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A CERTAIN NEW METHOD FOR SEISMIC NETWORK OPTIMIZATION AND ITS CONSEQUENCES

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ABSTRACT. The study is a trial to find a new approach to the problem of seismic site optimum location. The essence of the method is based on the theory of implicit function and mathematical statistics. In the study the results obtained, to some degree, independently of the knowledge of the statistical distribution density function are used. Also some new aspects resulted from the presented approach have been enclosed.

1. INTRODUCTION

The widely used solution of the seismic network optimization problem based on the theory of experimental optimum planning was, for the first time, presented by A.Kijko in 1977. This approach based on the theory of probability and mathematical statistics applies the gaussian error distribution to the seismic wave first arrival times. The above assumption is, practically, the principle of the method that allows estimating the covariance matrix of the source x_0, y_0, z_0 coordinate axes and the tremor origin time t_0.

The purpose of the study is to derive theoretical evaluation of the variance of time t_0 and the source coordinates without making any assumptions regarding the distribution type. The general concepts of the theory of probability such as the expected value and variance are only dealt with here. The method described hereafter has been called by the author the explication method due to its connection with the theory of implicit functions. The application of arbitrary distribution types to the seismic wave onset time errors is an essential featured of this method, different from Kijko's approach which, as mentioned above, is based on the gaussian error distribution. Because of that, the explication method is a more generalized one.

Despite overall differences between the proposed and Kijko's approaches there is an interesting fact that at some points the two solutions are concurrent, which will be discussed hereafter. As the character of the study is analytical one all the well-known results from mathematical analysis and statistics, needed to understand the proposed solution, will be presented in the following sections.

2. Basic Assumptions

2.1. Generalized Variance Formula

Suppose, there is an arbitrary function

$$\mathbf{U} = \mathrm{f}(\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n) \tag{1}$$

with n random variables $\mathbf{X}_1, \ldots, \mathbf{X}_n$.

Also suppose, the following parameters are known:
– expected values of random variables $\mathbf{X}_1, \ldots, \mathbf{X}_n$

$$\mathbf{EX}_1 = \nu_{\mathbf{X}_1}\,,\ \mathbf{EX}_2 = \nu_{\mathbf{X}_2}\,,\ \ldots\,,\ \mathbf{EX}_n = \nu_{\mathbf{X}_n} \tag{2}$$

– variances of random variables $\mathbf{X}_1, \ldots, \mathbf{X}_n$

$$\mathbf{D}^2\mathbf{X}_1\,,\ \mathbf{D}^2\mathbf{X}_2\,,\ \ldots\,,\ \mathbf{D}^2\mathbf{X}_n \tag{3}$$

– covariances of random variables $\mathbf{X}_1, \ldots, \mathbf{X}_n$

$$\mathrm{cov}(\mathbf{X}_1, \mathbf{X}_2)\,,\ \mathrm{cov}(\mathbf{X}_1, \mathbf{X}_3)\,,\ \ldots\,,\ \mathrm{cov}(\mathbf{X}_{n-1}, \mathbf{X}_n) \tag{4}$$

Let us denote point $\mathbf{P}$ as

$$\mathbf{P} = \mathbf{P}(\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n) \tag{5}$$

We would like to determine the variance value of random variable U as a function of variables $\mathbf{X}_1, \ldots, \mathbf{X}_n$. Such a task is, in general, impossible to perform. However, in the case, when the $(\mathbf{X}_1, \ldots, \mathbf{X}_n)$ variable distribution probability masses are concentrated in a close neighbourhood of point $\mathbf{P}$, the common center, it may be found through the linearization that the variance of variable U is given by the following equation:

$$\begin{aligned} D^2U \approx \left(\frac{\partial f}{\partial X_1}\right)^2_{\mathbf{P}} D^2X_1 + \left(\frac{\partial f}{\partial X_2}\right)^2_{\mathbf{P}} D^2X_2 + \cdots + \\ + 2\left(\frac{\partial f}{\partial X_1}\right)_{\mathbf{P}} \left(\frac{\partial f}{\partial X_2}\right)_{\mathbf{P}} \mathrm{cov}(X_1, X_2) + \ldots \end{aligned} \tag{6}$$

2.2 Implicit Functions and Derivatives

Let us take the system of m equations with $n+m$ variables

$$\begin{aligned} &G_1(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_m) \equiv 0 \\ &G_2(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_m) \equiv 0 \\ &\ldots \\ &G_m(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_m) \equiv 0 \end{aligned} \tag{7}$$

This system, under certain assumptions, may be considered to be a system of m equations of variables $y_1, \ldots, y_m$ defined as implicit functions of n variables $x_1, \ldots, x_n$. Our purpose is to calculate the following partial derivatives:

$$\frac{\partial y_1}{\partial x_1}, \ldots, \frac{\partial y_m}{\partial x_1}, \ldots, \frac{\partial y_1}{\partial x_n}, \ldots, \frac{\partial y_m}{\partial x_n}. \tag{8}$$

On differentiating the above identities with respect to arbitrary variables (for instance x_1) we obtain:

$$\begin{gathered}\frac{\partial G_1}{\partial x_1} + \frac{\partial G_1}{\partial y_1}\frac{\partial y_1}{\partial x_1} + \cdots + \frac{\partial G_1}{\partial y_m}\frac{\partial y_m}{\partial x_1} \equiv 0 \\ \ldots \\ \frac{\partial G_m}{\partial x_1} + \frac{\partial G_m}{\partial y_1}\frac{\partial y_1}{\partial x_1} + \cdots + \frac{\partial G_m}{\partial y_m}\frac{\partial y_m}{\partial x_1} \equiv 0\end{gathered} \tag{9}$$

This is the system of linear equations of derivatives of y with respect to x which may be written in the matrix form:

$$\begin{bmatrix} \frac{\partial G_1}{\partial y_1} & \cdots & \frac{\partial G_1}{\partial y_m} \\ \cdots & \cdots & \cdots \\ \frac{\partial G_m}{\partial y_1} & \cdots & \frac{\partial G_m}{\partial y_m} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial y_1}{\partial x_1} \\ \cdots \\ \frac{\partial y_m}{\partial x_1} \end{bmatrix} = - \begin{bmatrix} \frac{\partial G_1}{\partial x_1} \\ \cdots \\ \frac{\partial G_m}{\partial x_1} \end{bmatrix} \tag{10}$$

On inverting the above system of linear equations we obtain the solution of the task problem.

3. LOCATION AND OPTIMIZATION

The location problem solution by means of iterative reconstruction methods using seismic wave first arrival times (P or S waves) involves finding a point of minimum of the the functional:

$$F(x_0, y_0, z_0, t_0) = \sum_{i=1}^{n} (t_i - q_i)^2 . \tag{11}$$

It should be noted that our purpose is to find a minimum point of the sum of squares or to find the minimum using the so called $\mathbb{L}^2$ norm. Denotations of equation (11) have the following meaning:

x_0, y_0, z_0 – source coordinates,
t_0 – tremor origin time,
t_i – seismic wave first arrival time,
$q_i = t_0 + \dfrac{d_i}{v_i}$ – theoretical seismic wave travel time between the source and receiver site No. i,
d_i – distance from source to receiver site No. i,

v_i – seismic wave velocity along the path joining source and receiver site No. i.

At point of extremum the following system of equations should be satisfied:

$$\begin{aligned}
\frac{\partial F}{\partial x_0} &= -2\sum_{i=1}^{n}(t_i-q_i)\frac{\partial q_i}{\partial x_0} = 0 = G_1(t_1,t_2,\ldots,t_n,v_i,t_0,x_0,y_0,z_0)\\
\frac{\partial F}{\partial y_0} &= -2\sum_{i=1}^{n}(t_i-q_i)\frac{\partial q_i}{\partial y_0} = 0 = G_2(t_1,t_2,\ldots,t_n,v_i,t_0,x_0,y_0,z_0)\\
\frac{\partial F}{\partial z_0} &= -2\sum_{i=1}^{n}(t_i-q_i)\frac{\partial q_i}{\partial z_0} = 0 = G_3(t_1,t_2,\ldots,t_n,v_i,t_0,x_0,y_0,z_0)\\
\frac{\partial F}{\partial t_0} &= -2\sum_{i=1}^{n}(t_i-q_i) = 0 = G_4(t_1,t_2,\ldots,t_n,v_i,t_0,x_0,y_0,z_0)
\end{aligned} \tag{12}$$

The above system of equations may be considered to be that determining the source coordinate x_0, y_0, z_0 and time t_0 as implicit functions of the variables $t_1,\ldots,t_n$ and v_i. Such an approach allows us to use the generalized variance formula (6) applied to the variables x_0, y_0, z_0 and time t_0 on condition that the following derivative values are known:

$$\frac{\partial x_0}{\partial t_i},\ \frac{\partial y_0}{\partial t_i},\ \frac{\partial z_0}{\partial t_i},\ \frac{\partial t_0}{\partial t_i},\ \frac{\partial x_0}{\partial v_i},\ \frac{\partial y_0}{\partial v_i},\ \frac{\partial z_0}{\partial v_i},\ \frac{\partial t_0}{\partial v_i}.$$

The latter quantities could be derived from the relationship (10).

To calculate derivatives $\frac{\partial x_0}{\partial t_i}, \frac{\partial y_0}{\partial t_i}, \frac{\partial z_0}{\partial t_i}, \frac{\partial t_0}{\partial t_i}$ the following equations should be solved:

$$\frac{\partial G_m}{\partial x_{0l}} = \frac{\partial^2 F}{\partial x_{0l}\partial x_{0m}} = -2\left\{\sum_{i=1}^{n}(t_i-q_i)\frac{\partial^2 q_i}{\partial x_{0l}\partial x_{0m}} - \sum_{i=1}^{n}\frac{\partial q_i}{\partial x_{0l}}\frac{\partial q_i}{\partial x_{0m}}\right\} \tag{13a}$$

and

$$\frac{\partial G_l}{\partial t_m} = \frac{\partial^2 F}{\partial x_{0l}\partial t_m} = -2\frac{\partial q_m}{\partial x_{0l}} \quad \text{for } m=1,\ldots,n \tag{13b}$$

$$\frac{\partial G_l}{\partial t_m} = \frac{\partial^2 F}{\partial x_{0l}\partial v_i} = -2\left[-\sum_{j=1}^{n}\frac{\partial q_j}{\partial x_{0l}}\frac{\partial q_j}{\partial v_i} + \sum_{j=1}^{n}(t_j-q_j)\frac{\partial^2 q_j}{\partial x_{0l}\partial v_i}\right] \tag{13c}$$

After indexing the denotations have the following meaning:
$x_{01} \equiv x_0 - x$ coordinate of the source,
$x_{02} \equiv y_0 - y$ coordinate of the source,
$x_{03} \equiv z_0 - z$ coordinate of the source,
$x_{04} \equiv t_0$ tremor origin time,
and
$t_m = t_i$ for $m = 1,\ldots,n$ – seismic wave onset time,

$t_m = v_i$ for $m = n+1, \dots, 2n$ – seismic wave propagation velocity along the path joining source and receiver locations.

And so we have obtained final solutions using values of the variance for the variables x_0, y_0, z_0 and t_0, which are the functions of site coordinates.

4. THE EXPLICATION METHOD ANALYSIS

During the investigation of the behaviour of the explication method the author has noted some regularities.

Firstly, the explication method will become identical with Kijko's method if and only if, we assume that the seismic wave first arrival time errors do not correlate and the variance values are equal. This can be analytically confirmed:

$$\mathbf{D}^2\mathbf{X}_1 = \mathbf{D}^2\mathbf{X}_2 = \cdots = \mathbf{D}^2\mathbf{X}_n \tag{14}$$

$$\operatorname{cov}(\mathbf{X}_1, \mathbf{X}_2) = \operatorname{cov}(\mathbf{X}_1, \mathbf{X}_3) = \cdots = \operatorname{cov}(\mathbf{X}_{n-1}, \mathbf{X}_n) = 0\,. \tag{15}$$

The computation results obtained by using such assumptions are shown, as an example, in Fig. 1.

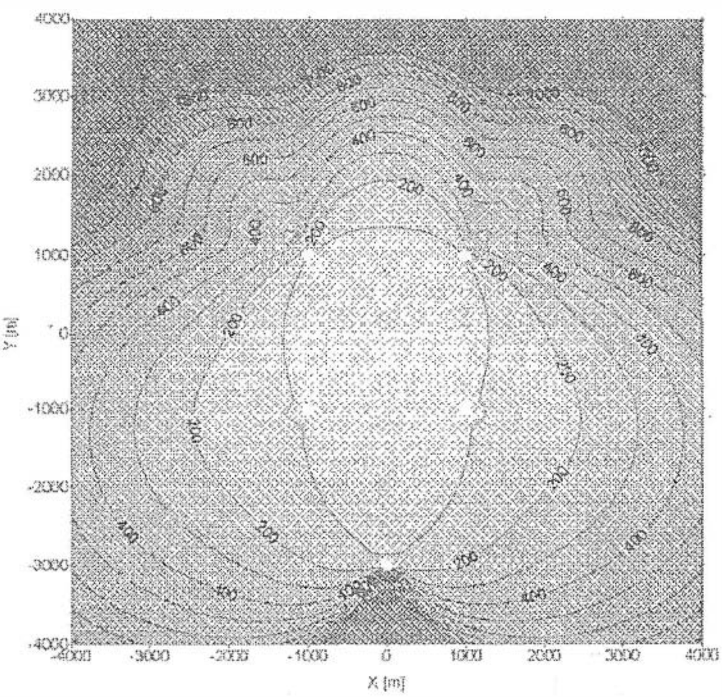

FIG. 1. Location errors map for the XY coordinates made on condition that the seismic wave first arrival time determination errors are committed at all sites. The standard deviation of timing error statistical distribution is everywhere equal to 15 msec. Covariances are equal to zero. Site locations are depicted by white points. This case corresponds to the solution derived by Kijko.

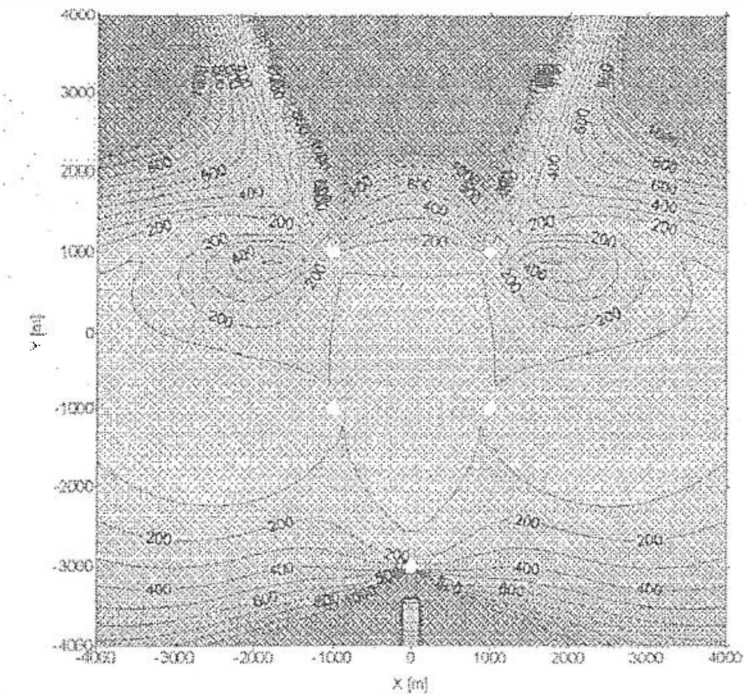

FIG. 2. Location errors map for the XY coordinates made on condition that the errors are committed in the velocity model. The standard deviation of velocity error statistical distribution is equal to 300 m/sec. Site locations are depicted by white points.

Secondly, the Kijko's method would be a complete failure if we hypothetically assumed that the seismic wave first arrival time determination errors were committed at one station (for instance, be it the first one) only, which would be tantamount to an assumption that the variances were equal to zero at all but that one station, and the covariances will equal to zero everywhere. The unreliability of the Kijko's method results from the necessity to calculate the inverse of the seismic wave onset time covariance matrix, which is non–existent in this case. however, the explication method is capable to perfectly manage the afore–mentioned problem yielding a solution that is part of the solution of the problem described by the sum (6). The numerical solution of such a problem is illustrated, as an example, in Figs. 3 and 4. The maps from Fig. 3 may be called the maps of the error amplification factor for a given station, because they show the spatial distribution of the location errors originated from that station. The total location error is the sum of the errors from individual stations (the variance of errors). This information could be very helpful in the location process. It allows selecting sites, which, despite the minor seismic wave first arrival time determination errors, can strongly amplify the location errors, and vice versa, the stable sites could be found, which despite, the major seismic wave first arrival time determination errors, can only slightly influence the location errors. And so the analysis of the location problem stability may entirely

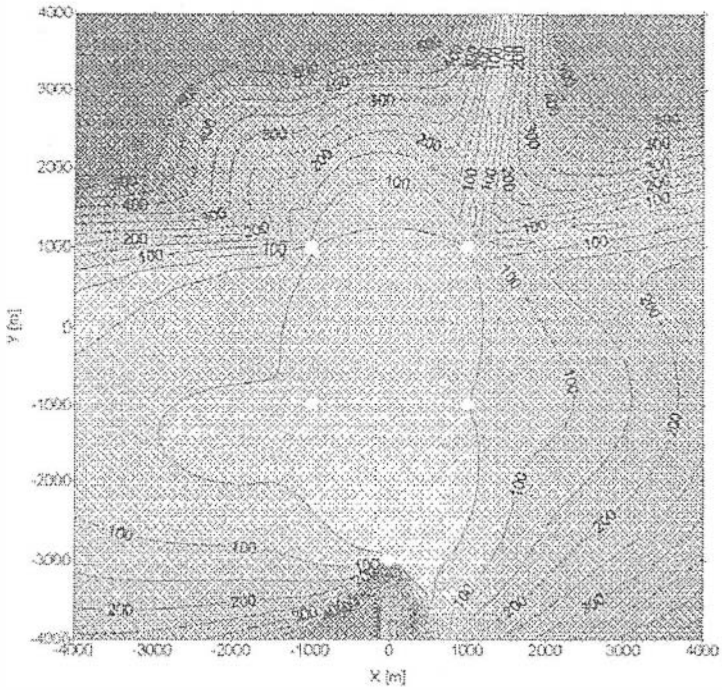

FIG. 3. Location errors map for the XY coordinates made on condition that the seismic wave first arrival time determination errors are only committed at a site of coordinates $X = 1000$, $Y = 1000$. The standard deviation of timing error is equal to 15 msec. Site locations are depicted by white points.

be defined in a different manner.

And thirdly, the explication method allows us to consider the velocity errors as an independent factor influencing the location errors during the computation of the functional minimum point (11). It should be noted that Kijko is trying to solve the problem by imposing a kind of artificial assumption stating that the velocity errors influence the time determination accuracy, whereas, factually, in the functional (11) we deal with an erroneous velocity model that itself causes additional location process perturbations. And so, in such a way the explication method is taking into consideration the velocity errors. An example of the velocity model error solution is shown in Fig. 2.

5. CONCLUSIONS

The discussed explication method is now the mostly used and generalized method allowing us to widely analyze tremor source location errors and to establish the seismic network optimum site location by solving the problem entirely in a different manner. At the same time, completely new concepts have emerged such as error maps for individual sites and the analysis of velocity errors as an independent factor perturbing the location directly in a minimized functional. Moreover the

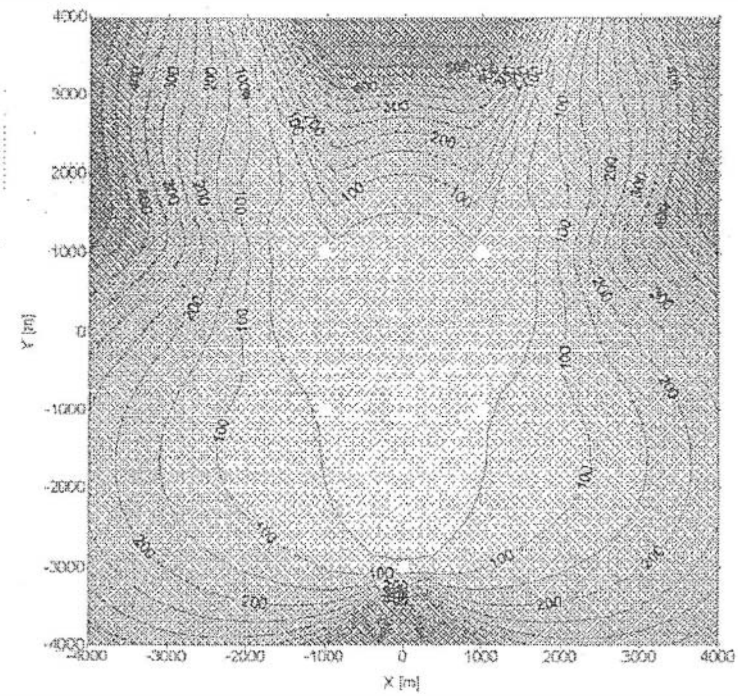

FIG. 4. Location errors map for the XY coordinates made on condition that the seismic wave first arrival time determination errors are only committed at a site of coordinates $X = 0$, $Y = -3000$. The standard deviation of timing error is equal to 15 msec. Site locations are depicted by white points.

whole method is based on the assumption that the error distribution at the seismic wave first arrival times is arbitrary. The author has noted some important features of the explication method and yet this does not rule out any possibility of the existence of the other featured missed by the author. The new studies in this field may allow us to better understand the practical application of the method.

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