STUDIES ON THE PHASE SPACE OF MULTISCALE SEISMICITY

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(*Received* September 2003*, accepted* November 2003)

ABSTRACT
Sets of seismic events of different origin and different scale of the observed failure processes were analysed. The data sets, from micro-scale - seismoacoustic emission recorded from a rock sample under stress through coal-mining induced seismicity to macro-scale represented here by earthquakes caused by the Yake-dake volcano activity and San Andreas fault were studied to seek the deterministic chaos. The goal was to characterize the fracturing processes in terms of some chaotic parameters: the phase space and the dimension of the reconstructed attractor estimated for time-space distribution and energy distribution of the events. The calculated values were used to determine whether fracturing processes have similar features in different scales and on this basis to verify the usefulness of the applied data analysis methods for description of studied seismic processes, and finally to draw the conclusions about the time evolution of this systems.

KEYWORDS: deterministic chaos, phase space, attractor, fracturing processes, induced seismicity, earthquakes, seismoacoustic emission, time-space and energy distribution

INTRODUCTION

Fractality of any physical variable could be treated as a numerical trace of the non-linear processes very sensitive for small disturbances in boundary or initial conditions. As a result of those processes deterministic chaos is appearing in the system – seemingly accidental, deterministic evolution of non-linear physical system. Natural and induced seismicity is considered in this paper as a process with deterministic chaos behaviour.

Several data sets were collected comprising seismic events of different origin, and first of all, of different scale of the fracturing and destruction processes observed. The analysis of the sets was conducted starting with micro scale – seismoacoustic emission recorded from a rock sample under stress through coal-mining induced seismicity to macro scale represented here by earthquakes caused by the Yake-dake volcano activity and San Andreas fault. The main goal of this work was to look for the presence of non-linear dynamics and deterministic chaos in those systems and quantitative characterization of some chaotic parameters (phase space dimension, attractor dimension), essential for determining how much fracturing and destruction processes are similar in various scales.

METHODOLOGY

According to Takens theorem (Takens, 1981) in order to study a dynamic system with multidimensional phase space, embedded spaces constructed on single variable time series are analysed. From the set of observations of a scalar quantity time series $\omega(t)$ (here: energy, time, *xy* epicentre and *xyz* hypocenter location) vectors $\bar{y}(t)$ are created in d-dimensional Euclidean spaces. The components of the vector are taken from the set of observations of particular values delayed by the time lag τ:

$$\bar{y}(t) = \{\omega(t),\ \omega(t+\tau),\ \ldots,\ \omega(t+(d\text{-}1)\tau)\}; \qquad (1)$$

where $\omega(t)$ – system variable, τ – time lag.

There are a number of different criterions for choosing the time lag and minimal embedded phase space dimension optimised for the lowest lost of information in the original system. Here τ is taken as a first zero of the autocorrelation function (Mortimer, 2001). The series of embedded spaces are examined with rising d dimensions. Then correlation dimension D_2 of the attractor of the phase space reconstructed in this way is calculated. This procedure is repeated until the saturation level of D_2 value is reached. The

Attractor dimension estimation was accomplished by calculating the correlation integral $C(R)$ (Grassberger and Proccacia, 1983):

$$C(R)=\lim_{N\to\infty}[\frac{1}{N^2}\sum_{i,j=1}^{N}H(R-\|x_i-x_j\|)] \qquad (2)$$

where x_i, x_j are the attractor points, N is the number of the points accidentally chosen from the whole data set and $H(y)$ is the Heaviside function equal to 1, when $y\geq 0$ and 0 when $y<0$. Correlation dimension D_2 calculated from the correlation integral is defined by the formula:

$$C(R)\propto R^{D_2},\quad R\to 0 \qquad (3)$$

The results of the analysis are displayed in D_2 vs. d relation graphs. The value of d, in which the D_2 value stabilizes, is treated as the estimate of d_{min} (minimal embedded dimension) - the phase space dimensions and the level of stabilization estimates the dimension of the reconstructed attractor.

Studies were carried out on few dozen of data sets. Here, results of phase space investigations for seismicity of natural and mining induced origin are presented: caused by San Andreas fault (California, USA) and Yake-dake volcano activity (Honshu, Japan) as well as generated by Katowice Coal Mine (Silesia, Poland) and seismoacoustic emission of syenite rock sample recorded during uniaxial compression test (conducted in Poland).

A. Cichy computer codes described by him in the scientific project (Mortimer et. al, 2000) were applied for calculations of Euclidean spaces dimensions. These spaces were eventually created from time series of: logarithm of tremors energy, time of event and epi/hypocenter distances of consecutive events. For each of these four variables the correlation dimension D_2 was estimated for embedded spaces with Euclidean dimensions d ranging from 1 to 15.

DATA AND RESULTS

San Andreas, California - USA. The data set was obtained from *Southern California Earthquake Centre*. The whole set recorded from 1.01.2002 to 15.08.2002 was divided into south and north regions. *South California* set consists of 1422 events. The energy of tremors ranges from $2.51\cdot10^6$ *J* to $6.97\cdot10^{16}$ *J*. The second set – *North California* consists 1994 events. The energy ranges from $4.11\cdot10^6$ *J*, to $3.89\cdot10^{14}$ *J*.

In figure 1 the results of conducted analysis for those two regions are set together. The phase space was reconstructed from the series of log of energy, *t*, *xy*, *xyz*.

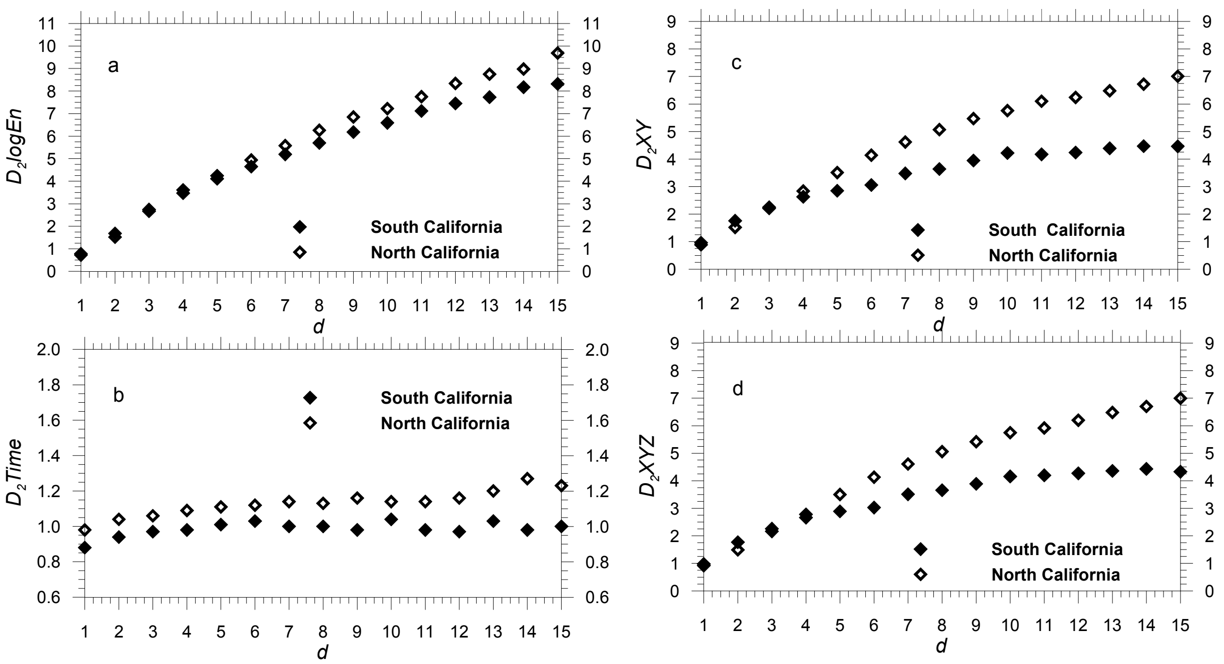

Fig. 1 Correlation dimensions D_2 of the attractors in dependence of Euclidean dimension d of the embedded space reconstructed from the distribution of logarithm of energy (a), time (b), epicentre (c), hypocenter (d). Tremors recorded in South and North California in 2002.

For the energy case the run of both curves is very similar. The level of stabilisation is weakly visible for *South California* at d = 14 - 15 and for the *North California* set near d = 13 - 14. The dimension of the reconstructed attractor related to this level is 8.2 and 8.9, respectively. For time t the curvers differ but the overall shape is preserved. The levels of stabilisation for *South* and *North California* are placed in the same d interval ($d > 7$). In the first case the D_2 value is very close to unity and in the second one it fluctuates around 1.15. The shapes and runs of the plots for both data sets considering spatial distribution *xy* and *xyz* are nearly identical what indicates insignificant influence of z coordinate on the results of calculations. This is because the Californian earthquake hypocenters are rather shallow and lie at the same underground surface. The levels of stabilisation in *xy* case occur for *South* and *North California* at the same place ($d > 10$) but the attractor dimensions differ. For the first set $D_2 = 4.3$ and for the second set $D_2 = 6.4$. These values stay nearly the same for *xyz* spatial distribution.

Yake-dake, Honshu - Japan. The seismicity of the Yake-dake volcano activity region was represented by the records of 7274 seismic events. Nearly half of them occurred during August of 1998 and the rest are the events recorded during a couple of next months in the same year. For this reason the whole data set was divided into two smaller subsets. The first one (*Volcanism 1*) consists of 3195 tremors. The weakest event has the energy of $4.4 \cdot 10^3$ *J*, and the strongest one $1.3 \cdot 10^{13}$ *J*. The second subset (*Volcanism 2*) consists of 3293 tremors and the energy range is *270 J* - $8.2 \cdot 10^{11}$ *J*. The phase space was reconstructed from the series of log of energy, *t*, *xy*, *and xyz*. The results of the analysis for these two subsets are presented together in figure 2.

The energy case here is similar to that observed for San Andreas seismicity. The runs of the both curves are nearly the same until the certain d point is reached. But when d is equal to 10 the plot for *Volcanism 1* reaches weak stability while for the other set the curve still raises, not reaching any stability level. In that case the results of the reconstruction of phase space from the series of logarithm of energy does not allow the determination of the dimension, but the slope of D_2 vs. d curves changes in the range of 8 – 10 in both cases. For the time series the stabilisation levels are close for two subsets and the D_2 value oscillates around 1.10. Slight divergences in the shape and run of the graphs for *xy*, *xyz* were observed.

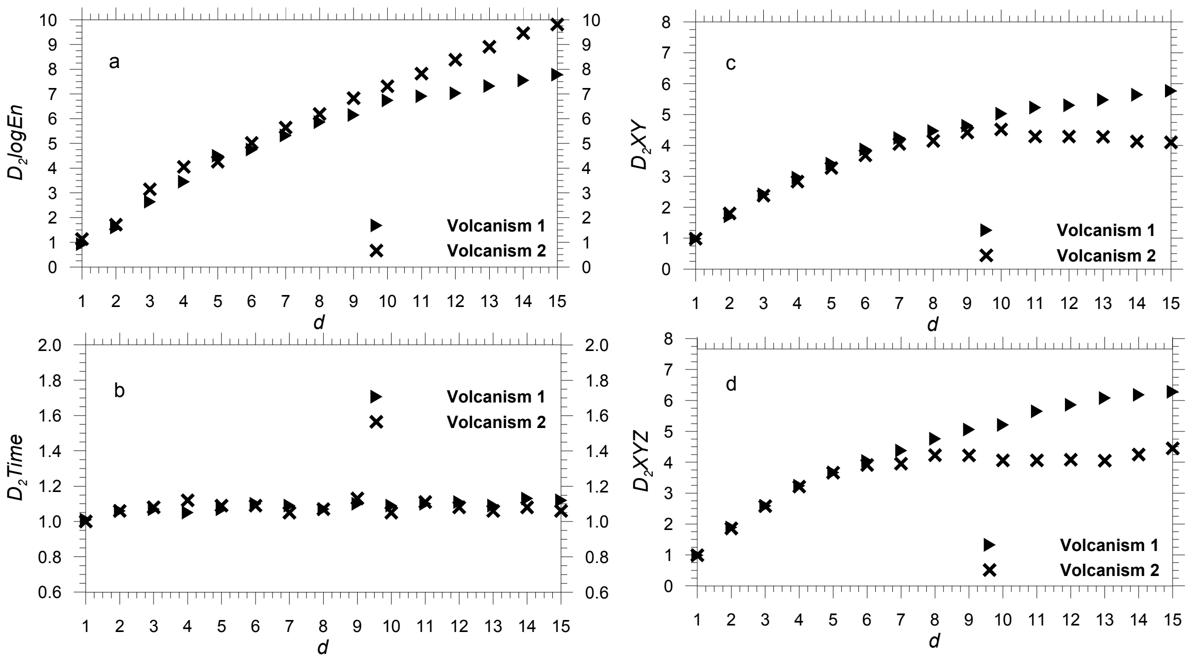

Fig. 2 Correlation dimensions D_2 of the attractors in dependence of Euclidean dimension d of the embedded space reconstructed from the distribution of logarithm of energy (a), time (b), epicentre (c), hypocenter (d). Tremors generated by Volcanism 1 and 2 in Japan, in 1998.

Generally, stabilization levels are more distinct for *Volcanism 1* curve. For *Volcanism 2* only the change of slope inclination was noticed.

Katowice Coal Mine, Silesia - Poland. This data set originates from the catalogue of the mining induced seismicity caused by extraction of longwall 532 in the Katowice Coal Mine. Nearly 1000 entries were collected approximately over the period of one year (1985-86). The energy ranges from *100 J* to $1 \cdot 10^6$ *J*. The phase space was reconstructed here from the series of three variables only: log of energy, *t, and xy*. Epicentral rather than hypocentral locations are analysed because the evaluation of the depth of shocks is far more biased than that of their horizontal coordinates (Lasocki and Mortimer, 1998). In figure 3 the results of conducted analysis are presented.

The plot for the energy distribution reaches rather weak stability level at d = 10 but then it still rises until d = 13 – 15. D_2 values ranges between 8 and 9. The D_2 saturation level is easily observed for phase space reconstruction from the time series. It starts near d = 5 what gives the attractor dimension equal to 1.2. From d = 8 to 15 the curve shows some slight fluctuations which are difficult to interpret. They might be caused by some exterior factors such as the changes in longwall extraction movement – typical for induced seismicity. The stabilisation level for *xy* spatial distribution of shocks is not clearly noticed. Only a change in inclination of the plot is observed. It appears near d = 9 – 10 and the stabilisation level might be starting from d = 14. D_2 is then about 6 or nearly 8 if the beginning of flattening is taken into account. Generally the view of the graph is very similar to that of energy distribution what indicates the possible close relation between energy and spatial distribution of that seismicity.

Sample P3, Syenite from Przeborow – Poland. Two data sets were analysed related to the pressure tests conducted on syenite rock samples from Przeborow with the dimensions 31.5 x 64.3 mm and the density of 2.92 g/cm^3. The data consist of the records (6031 entries) of acoustic emission energy rate variations occurred during the 2h40m35s of the test. The reconstruction of the attractor in the embedded space has usually a different character in different stadium of the sample fracturing and destruction processes (Majewska and Mortimer, 2000). To find a difference in presumable manifestations of the non-linear dynamics and chaotic behaviour, two stages of sample destruction were analysed: part b) – during destruction and part c) – after destruction. The first part has 4795 and the second one 1377 recorded events. Only time series of the acoustic emission energy rate in those two stages were studied. The results of conducted analysis are presented in figure 4.

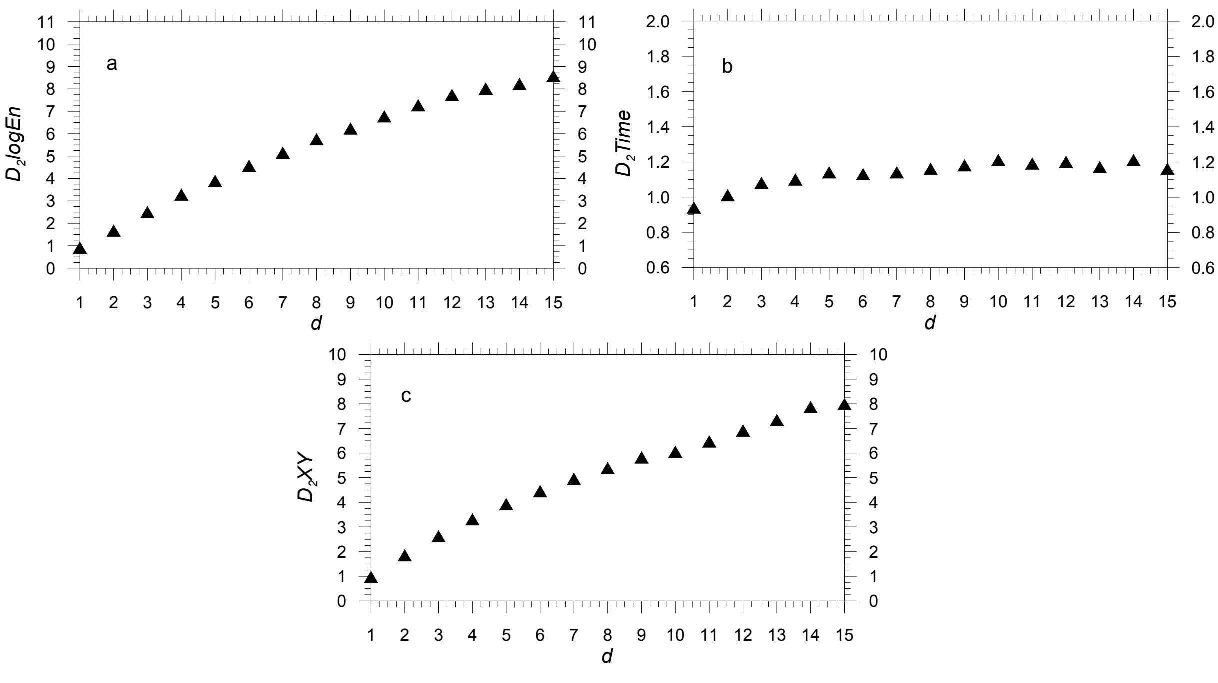

Fig. 3 Correlation dimensions D_2 of the attractors in dependence of Euclidean dimension d of the embedded space reconstructed from the distribution of logarithm of energy (a), time (b), epicentre (c). Tremors generated by mining in Katowice Coal Mine in Poland, in 1985-86.

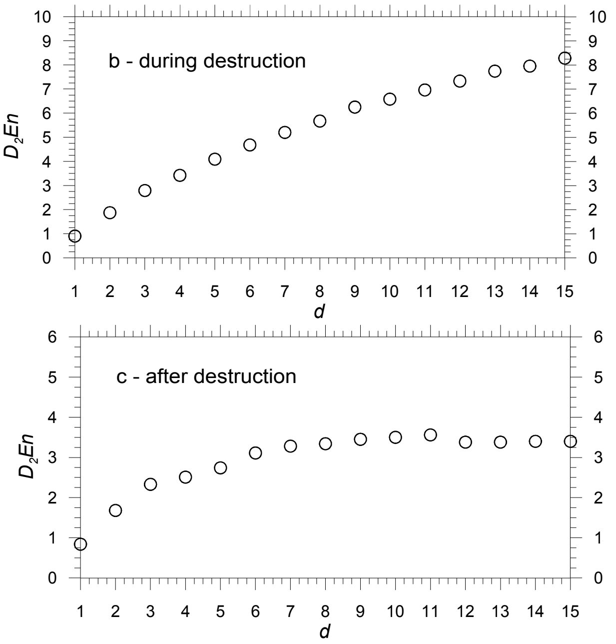

Fig. 4 Correlation dimensions D_2 of the attractors in dependence of Euclidean dimension d of the embedded space for AE recorded during two stages of pressure test of the syenite sample P3 in 2h40m35s.

In case c) the stabilisation level is clearly visible. It starts from d = 9 and the corresponding D_2 dimension for this part of the test is about 3.4. Reconstruction of the attractor from the series of energy rate in the stage b) does not allow the determination of dimension. D_2 vs. d curve reaches the value of 8.3 for d = 15. In this stage similar results were obtained for other tested samples. Phase spaces reconstructed from the time series of different variables should describe the studied seismicity in the same way. For the time series of energy results were different than those obtained for time or spatial distribution of sources. Similar results were obtained for reconstruction of the phase space from the time series of xy and xyz. This could be caused by the accuracy of pointing out particular values, limiting the data set by chosen energetic steps and in this way the existence of the so called "false neighbours". The influence of other factors on e.g. time distribution such as the change in longwall succession for induced seismicity is also possible.

The attempt to bypass this problems is to apply the holistic event sequence analysis in the phase space. The coordinates of the space are seismic parameters available for a given event set. The most frequent those are the hypo/epicentre distances, energy or magnitude and time of the event occurrence or the interval of time from the previous event. So this is a "time" series measured with the succession in time.

EVENT SEQUENCE ANALYSIS

The studies presented here combine the description of non-linear dynamics of seismic events with modified ET (*event thread analysis)* (Suteanu, 1998) and SLC (*single-link cluster analysis*) methods (Frohlich and Davis, 1990).

The seismic event set evolution is resolved by analysing the displacement of the event on the trajectory in the multidimensional phase space of the process or its embedded phase space.

The embedded phase space dimension is defined by the number of available parameters that characterize the event set. In this study, the five-dimensional embedded space is defined by the position axes, magnitude axis and time intervals axis. Apart from those quantities, the seismic moment or stress drop could be taken into consideration. In order to get a possibility of joint analysis of particular variables each of the time series $\{s_i\}$ was normalized to fit into [0,1] section and to be dimensionless.

The D_2 dimension for 5-D embedded space was estimated from trajectory points' distances d_{ij} according to the formula [4]:

$$d_{ij} = \sqrt{\sum_{k=1}^{5} w_k \left(s_i^{(k)} - s_j^{(k)}\right)^2} \qquad (4)$$

where w_k - weight – scaling constants connecting particular variables. In the present stage of the researches the value of weight w_k = 1 was taken.

From the seismic hive recorded near Hida mountaines in Japan between August and December of 1998 series H1 with 1686 events was analysed. Each event magnitude has to fulfil the condition: $M \geq M_{min} = 1.5$ and has to have complete hypocenter depth data. Apart from that, mining seismicity induced by Katowice Coal Mine extraction of longwall 532 was also studied.

The trajectory of the process was constructed in pursuance of the formula [4] in 5-D space spread out onto variables of time, hypo/epicentre and energetic (magnitude) event distribution. The estimates of the correlation dimension D_2 of the embedded spaces attractors' are 1.12 and 2.79 for H1 and the coal mine respectively.

Those values are closest to the dimensions received for the embedded spaces reconstructed from the time distribution of events. They suggest low-dimensionality of the shock-creating processes much better than it is possible to obtain based on the reconstruction of the phase spaces from the separate variables. Besides, the values of the attractor dimensions for the H1 hive are characteristic, because even if one would not know the data, the estimates show clearly the close clustering and strong selfsimilarity of the events.

The future works will be aimed at analysing the miscellaneous choosing of the weights w_k and its justification and influence on the results.

SUMMARY

The summarised results of all conducted analysis are presented in Table 1 below.

Table 1 Estimation of phase space dimensions d_{min} and the dimension of the reconstructed attractor D_2

No.	Data set name	**Distribution of:**	**Energy**		**Time**		**XY distance**		**XYZ distance**	
		Number of events	d_{min}	D_2	d_{min}	D_2	d_{min}	D_2	d_{min}	D_2
1	North California	1994	**13**	8.9	**7**	1.0	**10**	6.4	**10**	6.3
2	South California	1422	**14**	8.2	**7**	1.2	**10**	4.3	**10**	4.3
3	Volcanism 1	3195	**10**	7.2	**4**	1.1	**11**	5.5	**11**	6.0
4	Volcanism 2	3293	**> 15**	>9.8	**4**	1.1	**9**	4.3	**8**	4.1
5	Katowice Mine	994	**12**	8.15	**5**	1.2	**14**	7.9	-	-
6	P3 syenit (b)	4795	**> 15**	>8.3	-	-	-	-	-	-
7	P3 syenit (c)	1377	**9**	3.4	-	-	-	-	-	-

For all studied data sets:

- Parameters of the non-linear dynamics can describe the seismicity and the fractal parameters describe the phase space geometry.
- In most cases multi-dimensionality (>8) was observed for the reconstructed phase spaces of the fracturing and destruction processes in different scales. In eight cases d_{min} was < 10 what indicates that the time evolution of this system might be predicted. For the rest of sets chaotic behaviours are not predictable.
- Good recurrence for the results of analysis performed for different data sets from the same region was obtained (*Volcanism*, *California*).
- Attractor dimensions received from calculations for particular type of the studied seismicity indicates chaotic behaviour. Only for the time distribution D_2 is sometimes close to unity what announces that this variable is much more complicated and close to random.
- A significant advantage of the presented methodology of event sequence analysis is the possibility of full usage of available data, which were previously incompatible.

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