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NEW COMPUTING METHOD OF WIND WAVE PARAMETERS

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Abstract. The study deals with problems of generalization of measurement characteristics of the wave regime on water reservoirs and with the calculation of wind wave parameters. It compares the generalization results according to Buckingham's π–theorem with those according to Reynolds' number (Kratochvíl, 1982).

The solution of the motion equation of the progressive wave results in the conclusion that the hitherto used dimensionless arguments gh/w^2 and $gT/(2\pi w)$ can be replaced by a single one, the quotient c/w, where c is the velocity of progressive wave motion and w is the velocity of wind.

1. Introduction

In order to evaluate the resistance of dams and banks of water reservoirs against the wave motion effect, it is particularly important to determine the energy of this wave motion acting on 1 m of the bank line. The energy of waves is determined in (Lukáč and Abaffy, 1980) by the height of waves, their length, the period and duration of their activity towards embankment. These wave parameters depend, above all, on the wind velocity and the length of its track. The Table 1 quotes 12 factors, which certainly affect the character of the wave motion. The effect on other, namely morphological factors, is not considered, because these cannot be reliably quantified. Usually the first six factors, concentrated into three dimensionless parameters are considered, as they were determined according to Buckingham's theorem (Kratochvíl, 1982):

$$\pi_1 = \frac{gD}{w^2}\,, \qquad \pi_2 = \frac{gh}{w^2}\,, \qquad \pi_3 = \frac{gT}{2\pi w} \tag{1}$$

In so doing, it is not certain, whether this choice is the only suitable one, whether, for example, the density or viscosity have not the same or even a higher effect than the acceleration due to gravity. We could consider these quantities as well, however at the price that we should have five dimensionless parameters instead of three. The number of dimensionless parameters is namely by at most three of them lower

TABLE 1. Factors affecting the wind wave parameters

Nr.	designation	symbol	unit	dimension
1	Wave height	h	m	L
2	Starting length of wind	D	m	L
3	Wind velocity	w	ms^{-1}	LT^{-1}
4	Acceleration of gravity	g	ms^{-2}	LT^{-2}
5	Progressive velocity of wave	c	ms^{-1}	LT^{-1}
6	Wave period	T	s	T
7	Water density	ρ	kgm^{-3}	ML^{-3}
8	Water viscosity	μ	$kgs^{-1}m^{-1}$	$MT^{-1}L^{-1}$
9	Air viscosity	μ_0	$kgs^{-1}m^{-1}$	$MT^{-1}L^{-1}$
10	Air density	ρ_0	kgm^{-3}	ML^{-3}
11	Perimeter of banks	o	m	L
12	Reservoir depth	H	m	L

than the number of considered factors, provided these factors involve all three basic dimensions, i.e. the length L, mass m and time T – see Table 1.

As a possible alternative, the choice of the following four quantities from Table 1 proved suitable, namely D, w, ρ_0, μ_0. These can be expressed by means of a single dimensionless parameter

$$\pi_1 = Dw^x \rho_0^y \mu_0^z \,. \tag{2}$$

When substituting, into the equation (2), dimensions of individual quantities from Table 1, then, to be dimensionless, it must hold $\pi_1 = L^0T^0M^0 = 1$ and thus

$$L^0T^0M^0 = (L)(LT^{-1})^x(ML^{-3})^y(MT^{-1}L^{-1})^z$$

From this expression, $x = 1$, $y = 1$, $z = -1$, are determined; detailed explanation in (Kratochvíl, 1982; Lukáč and Abaffy, 1980). By substituting into equation (2), $\pi_1 = Dw\rho_0\mu_0^{-1}$ is obtained. The quotient $\mu_0/\rho_0 = \nu_0$ is kinematic viscosity of the air, and the parameter $\pi_1 = Dw/\nu_0$. This is the Reynolds' number, characterizing the flow of any Newtonian fluid. Let us use, for the air, the designation $\mathrm{Re}(w) = Dw/\nu_0$, for water $\mathrm{Re}(h) = hc/\nu$, and $\mathrm{Re}(\lambda) = \lambda c/\nu$, since both the height h and length λ may be used as length characteristics of the wave. The Fig. 1 illustrates relationships generalized according to the π–theorem, Fig. 2 those according to Reynolds' numbers. Data in Table 2 are quoted from the literature (Lukáč and Abaffy, 1980) except the last one (eight line), which were obtained by measurements on the water reservoir Nechranice.

Both expression methods for relationships between dimensionless quantities are practically equivalent. The use of Reynolds' numbers has long tradition, which

TABLE 2. Measurement results on water reservoirs Liptovská Mara, Vihorlat, Orava and Nechranice

Nr.	Wind speed w [ms−1]	Wave height h [m]	Wave length λ [m]	Period T [s]	Starting length of wind D [m]	Wave speed c [ms^{-1}]	Ratio h/λ	Ratio $h/c^2 = k_2$
1	12.0	0.70	13.4	2.9	6000	4.68	0.0522	0.0327
2	12.0	0.58	10.4	2.6	3640	4.00	0.0558	0.0362
3	8.9	0.54	10.8	2.65	1500	4.07	0.0500	0.0325
4	8.9	0.45	8.2	2.3	3950	3.56	0.0548	0.0355
5	7.8	0.48	9.8	2.5	7100	3.92	0.0490	0.0312
6	7.8	0.40	8.0	2.25	4700	3.55	0.0500	0.0317
7	7.8	0.31	5.5	1.8	2600	3.05	0.0563	0.0333
8	10.0	0.196	4.78	1.75	3000	2.73	0.0410	0.0263

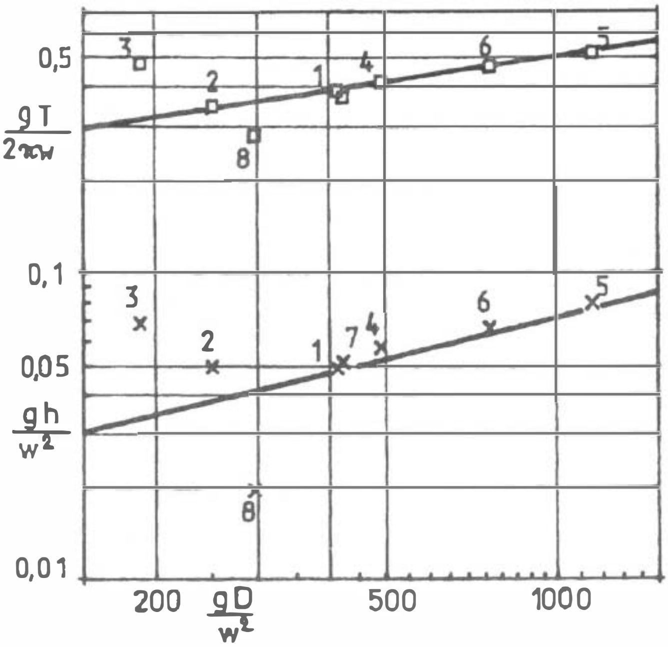

FIG. 1 Measurement results from Table 2 generalized according to π–theorem

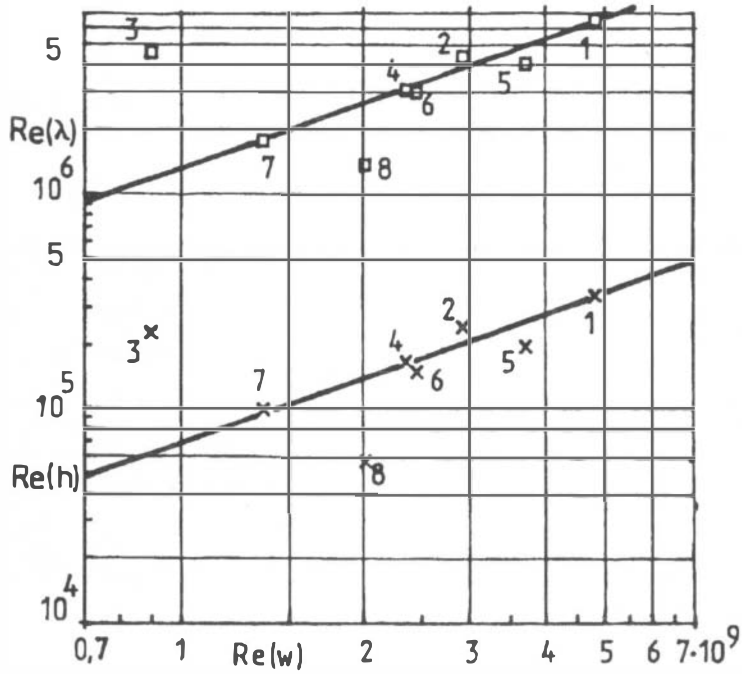

FIG. 2 Measurement results from Table 2 generalized according to Reynolds' number

is supported, in case, by the following reasoning: water movement is caused by the movement of air; each of these movements is characterized by its Reynolds' number and it is therefore logical to look after connections between these very characteristics.

Nevertheless, also the dimensionless arguments from the equation (1) determined according to the π-theorem have their "raison d'être", as it will be shown in the following chapter.

2. Motion of Surface Waves

Fig. 3 illustrates two wave profiles, a) sinusoidal symmetric and b) sinusoidal asymmetric. The later one agrees better with the real wave form and it is therefore considered in further deductions. The height of the wave y at the distance x from the point O is

$$y = \frac{h}{2}\left(1 - \cos 2\pi \left(\frac{x}{\lambda}\right)^n\right). \tag{3}$$

For the calculation of the wave motions and their energy, the sectional area $S = \int_0^\lambda y \, \mathrm{d}x$ and static moment $M = \int_0^\lambda \frac{1}{2} y^2 \, \mathrm{d}x$ should be known. Both can be determined only by numerical integration, which leads for $n = 2$ to these results:

$$S \doteq 1.008 \cdot \frac{3}{8}\, h\lambda \qquad M \doteq 1.099 \cdot \frac{h^2\lambda}{8}\,. \tag{4}$$

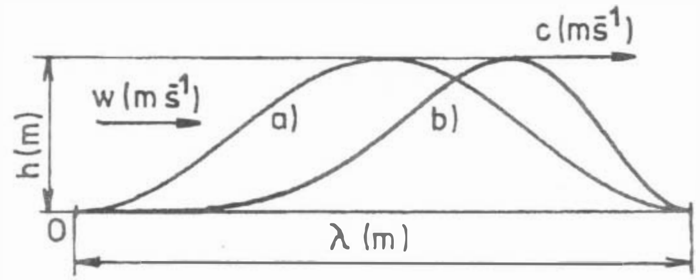

FIG. 3. Scheme of the wind wave

The proceeding wave is subjected to the action of the wind force F_w, then the resistance F_t caused by turbulence, and finally the resistance of internal forces F_s. It must hold, for every moment, that

$$F_w = F_t + F_s \tag{5}$$

Individual forces are computed from equations

$$\begin{aligned} F_w &= \frac{1}{2}\xi_0\rho_0 hb(w-c)^2 \\ F_t &= \frac{1}{2}\xi\rho\lambda bc^2 \\ F_s &= \rho Sb\,\frac{\mathrm{d}c}{\mathrm{d}t} \end{aligned} \tag{6}$$

F_w and F_t are derived from the relationships for hydraulic resistance (Černoch, 1977; Kratochvíl, 1982; Michejev, 1953), F_s represents the 2nd Newton's law.

It is assumed that the momentary length and height of the wave are given by equations

$$\begin{aligned} \lambda &= k_1c^2\,; \qquad k_1 = \frac{2\pi}{g} \\ h &\doteq k_2c^2\,; \qquad k_2 \doteq 0.0334\,, \end{aligned} \tag{7}$$

k_1 being adopted from the standard ČSN 750255, k_2 being the average from values quoted in Tab. 2 (with the exception of extremes quoted in the 3rd and 8th files – see Fig. 1 and 2). As k_2 shows only a 6 % deviation from the average, this k_2 is considered constant. It results that also the ratio $h/\lambda = 0.052$ must be considered constant. According to some measurements (Kratochvíl, 1982; Lukáč and Abaffy, 1980) the h/λ depends on the argument gD/w^2, however this dependence is too indeterminate (see Fig. 5), so we prefer to consider $h/\lambda = \text{const}$.

By introducing expressions from equations (6), (7) into the equation (5) we obtain a differential equation for relationship between $\mathrm{d}c/\mathrm{d}t$ and c, whose solution (Rektorys, 1968) may be transformed into:

$$\frac{gD}{w^2} = \frac{g}{A}\left\{ \frac{1}{1-B}\cdot\frac{c}{w} + \frac{1}{(1-B)^2}\cdot\ln\left|(1-B)\left(\frac{c}{w}\right)^2 - 2\,\frac{c}{w} + 1\right| + \right.$$
$$\left. + \frac{1+B}{2\sqrt{B}(1-B)^2}\cdot\ln\left|\frac{1-(1-\sqrt{B})\frac{c}{w}}{1-(1+\sqrt{B})\frac{c}{w}}\right|\right\} \tag{8}$$

where

$$A = A_0\left(0.67 + 4.12\,\mathrm{e}^{-12c/w}\right), \quad A_0 = \frac{\xi_0\rho_0}{2(3/8)\cdot\rho k_1}, \qquad B = \frac{\xi\rho k_1}{\xi_0\rho_0 k_2}$$

For values quoted in the List of symbols and for equations (7), it holds $A_0 \doteq 0.002$, $B \doteq 0.89$. Values of the resistance coefficients were determined from formulae for resistance of fluid–locked bodies and from the Nikuradse's formula for $\mathrm{Re} \gg 10^5$ (Michejev, 1953), taking in account results quoted in Tab. 2 and (ČSN 750255) and (Kratochvíl, 1982).

It results, from the graphic representation of the relation (8) in Fig. 4, that for $0 < B < 0.89$ are courses of the function c/w practically identical up to $gD/w^2 \leq 500$. The highest velocity of the wave, obtainable at the wind speed w is

$$\frac{c_\infty}{w} = \frac{1}{1+\sqrt{B}}\,. \tag{9}$$

The relation (8) in Fig. 5 is illustrated, for $B = 0.89$, by the curve 1. The curve 2 illustrates the course of the same function according to Kratochvíl (Kratochvíl, 1982, Fig. 5, p.332) transformed from arguments gh/w^2, $gT/(2\pi w)$ into argument c/w, because, according to equations (7) is

$$\frac{gh}{w^2} = \frac{c^2}{w^2}\cdot gk_2\,; \qquad \frac{c}{w} = \frac{gT}{2\pi w} \tag{10}$$

The diagram illustrates, namely, the measurement results according to Kratochvíl (K), Berkeley (B), Lukáč (L), and Sverdrup (S) as quoted in the mentioned publication (Kratochvíl, 1982). It is evident, from the last two equations (10) that two parameters, gh/w^2 and $gT/(2\pi w)$ can be substituted by a single one, namely ratio c/w, which has also a higher expression ability. From diagrams quoted in the publication (Kratochvíl, 1982), we may read, for example, for the argument $gD/w^2 \doteq 10^5$ the value of $gT/(2\pi w) \doteq 1.5$. Knowing from equations (10) that this gives also the ratio c/w, the progressive velocity of waves should be by 50 % higher

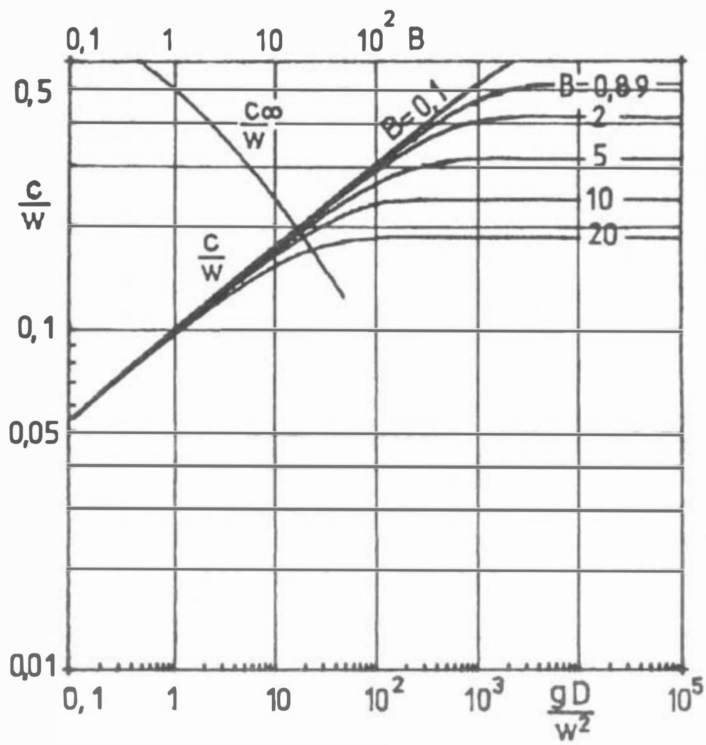

FIG. 4. Criterial dependence $c/w = f(gD/w^2, B)$, $c_\infty/w = F(B)$.

than the wind speed, which is, however, for the wind-caused wave motion, highly improbable.

Within the interval $0.1 < gD/w^2 < 1000$, the ratio c/w can be calculated directly from the equation

$$\frac{c}{w} \doteq 0.096 \left(\frac{gD}{w^2}\right) . \tag{11}$$

We quote, for completeness sake, also the formula used in (Lukáč and Abaffy, 1980) for the calculation of wave-motion energy

$$E = \frac{1}{8} \rho g h^2 \lambda \frac{t}{T} . \tag{12}$$

Example

On the water reservoir Vihorlat (Lukáč and Abaffy, 1980), the wind speed of $w \doteq 7.8\,\mathrm{ms}^{-1}$ and the corresponding starting length of the wind $D = 7100$ m have been measured within the studied time interval. Let us determine the parameters of the resulting wave motion and its energy pertaining to 1 m of the ban line during a year with 42 wind days, the wind blowing in the average for 4 hours a day. For a given D

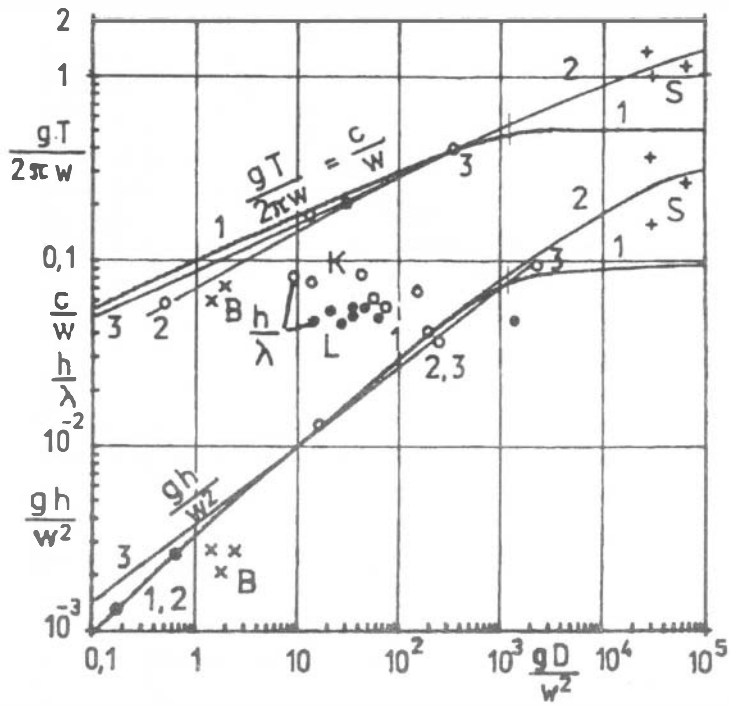

FIG. 5 Nomograph for determination of c/w according to: 1 – equation (8), 2 – (Kratochvíl, 1982), 3 – ČSN 750 255, and measurement results according to Kratochvíl (K), Berkeley (B), Sverdrup (S) and Lukáč (L).

and w is $gD/w^2 \doteq 1145$. Let's read the value of $c/w \doteq 0.5$ from the graph in Fig. 5. At the wind speed of $w \doteq 7.8\,\mathrm{ms}^{-1}$, the wave velocity will be $c \doteq 3.9\,\mathrm{ms}^{-1}$. The wave height $h \doteq 0.45\,\mathrm{m}$, its length $\lambda \doteq 9.7\,\mathrm{m}$ and the period $T \doteq 2.5\,\mathrm{s}$ are determined from equations (8) or Fig. 5, resp. 42 windy days represent, in this case, the time $t = 42 \cdot 4 \cdot 3600 = 604800\,\mathrm{s}$. It can be computed, from the formula (12), that the energy $E = 609000\,\mathrm{kJ}$ pertains in one year, to 1 m of the embankment line.

3. COMPARISON WITH OTHER METHODS

The comparison of calculations according to the equation (8), Kratochvíl, and ČSN 750 255 is carried out graphically in Fig. 5 and numerically in Table 3, the standard ČSN 750 255 being defined only within $c/w < 0.4$ or $gh/w^2 < 0.1$, resp. – see points 3 in Fig. 5. The dispersion variance of h/λ values according to only two author (points L, K) is as high that it does not justify to introduce this quantity into the calculation otherwise than as an average value. In addition to that Kálal, who according to (Kratochvíl, 1982) modified the formulae from different authors, obtained the result of $\lambda/h = 20.15\,D^{1/6}w^{-1/4}$, which cannot be transformed into a relation of the type $h/\lambda = f(gD/w^2)$. Several empirical relations have been derived for the determination of the wave height and length which, however, yield very disparate results. So, for example, for $D = 3000$ m and $w = 20\,\mathrm{ms}^{-1}$, the wave height is 0.7 m according to (Kratochvíl, 1982) and 1.4 m according to Lobzovsky. Compared with that, is the difference of 0.9–0.94 m, determined from curve 1, 2 and 3 in Fig. 5, practically negligible.

TABLE 3. Computing results according to: 1 – equations (8), 2 – (Kratochvíl, 1982), 3 – ČSN 750 255

Starting length wind D (m)	Wind speed w (ms^{-1})	Progressive wave velocity c (ms^{-1})	Wave height h (m)	Wave length λ (m)	Wave period T (s)	Calculated according to:	gD/w^2
100	30	3.00	0.30	5.76	1.92	1	1.09
		2.04	0.30	2.67	1.31	2	
		2.76	0.34	4.89	1.77	3	
400	20	3.3	0.36	6.97	2.11	1	9.81
		3.0	0.40	5.76	1.92	2	
		3.2	0.40	6.55	2.05	3	
3000	10	3.75	0.47	9.00	2.40	1	294.3
		3.80	0.39	9.20	2.43	2	
		3.80	0.39	9.20	2.43	3	
30000	5	2.50	0.23	4.00	1.6	1	11772
		5.00	0.48	16.00	3.20	2	11772
		—	—	—	—	3	beyond the range ČSN 750 255

The suggested calculation method could be of considerable advantage too, because

a) it gives the same results as ČSN 750 255 and
b) is, at the same time, defined within the whole interval of $0.1 < gD/w^2 < 10^5$,
c) is not described only graphically, but also by the equation (8) – this removes inaccuracies, which can never be entirely eliminated in graphical methods.

The high dispersion variance of results from individual authors suggests that a successful generalization will require to be issued from a precise and complete description of the dam reservoir to the description of location and type of the used measurement devices.

4. CONCLUSIONS

A significant information consists in finding out that measurement result of parameters of wind-induced waves generalized according to the Buckingham's π–theorem do not practically differ from those generalized according to the Reynolds' number. Another information results from the equation (9), i.e. that in our conditions is $c/w < 0.5$, so the progressive wave velocity should not exceed the half of the wind speed.

The new computing method of wind wave parameters is issued from further information, that relations hitherto expressed by two dimensionless arguments gh/w^2 and $gT/(2\pi w)$, can be formulated by means of the single argument c/w – see equation (10). This makes calculations considerably simpler and their reliability in increased by the fact, that we can see immediately if the argument c/w can agree with the reality (c/w must always be lower than 1), which is not possible for the argument $gT/(2\pi w)$.

List of Symbols

$h\,(\mathrm{m})$	wave height
$\lambda\,(\mathrm{m})$	wave length
$b\,(\mathrm{m})$	wave width
$D\,(\mathrm{m})$	length of wind's track
$w\,(\mathrm{ms}^{-1})$	wind speed
$c\,(\mathrm{ms}^{-1})$	progressive wave speed
$t\,(\mathrm{s})$	duration of the wind action
$T\,(\mathrm{s})$	wave period
$g\,(\mathrm{ms}^{-2})$	acceleration of gravity
$\rho = 1000\,\mathrm{kgm}^{-3}$	density of water
$\rho_0 = 1.2\,\mathrm{kgm}^{-3}$	density of air
$\mu = 10^{-3}\,\mathrm{kg.m}^{-1}\mathrm{s}^{-1}$	viscosity of water
$\mu_0 = 18 \cdot 10^{-6}\,\mathrm{kgm}^{-1}\mathrm{s}^{-1}$	viscosity of air
$\nu = 10^{-6}\,\mathrm{m}^2\mathrm{s}^{-1}$	kinematic viscosity of water
$\nu_0 = 15 \cdot 10^{-6}\,\mathrm{m}^2\mathrm{s}^{-1}$	kinematic viscosity of air
$\xi = 3.35 \cdot 10^{-5}$	resistance coefficient of water
$\xi_0 = 0.6$	resistance coefficient of air

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