

Introducing this equation to (12) we obtain a modified expression devoid of parameter t_0:

$$\sigma = \sum_{j=1}^{s} \left(\varphi_j - t_j^{\text{ex}}\right)^2 - \frac{1}{s} \left[\sum_{j=1}^{s} \left(\varphi_j - t_j^{\text{ex}}\right)\right]^2 . \tag{14}$$

Such an elimination method of t_0 for the equations of differential tomography was presented by Garus (1990) in the problem of tomographic reconstruction of geophysical fields. The above method with regard to localization of mining tremors was discussed by Wanat (1992).

Earlier, t_0 was eliminated by Matsu'ura (1984) in the Bayesian estimation model, by introducing boundary probability of remaining parameters.

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