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DETERMINATION OF REPRESENTATION ERRORS IN SEISMIC TOMOGRAPHY

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ABSTRACT. Representation errors in the revers problem of the straight–line and curvilinear seismic tomography have been analyzed based on the assumed arrival time errors of given wave groups. For the analysis a hypothetical medium with a clear low velocity anomaly was used. A modified algebraic reconstruction algorithm, the so called SIRT algorithm which takes into account a curvilinear seismic ray propagation, was verified. The test was done by disturbing wave front arrival times while the process was assumed stochastic.

1. Introduction

The seismic tomography method, which as being widely applied in practice for its capability of providing a lot of information, can especially be of use for studying the inhomogeneity of the rock mass environment. However, this method requires high quality measurement data and the appropriately performed interpretation able to precisely distinguish the wave groups. It is of high importance to know how big errors in the determination of arrival times of particular wave trains might be committed so that the task of seismic tomography should still yield correct solutions.

2. Formulation of the Problem

There are the following kinds of tomographic algorithms:

- based on Fourier transform;
- based on Radon transformation;
- generalized inverse of matrix;
- algebraic reconstruction.

In geophysics the algorithms based on computational iterative process, the so called algebraic reconstruction, are now in common use. In the study the action of a modified algebraic reconstruction algorithm – the simultaneous iterative reconstruction technique SIRT – has been tested. The method consists in dividing the area under study into the constant velocity rectangular mesh elements with the corrections included after a certain number of iterations had been completed for each mesh element. The general formula for the corrections can be written as follows:

$$\triangle p_{ij} = \frac{\triangle t_i d_{ij}}{N_p \sum\limits_{k=1}^{M} d_{ik}^2}, \tag{1}$$

where:
$\triangle p_{ij}$ – part of a correction for the jth mesh element derived from the ith ray,
$\triangle t_i$ – difference between the time computed for a given iteration and the measured time,
d_{ij} – path of the ith ray propagating through the jth mesh element,
d_{ik} – path of the ith ray propagating through the kth mesh element,
N_p – number of rays propagating through the jth mesh element,
M – number of mesh elements,
N – number of the measured seismic wave first arrival times,
$1 \leq i \leq N$, $1 \leq j \leq M$.

The total correction of an iteration series is as follow:

$$\triangle p_j = \sum_{i=1}^{N} \triangle p_{ij}, \tag{2}$$

where:
N – number of the measured seismic wave first arrival times,
$\triangle p_{ij}$ – corrections of individual rays.

The two following approaches have been applied to the above algorithm: straight-ray model and curved–ray model for the seismic wave propagation. The curved–ray model is based on Huygen's principle, according to which the wavefront propagation is computed first and the seismic ray path between the excitation and reception points is computed later.

The study of the error effects is based on the theories of Monte Carlo and analysis of experimental disturbance methods.

The computations were performed in the following way:

(1) Determining the velocity model of a medium.
(2) Determining the measurement configuration ensuring that the excitation and reception points be arranged so that the area under study should be fully covered (every 20 m from all sides).
(3) Determining the theoretical wave travel times at the excitation point – reception point distance with taking ray path curvature into account.
(4) The disturbance was done by using pseudorandom numbers of theoretical seismic wave first

It has been assumed that the "timing" process is the random process with the probability density function written in the form:

$$p_t(d) = \frac{1}{cd\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left[\frac{t-m}{cd}\right]^2\right), \tag{3}$$

where:
d – excitation point-reception point distance,
c – constant,
$\sigma = cd$,
$m = E(t)$.

In such a process the standard deviation density comes into prominence. In the next disturbance series the above quantity obtained the following values: 0.25 msec/100 m, 0.5 msec/100 m, 1 msec/100 m, 2 msec/100 m.

To generate the Gaussian distribution pseudorandom numbers the following Box–Muller transformation has been used:

$$\begin{aligned} N_1 &= \sqrt{-2.0 \ln x} \cos(2\pi y) \\ N_2 &= \sqrt{-2.0 \ln x} \sin(2\pi y) \end{aligned} \tag{4}$$

where:
N_1, N_2 are the Gaussian distribution random numbers,
x, y are the constant distribution random numbers from interval $[0, 1)$.

3. RESULTS OF MODELLING

The two phase medium shown in Fig. 1 has been chosen as a computational model. An anomalous region characterized by a velocity of 3000 m/s was located inside the area in the form of a square with the side equal to 200 m and the elastic wave propagation velocity of 4000 m/s. The region occupies 36 % of the model area. At each side of the area hypothetical vibration sources and receivers were placed every 10 m. The reconstruction of the velocity was performed after a grid had been imposed on the model dividing it into squares with the side equal to 10 m. As a measure of accuracy of the obtained reconstruction the velocity mean quadratic error has been assumed.

TABLE 1. Summary of the results of modelling

time error ms/100 m	0		0.25		0.5		1		2	
reconstruction	p	k	p	k	p	k	p	k	p	k
RMS, m/s	320	270	333	276	386	339	582	543	762	683
RMS, %	10.6	9.0	11.1	9.2	12.8	11.3	19.4	18.1	25	22.7

p – straight-ray approximation,
k – curved-ray approximation.

The case of inversion problem solution without time perturbation was considered as a test for the algorithm uniqueness and the lower bound of reconstruction uncertainty estimation. The velocity mean errors obtained for the straight-ray approximation and for the curved-ray approximation are 320 m/s and 270 m/s, respectively. The error range depends on a division of the medium into rectangular

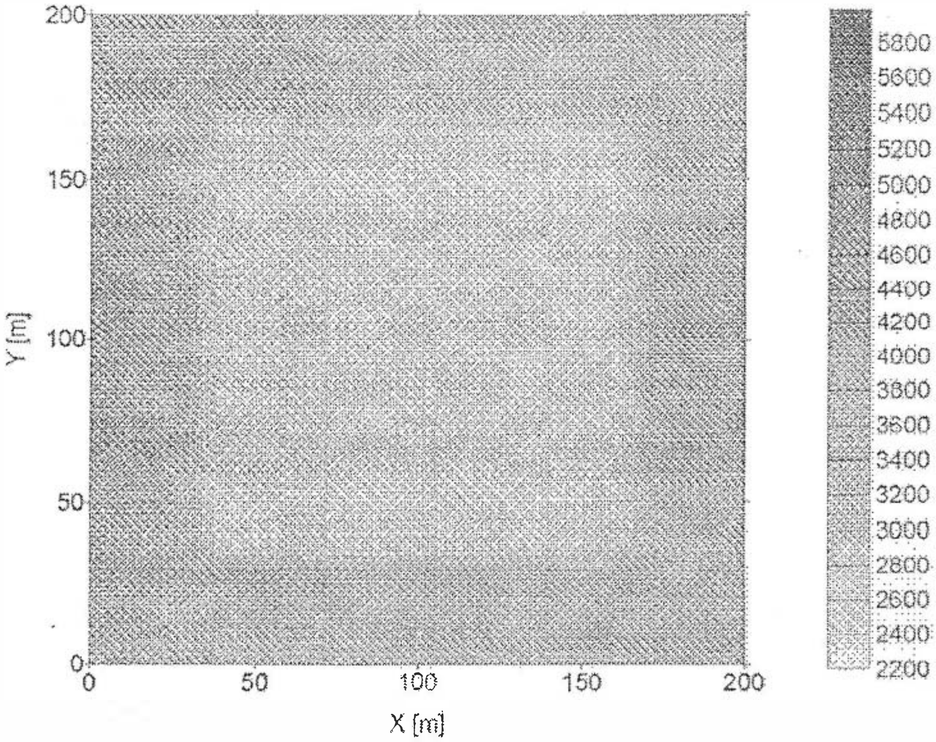

FIG. 1. Velocity model of the medium

pixels and a number of performed iterations. With the accepted assumptions the velocity field reconstruction errors for the straight–ray and curved–ray calculations are 10.6 % and 9.0 %, respectively. The extent and shape of the anomalous zone have been reconstructed better with the curved–ray approach then with the straight–ray approach, which is intuitively consistent. The error range of 10 % results from the assumed instrumental configuration (infinitely continuous measurement is impossible) and division of the medium. However, this is realistic for practical reasons. The applied observation schemes fit the assumed one best with the poor seismic ray coverage obtained. The seismic wave travel times perturbation makes the obtained reconstruction quality worse. The purpose of the augmentation of the time errors standard deviation density medium value from 0.25 ms/100 m to 2 ms/ 100m was to determine the error acceptable quantity (according to the observer's view point) and to correlate it with the real errors occurring in the interpretation process. An error on the order of 0.25 ms/100 m may produce a negligible increase in velocity field reconstruction error, i.e., up to 11.1 % for the straight–ray calculation and 9.2 % for the curved–ray calculation. The anomalous zone geometry has been perturbed insignificantly. Much larger changes in reconstruction quality occur for errors on the order of 0.5 ms/100 m. Large differences between calculated and measured velocity values (on the order of 30 %) may occur. The reconstruction errors are: 12.8 % for the straight–ray calculation and 11.3 % for the curved–ray calculation. A significant deterioration of the obtained reconstruction quality comes after the time error has

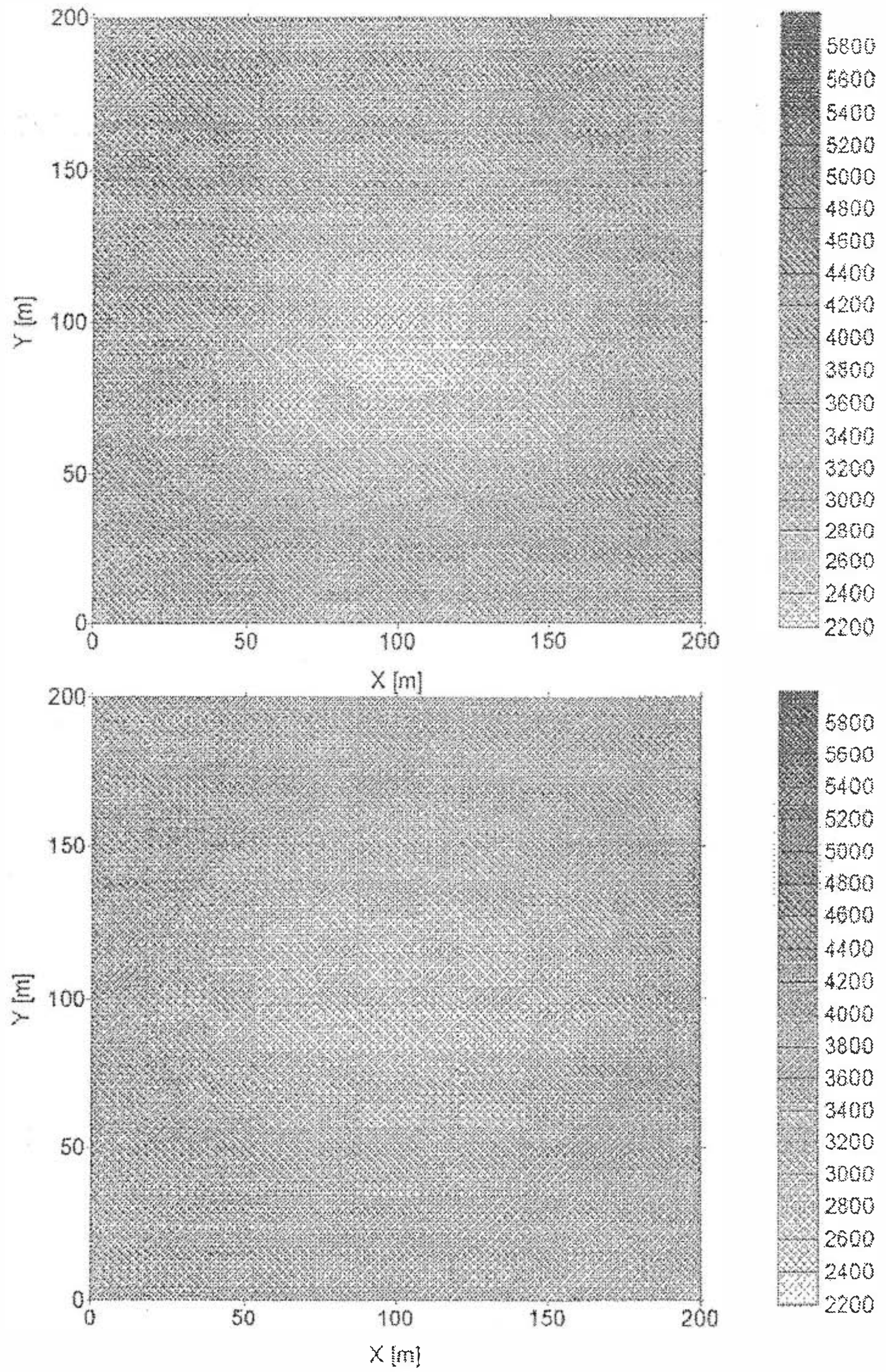

FIG. 2. Maps of straight–ray and curved–ray tomographic representation without time disturbance

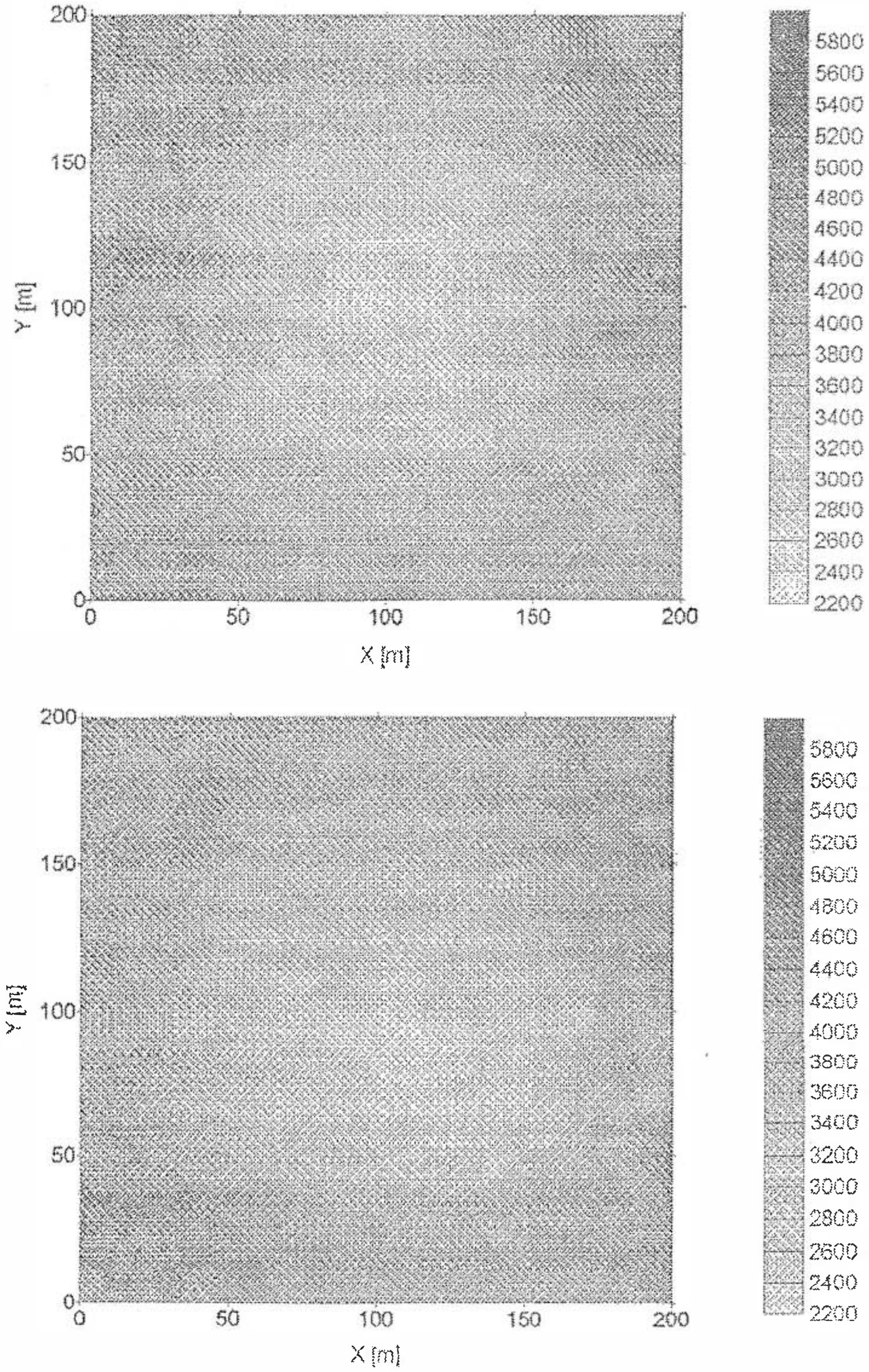

FIG. 3. Maps of straight–ray and curved–ray tomographic representation with time disturbance 0.25 msec/100 m

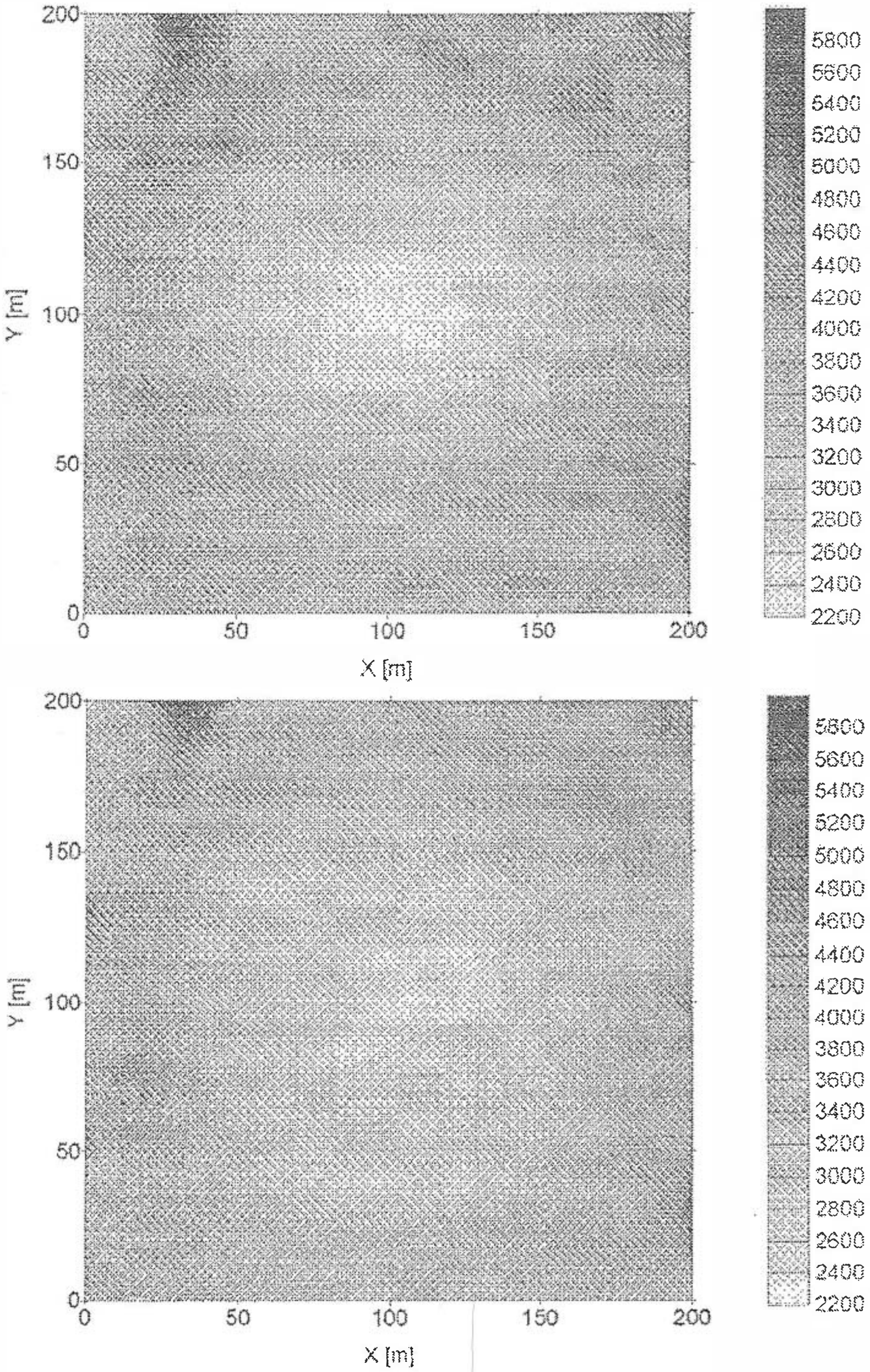

FIG. 4. Maps of straight–ray and curved–ray tomographic representation with time disturbance 0.5 msec/100 m

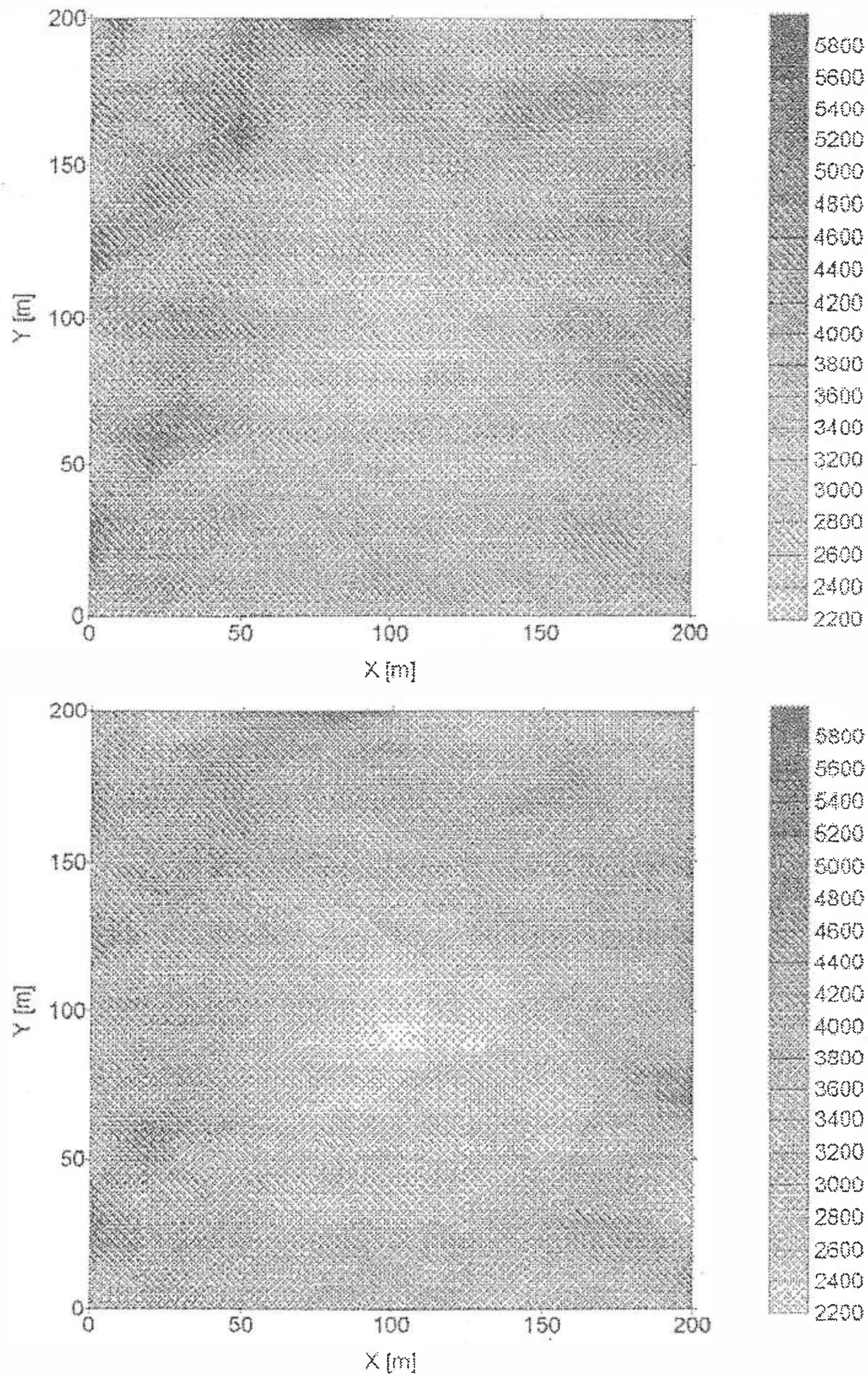

FIG. 5. Maps of straight–ray and curved–ray tomographic representation with time disturbance 1 msec/100 m

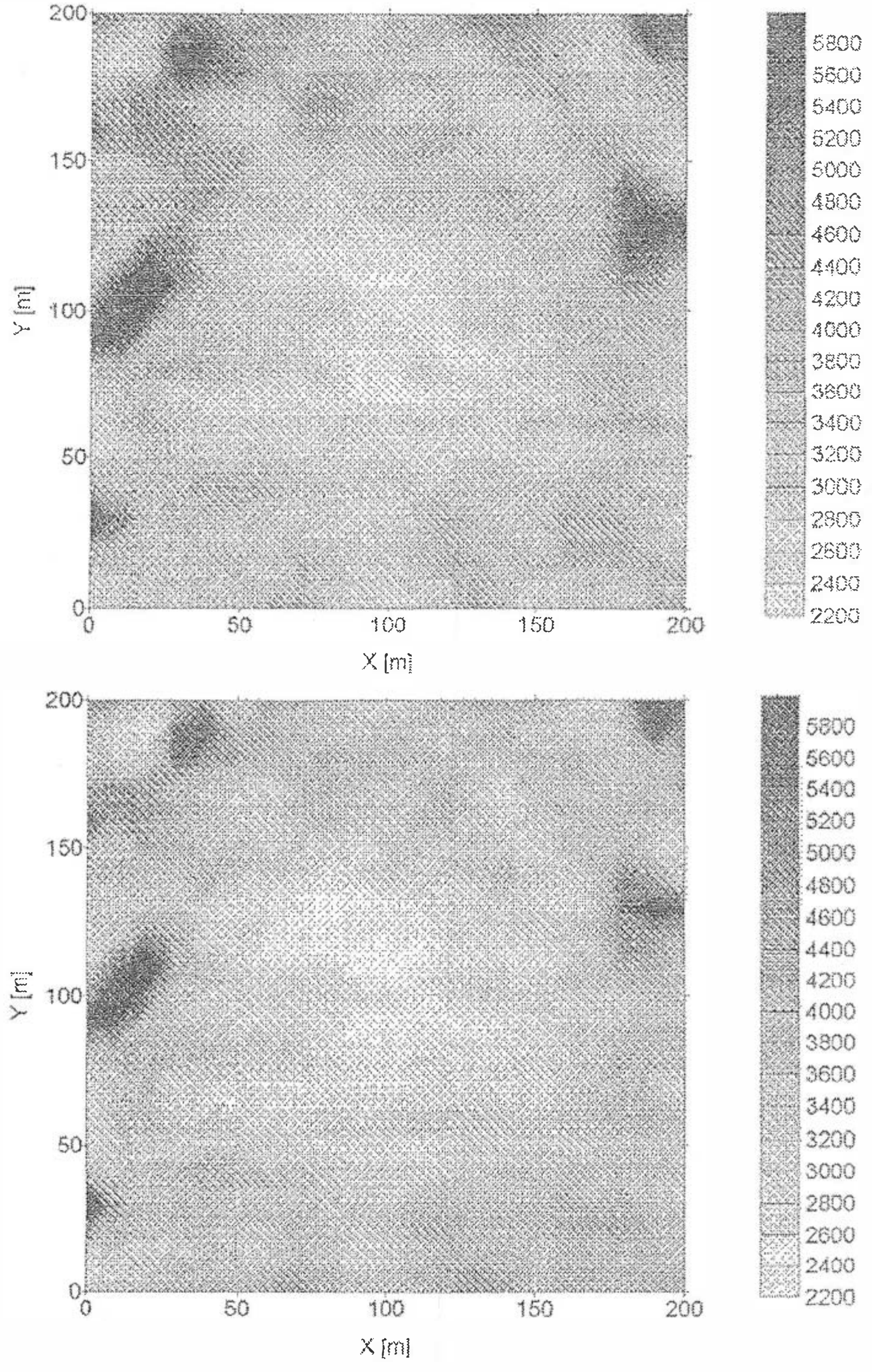

FIG. 6. Maps of straight–ray and curved–ray tomographic representation with time disturbance 2 msec/100 m

been increased up to 1 ms/100 m. the velocity errors rose to 19.4 % for the straight–ray calculation and to 18.1 % for the curved–ray calculation. The low velocity zone geometry has become false and a number of additional anomalies appeared. With errors as high as 2 ms/100 m the obtained image is unfit for interpretation. The reconstruction errors exceed 20 % and the location of anomalous regions may, in places, be close to the random one.

For all the studied quantities of perturbation more accuracy in velocity field reconstruction can be obtained by using curved–ray tomography. For the assumed error model of seismic wave first arrival times the critical value (from the interpretation point of view) of the standard deviation density is 0.5 ms/100 m. An approximate resolution is still possible for the errors as high as 1 ms/100 m. For the higher errors the reconstruction will be pronounced unfit.

4. Conclusions

1. From the study on velocity model described above it follows that the results of using curved–ray tomography appear to be better than the result of using straight–ray tomography. The variation in velocity mean quadratic error is small; it varies from 1.5 % to 2.3 %.

2. Perturbing the synthetic times will, surely, reduce the accuracy of obtained reconstruction. For the assumed error model a correct resolution of heterogeneities is possible with errors as high as 0.5 ms/100 m. For an error of 1 ms/100 m an approximate resolution could only be possible.

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