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THE PRINCIPLES OF THE AMPLITUDE TOMOGRAPHY

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ABSTRACT. A study of the space distribution of rock–mass physical parameters is a goal of the seismic tomography. Besides well known travel time (velocity) tomography the amplitude tomography becomes more and more popular. It provides information about the distribution of the waves attenuation coefficient Q. In this paper, the foundations of the spectral approach to the amplitude tomography are described in the context of the mine applications.

1. INTRODUCTION

The investigations of a rock-mass state, existing stress fields and their temporal changes are very important for a mining practice and for better understanding of physical processes leading to seismic, induced by mining, events. The seismic tomography provides the spatial distribution of some physical parameters like the P and S wave velocities, the quality factor Q, etc. inside rock-mass. The obtained images interpreted according to empirical relations between rocks parameters and external loads, allow to estimate the stress fields in the rock-mass and their temporal variations.

The seismic tomography uses seismic waves coming from artificial (active tomography) or natural (passive tomography) sources to "probe" the structure of the medium. The waves propagating through the medium (see fig. 1) accumulate information about physical parameters of the passed region. Recorded on the surface of the region they are inverted for this information.

The amplitude tomography makes use of the effect of the attenuation of seismic waves due to the absorption of an energy by the medium which leads to a (additional to the geometrical spreading) frequency dependent vanishing of the amplitudes of seismic waves with the source-receiver distances. This measurable effect can, in principle, be used for an estimation of the spatial distribution of the attenuation factor Q but in practice it is not used due to a sensitivity of such a method to a noise. Instead of that, the effect of the frequency dependence of the energy absorption is utilized. Namely, the spectra of recorded seismic waves differ from the emitted (source) ones due to the frequency depending nature of the attenuation

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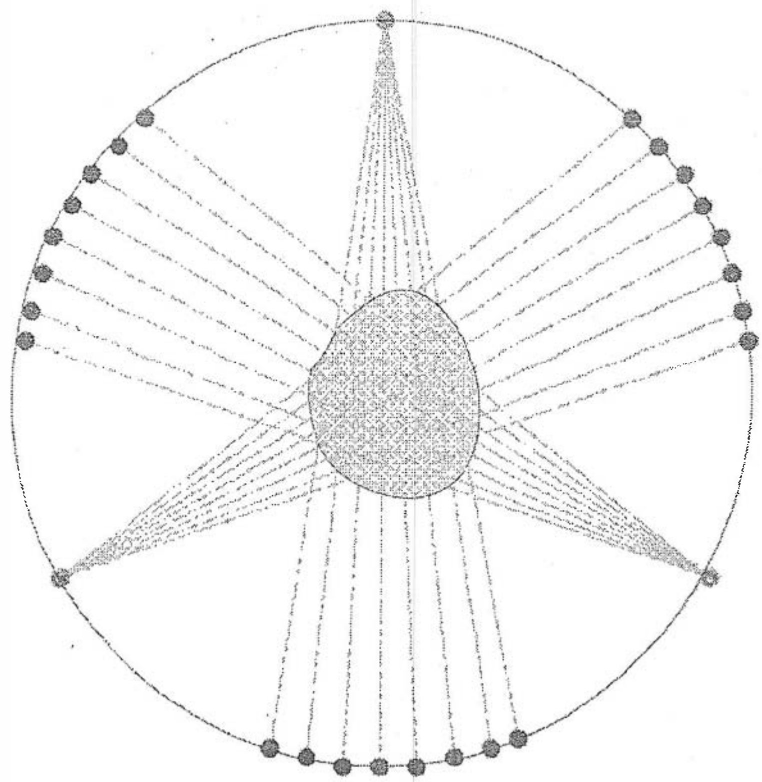

FIG. 1. The tomographic investigation consists in multiple "illuminating" of the explored region by means of seismic waves and recording of passed waves on its surface. The recorded seismic traces are inverted for the spatial distribution of studied parameters.

[Gibowicz, Kijko 1994; Aki, Richards 1987] (see fig. 2). For the linear absorbing medium, the relation between these two spectra is given by the formula

$$A(f,r) = A_s(f,r) \cdot \exp\left\{-f \int_R \frac{ds}{Q(s)v(s)}\right\}, \tag{1}$$

where $A_s(f,r)$ is a modified by a geometrical spreading spectrum of "source", Q is the quality factor and v is the velocity distribution. The integral is calculated along the path R of the wave propagation from the source to the receiver.

The method based on the analysis of the spectra of recorded seismograms we shall call the spectral approach to the amplitude tomography.

The main difficulty in an application of the amplitude tomography comes from the fact that amplitudes of the recorded waves depend on both the attenuation of the medium and an amount of the energy effectively emitted by the source. The spectrum of the source (term A_s in equ. (1)) is usually not known and, to make the matter worst, depends on an angle of the wave emission. So using of this method requires some additional assumptions. In case of the active tomography

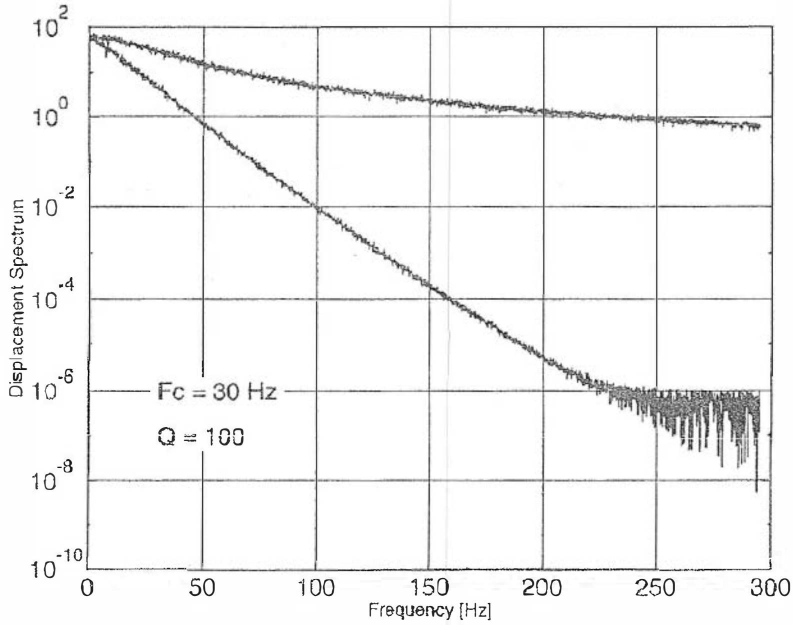

FIG. 2. The modeled spectrum of the recorded seismic waves for the source described by the Brune model. The upper curve corresponds to the perfect elastic medium (there is no attenuation) while the lower one has been obtained for $Q = 100$. The modeled spectrum includes 10% of the Gaussian, ($1/f^2$ type) noise and the white noise with a relative amplitude of order of 10^{-6}.

when sources are explosions, all sources are assumed to emit the same amount of the seismic energy isotropically. On this assumption, it is possible to use immediately the amplitudes of waves with a chosen frequency to obtain the image of Q distribution. This method, is however, seldom used due to its sensitivity to a noise in the data. The more robust approach [Evans, Zucca 1988] takes a spectrum recorded by one of the geophon (seismometer) as a reference one and uses the ratio

$$a_r = \frac{A(f)}{A_{ref}(f)} .$$

to get rid of the source depending term. This approach cannot be used in the case of the passive tomography when the emitted energy changes from case to case and is emitted significantly non-isotropically. This is a situation encountered in the mines environment. The most popular method used in such a situation, called the spectral ratio decay technic [Sanders 1993], is founded on the assumption that the high frequency part of the displacement source spectrum corrected for the geometrical spreading behaves like

$$A_s(f) \sim f^{-\gamma} \qquad \text{for} \quad f > f_c ,$$

where f_c is the corner frequency and the slope coefficient $\gamma \approx 2$ (e.g. for Brune model [Brune 1970] $\gamma = 2$). Then for $f > f_c$ the spectrum recorded by the i^{th} receiver can be approximated by

$$\ln A^i(f) = \text{const.} - \gamma \ln(f) - d^i f\,, \tag{2}$$

where the coefficient d^i

$$d^i(Q;f) = \int_{R^i} \frac{ds}{Q(s)\,v(s)} \tag{3}$$

does not depend on the source parameters but only on the distribution of Q and a wave propagation path[1]. The coefficients d^i occur to be the convenient, measurable parameters that can be inverted for Q and in following they are referred to as the measured *data*. In this approach th
on the accuracy of retrieving of d^i coefficients from the data. It should be kept in mind that the coefficients d^i depend on the frequency due to the frequency dependence of Q factor [Dębski 1996]. This fact can be disregarded when only rough tomography inversion is performed but has to be taken into account for more detailed analysis.

2. Tomography as the Inverse Problem

The task of construction of the spatial distribution of Q from the measured data is a typical inversion problem [Tarantola 1987; Dębski]. The solution of this problem requires experimental data as well as theoretical information on a solution of, so called, forward problem, i.e. ability of a theoretical modelling of d^i for a given Q distribution. The equations (2), (3) solve the forward problem. Let us notice that the relation among d^i and Q in equ. (3) is nonlinear. However, using instead of Q its reciprocity

$$m(x,y,z) = \frac{1}{Q(x,y,z;f)}\,, \tag{4}$$

we come to the linear form of the forward problem

$$d^i = \int_{R^i} m(x(s))\frac{ds}{v(x(s))}\,. \tag{5}$$

The above linear relation can be symbolically written as

$$\mathbf{d} = \mathbf{G}\cdot\mathbf{m}\,,$$

where the linear operator $\mathbf{G}$ describes the integration procedure.

The most general solution of any inverse problem (so also the amplitude tomography problem) consists in a defining an *a posteriori* probability density over the space of all possible models $\mathbf{m}$. It can be shown [Tarantola 1987] that for the

[1] In following we shall assume that the velocity distribution v is known e.g. from earlier performed travel time tomography.

Gaussian measurement and *a priori* uncertainties the required probability density reads

$$\sigma(\mathbf{m}) = e^{-S(\delta\mathbf{m})},$$

where the function $S(\mathbf{m})$ is given by

$$\begin{aligned} S(\mathbf{m}) =& (\mathbf{G}\cdot\mathbf{m}-\mathbf{d}_{obs})^T\mathbf{C}_D^{-1}(\mathbf{G}\cdot\mathbf{m}-\mathbf{d}_{obs}) \\ &+(\mathbf{m}_{apr}-\mathbf{m})^T\mathbf{C}_M^{-1}(\mathbf{m}_{apr}-\mathbf{m}) \end{aligned} \tag{6}$$

and $\mathbf{C}_D$, $\mathbf{C}_M$ are the covariance operators describing data errors and uncertainties of the choice of the *a priori* model $\mathbf{m}_{apr}$ respectively.

The above formula together with equ. (4) and (5) provides the theoretical solution of the discussed amplitude tomography problem in the framework of the made assumptions. However the explicit calculation of $S(\mathbf{m})$ is a very complicated numerical task due to the size of the problem. In real situations when the observational data and parameters m^i number in the thousands the only characteristic of the *a posteriori* probability density that can be calculated is the model $\mathbf{m}^{ML}$ for which $\sigma(\mathbf{m})$ reaches its maximum. This model, called the maximum likelihood solution, minimizes the sum of the squares of differences between predicted (modeled) and observed data. The numerical searching of $\mathbf{m}^{ML}$ requires discretization of the distribution $\mathbf{m}(\mathbf{x})$ which is achieved by the division of the spaces into blocks and assigning to each of them constant values of m^i. Thus, the spatial distribution $\mathbf{m}(\mathbf{x})$ is replaced by the vector $\{m^i\} = \{m^1, m^2, \dots\}$ and the operators $\mathbf{G}$, $\mathbf{C}_D$ and $\mathbf{C}_M$ become matrices.

3. THE *Steepest Descent* ALGORITHM

There exist many ways of finding of the likelihood model $\mathbf{m}^{ML}$ based on various optimization algorithms. One of the most popular approaches is the method founded on the preconditioning steepest descent algorithm. To find the maximum of $\sigma(\mathbf{m})$ according to this method the following steps are iterated

- The *a priori* model is taken as the initial one

$$\mathbf{m}_0 = \mathbf{m}_{apr}$$

- For the given model $\mathbf{m}_n$ the gradient of $S(\mathbf{m})$ is calculated

$$\gamma_n = \mathbf{C}_M\mathbf{G}^T\mathbf{C}_D^{-1}(\mathbf{G}\cdot\mathbf{m}_n-\mathbf{d}_{obs})+(\mathbf{m}_n-\mathbf{m}_{apr})$$

- The gradient γ_n is preconditioned by the operator being an approximation of the operator of the second derivatives of $S(\mathbf{m})$

$$\phi_n = \hat{\mathbf{H}}_o\cdot\gamma_n$$

  where

$$\hat{\mathbf{H}}_o \approx (\mathbf{I}+\mathbf{C}_M\mathbf{G}^T\mathbf{C}_D^{-1}\mathbf{G})^{-1}$$

- The model $\mathbf{m}_n$ is updated

$$\mathbf{m}_{n+1} = \mathbf{m}_n - \mu_n \phi_n$$

where μ_n reads

$$\mu_n = \frac{\gamma_n^T \mathbf{C}_M^{-1} \phi_n}{\phi_n^T \mathbf{C}_m^{-1} \phi_n + (\mathbf{G}\phi_n)^T \mathbf{C}_D^{-1} (\mathbf{G}\phi_n)}$$

- The iterative loop is repeated until some termination criterion is attained.

The most important elements of this algorithm is calculation of the gradient γ. It is calculated in the following steps

1. For a given model $\mathbf{m}_n$ the forward problem is solved and the residua between calculated and observed data are calculated

$$\Delta d_n^i = \sum_j m_n^j \xi^{ij} - d_{obs}^i$$

where

$$\xi^{ij} = \begin{cases} \frac{\Delta s^j}{v^j} & \text{if } i^{th} \text{ ray intersects } j^{th} \text{ block} \\ 0 & \text{in other cases} \end{cases}$$

Δs^j is a length of the ray path inside j^{th} block and v^j is a velocity assigned to j^{th} block.

2. The residua are weighted

$$\Delta \hat{d}_n^i = \sum_j (\mathbf{C}_d^{-1})^{ij} \Delta d_n^j .$$

3. Next they are back-projected to the model space and spread out around the ray path

$$\Delta m_n^j = \sum_i \Delta \hat{d}_n^i \Psi^{ij}$$

where

$$\Psi^{ij} = \sum_k \xi^{ik} \mathbf{C}_M^{kj} .$$

4. Finally the *a priori* information is added

$$\gamma_n^j = \Delta m_n^j + (m_n^j - m_{apr}^j) ,$$

which ends the calculation of the gradient γ.

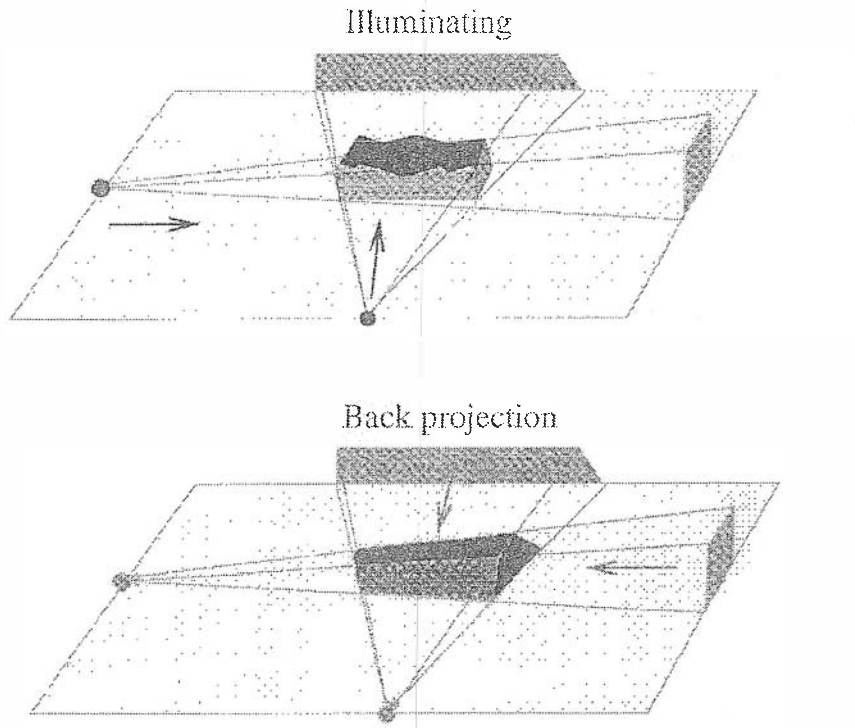

FIG. 3. Illuminations of the study object by the seismic waves (upper part) produce the projections (modelling data). The back-projection procedure (lower part) takes these data and transport along the ray path in the opposite sense (from receivers to sources). The intersection of so transported data forms the object which is an approximation of the real object.

Two steps of the above mentioned algorithm require some explanations. The first one is the procedure of the back projection.

The solution of the forward problem consists in "illuminating" of the studied object by seismic waves and recording obtained projection images. The back projection operation takes as an input the above projections (or more precisely, the difference between observed and modelled projections) and transports them along the wave path from receivers toward the source (see fig. 3). In this operation the residua are inverted into disturbances of the parameters m^i of blocks around the ray path. The matrix Ψ^{ij} describes how many blocks around the ray are disturbed. If the covariance matrix $\mathbf{C}_M$ is diagonal

$$\mathbf{C}_M^{ij} \sim \delta^{ij}$$

then only blocks through which the ray is passing give nonzero contributions to the gradient matrix γ

The second comments concerns the operation of the gradient preconditioning. The gradient vector γ points the local direction of the steepest descent. In case of

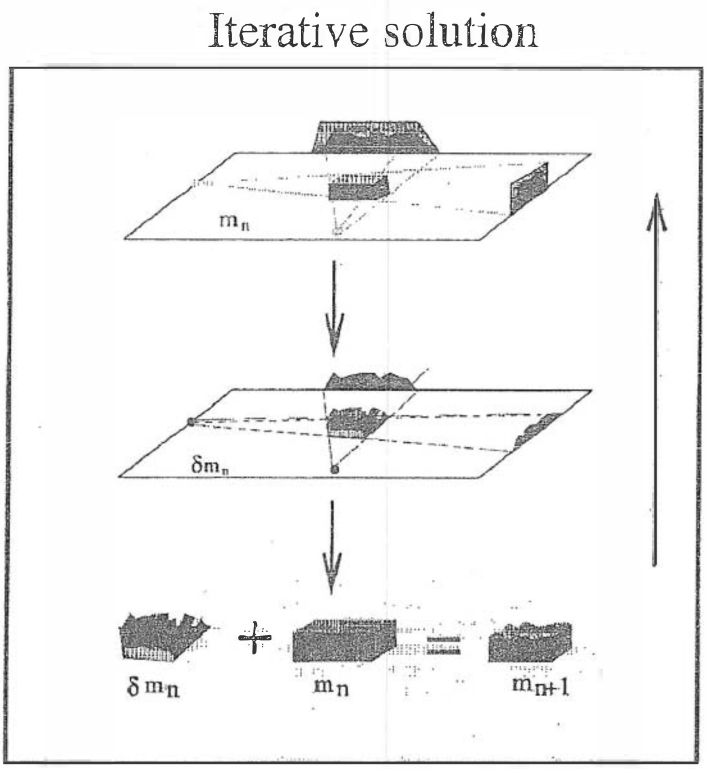

FIG. 4. The basic elements of the preconditioned steepest descent algorithm: modelling, calculation of residua and preconditioning (upper part), back propagation (middle part) and model updating (lower part).

linear forward problems it is the direction towards the global minimum of $S(\mathbf{m})$. The goal of the preconditioning is to modify a "length" (and a direction in case of non-linear forward problems) of γ in order to put its "end" to the real minimum as close as possible. The choice of the preconditioning operator is crucial for the algorithm convergence. In the simplest case taking $\hat{\mathbf{H}} = \mathbf{I}$, a very slow convergence and large oscillations around minimum can be expected. On the other hand the choice

$$\hat{\mathbf{H}}_0 = (\mathbf{I} + \mathbf{C}_M \mathbf{G}^T \mathbf{C}_D^{-1} \mathbf{G})^{-1}$$

causes that the minimum is reached in a single iteration step.

The figure 4. shows the most important steps of the described algorithm.

4. Summary

The amplitude tomography provides information about the state of the rock-mass complementary to that supplied by the velocity tomography. Its application requires either full control of the wave sources or additional assumption concerning spectra of the emitted waves. Due to this the method is not as much popular as the velocity tomography.

Construction of the tomografic image of the attenuation factor is the inverse problem and its most general solution relies on assigning of a given probability to any possible distribution of the Q factor. This *a posteriori* probability expresses our confidence that a given distribution is the real one. In most cases such a probability cannot be found explicit for a computational reason. Then only the likelihood distribution is calculated by means of iterative optimization methods. Let us point out that the iterative, gradient optimization methods suffers from the well known drawback: the found solution can correspond to some local maximum of the *a posteriori* probability density, and not necessarily to the global one. However the tomography problems are so large that using global optimization methods (Simulating Annealing, Neural Network, Genetic Algorithms, etc.) for solving the real problems is still not possible.

The other point is that the application of the amplitude tomography in the mine environment requires using waves of relatively large frequencies to resolve the small heterogeneity in the distribution of Q. Very often this condition cannot be fulfilled for the passive tomography. In such a situation the spatial distributions of rock-mass parameters can still be obtained but then using the more general waveform inversion method based on the full solution of the wave equation [Tarantola 1988] is required.

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