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SOME ASPECTS OF THE APPLICATION OF THE WIECHERT-HERGLOTZ METHOD TO REFRACTION DATA FROM WESTERN BOHEMIA

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ABSTRACT
The Wiechert-Herglotz method assumes the medium to be laterally homogeneous, and the travel-time curve of refracted wave to be continuous with monotonous derivatives. It is usually difficult to satisfy these requirements in refraction studies of shallow structures as a consequence of lateral inhomogeneities and large scatter of observed data. Nevertheless, we shall demonstrate that the Wiechert-Herglotz method can be used even in geologically complicated regions if the observed data are selected from identical geological units and smoothed considerably. For the smoothing, polynomial and rational approximations are analyzed in greater detail. The procedure is applied to *P*-wave travel times from the Libá refraction profile in western Bohemia. The derived velocity model is characterized with very low superficial velocities. This significant low-velocity zone was not recognized by the previous deep seismic soundings.

KEYWORDS: West-Bohemia, upper crust, refraction measurement, Wiechert-Herglotz method, polynomial approximation, rational approximation

1. INTRODUCTION

The region of West-Bohemia/Vogtland is well known for the occurrence of earthquake swarms. Especially, the last strong earthquake swarm at the turn of the years 1985/86 arose great interest of many investigators to this seismoactive region.

Many investigations of the crustal structure were performed in the region in the 1980s and 1990s, including exploitation of earthquake data (Málek et al., 2000), refraction methods (Nehybka and Skácelová, 1997) and reflection methods (Tomek et al., 1997). For further references we refer the reader to the reviews in Novotný (1996), Málek et al. (2001) and Málek et al. (2004a). Despite these numerous studies, especially the shallow crustal structure of the region was not known sufficiently for a long time. Consequently, many investigators used in their analyses of the swarms very simple models of a homogeneous half-space (Horálek et al., 1995), models composed of homogenous layers or layers with constant velocity gradients (Málek et al., 2000, 2004b).

The international CELEBRATION 2000 seismic refraction experiment (Guterch et al., 2001; Málek et al., 2001) provided a new opportunity to study the crustal structure of this interesting region, because one profile (CEL09) crossed it. A simplified 2-D velocity cross-section along this profile was described by Hrubcová et al. (2002), but no special attention was paid to details of the shallow structure.

Detailed analyses of the observed CELEBRATION 2000 travel times from the seismoactive region, with consideration of the very complex geological structure, and their interpretation in terms of simple vertically inhomogeneous models (1-D models) for individual geological units were performed by Málek et al. (2004a). To interpret the travel-time curves of refracted waves, the Wiechert-Herglotz method was used. In the present paper we discuss some aspects of the application of this method in greater detail.

The usage of the method will be demonstrated on data from the Libá additional profile (Fig. 1). The profile began at the Libá shot point in the Smrčiny-Fichtelgebirge pluton, then traversed the northern part of the Cheb basin, Vogtland-Saxonian Palaeozoic, Karlovy Vary pluton, and terminated in the region of the Nejdek pluton.

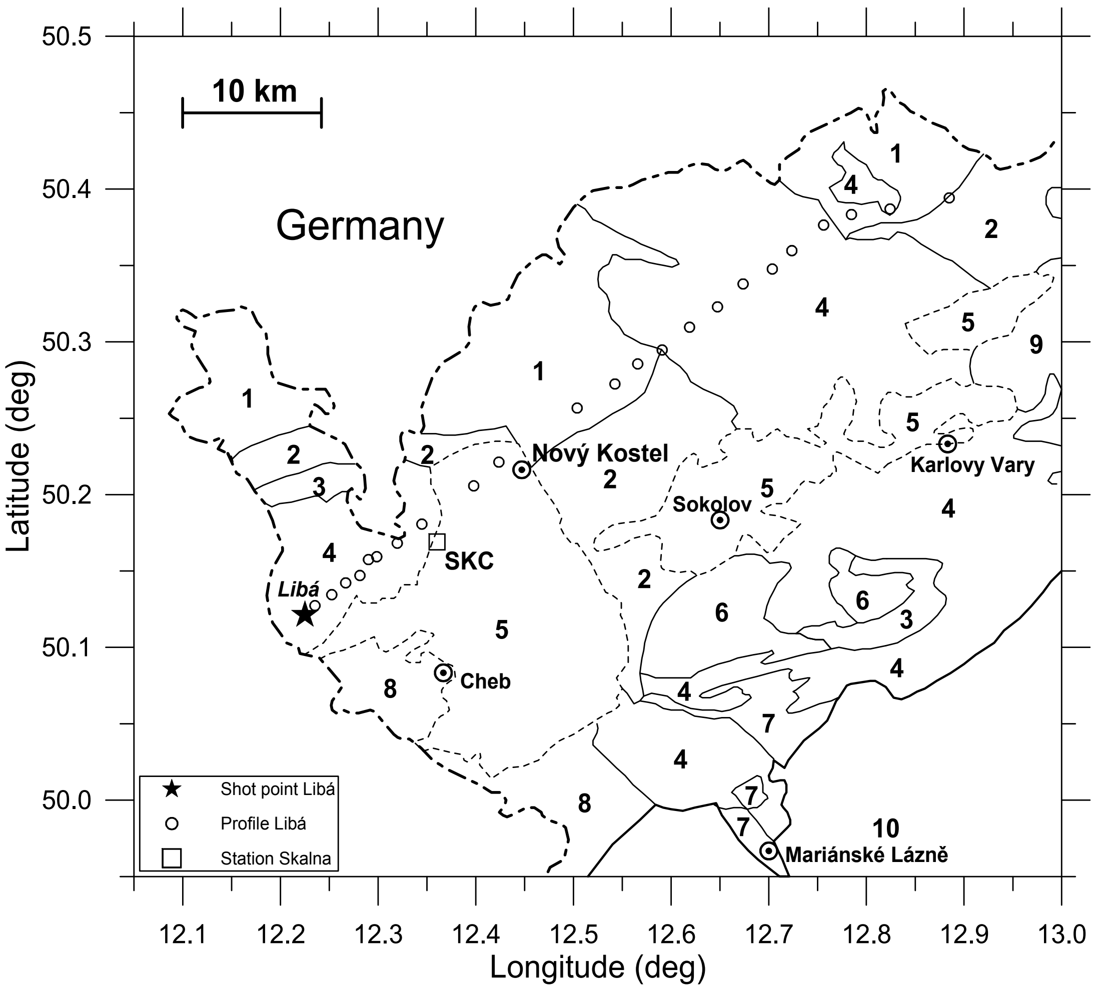

Fig. 1 Geological sketch map of the Saxothuringian Zone in western Bohemia (simplified after Mlčoch et al. (1997)) and the situation of the Libá profile: 1 - Vogtland and Saxony Palaeozoic; 2 - Smrčiny and Ore Mts. Crystalline Units; 3 - orthogneisses; 4 - Variscan granitoid plutons (Smrčiny-Fichtelgebirge, Karlovy Vary, Nejdek and Žandov plutons); 5 - Tertiary sediments (Cheb and Sokolov basins); 6 - Horní Slavkov Crystalline Unit; 7 - Kladská Unit; 8 - Cheb-Dyleň Crystalline Unit; 9 – Tertiary volcanites; 10 – Mariánské Lázně Complex.

2. SOME ASPECTS OF THE WIECHERT-HERGLOTZ METHOD

The Wiechert-Herglotz (WH) method is a classical method of studying the deep Earth's structure. However, its application to studies of shallow structures encounters some problems, because the properties of observed travel-time data are usually far from the assumptions of the WH method.

Assume the medium to be vertically inhomogeneous, i.e. the seismic velocity to be dependent on the depth only, $v = v(z)$. Consider a refracted ray impinging at an epicentral distance *r*. The depth of its turning point, *z*, is given by the well-known WH formula in the form

$$z = \frac{1}{\pi}\int_0^r \operatorname{arcosh}\frac{p(\xi)}{p(r)}\,d\xi \qquad (1)$$

where $p(\xi) = dt/d\xi$ is the derivative of the travel time *t* with respect to the epicentral distance ξ. In practice, the integral in formula (1) is evaluated by numerical integration. The velocity at the turning point is determined as the reciprocal derivative of the travel time at the epicentral distance *r*, $v(z) = 1/p(r)$.

a)Discrete data. First, let us mention the discrete character of the observed travel-time curves. This was a general problem, faced in application of the WH method even to gross Earth data, which are usually considered to be sufficiently smooth. The problem was solved by introducing the so-called “extremal inversion” approach (Bessonova et al., 1974); see also Kennett (1976). This method computes the upper and lower limits to the possible velocity-depth distribution. The extremal inversion solves only a partial

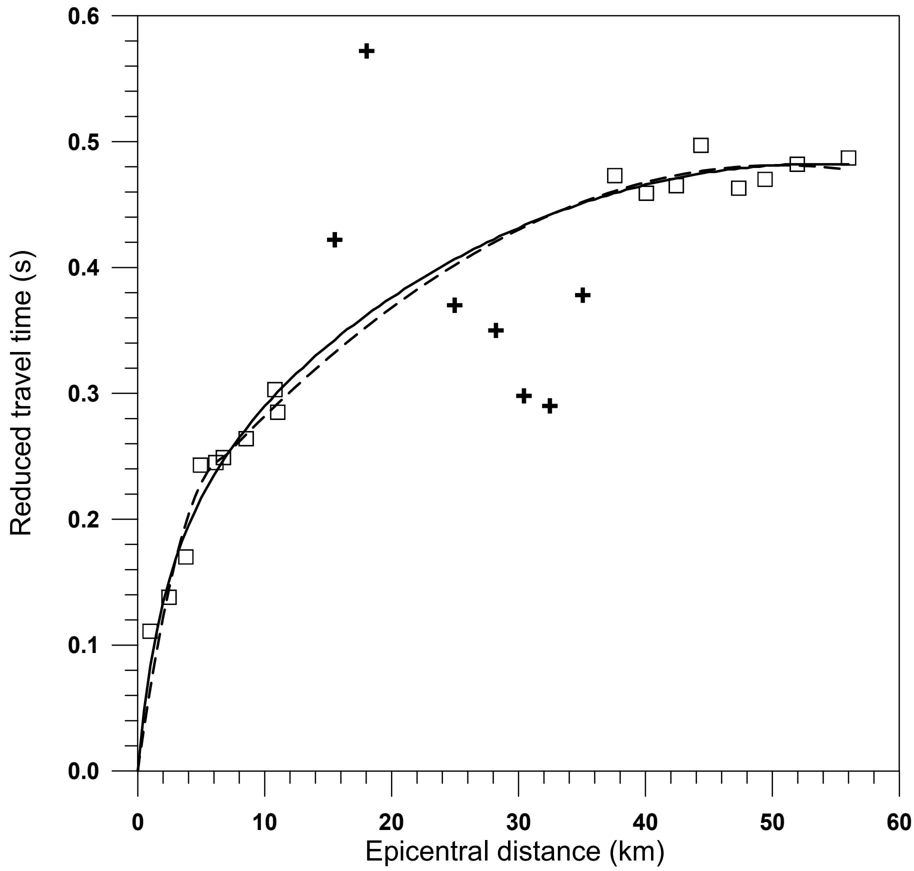

Fig. 2 *P*-wave travel times for the Libá profile and Skalná station (SKC) reduced with a velocity of 6.0 km/s: measured data on the plutons (squares); other measured data (+); polynomial approximation (dashed line); and the rational approximation (solid line).

problem, namely the effect of the absence of information between the points of observation. On the other hand, the extremal inversion does not suppress the scatter of data due to lateral inhomogeneities of the geological structure. Since such inhomogeneities are significant in the region under study (Mlčoch et al., 1997), we shall not apply the extremal inversion in the present paper.

b) Geological similarity. The observed travel-time points from a profile that crosses different geological blocks, such as the Libá profile, frequently display large "oscillations" (Fig. 2). This scatter of points is caused predominantly by lateral variations in the shallow crustal structure. It would be formally possible to use all such scattered data as input for the WH method in order to get some "average" velocity-depth distribution, but a radical smoothing of the data would be necessary in this case. Moreover, such results would depend strongly on the method of smoothing and, consequently, would not be very reliable. Málek et al. (2004a) solved this problem by deriving a velocity-depth distribution individually for each geological unit. Such an approach is better substantiated physically, and seems to be the only one to be used in interpreting refraction data from geologically complicated structures, if the WH method is to be applied. In other words, as the first step towards a reduction of the scatter in observed travel times we recommend to take the geological information into account, and to select the measurements from identical or similar geological units.

For the WH application to the Libá profile we have selected the data corresponding to wave propagation predominantly through the plutonic geological blocks (blocks No. 4 in Fig. 1). The selected data are shown in Fig. 2 with squares. Thus, from the original data we have excluded the measurements from the Cheb basin, Vogtland-Saxonian Palaeozoic and their close vicinities in the central part of the profile. In this way, the original data set was reduced to 17 points.

In order to obtain a long travel-time curve, we have joined data from separate plutons. This is partly substantiated by some laboratory measurements which indicate a convergence of seismic velocities in various rocks with increasing pressure (Martínková et al., 2000). However, if the velocities in deeper parts of the plutons and surrounding rock are significantly different, our structural models will be inaccurate.

c) Topographic corrections. In order to reduce the scatter of the observed travel-time points further, Málek et al. (2004a) introduced simple topographic corrections to eliminate the effect of different altitudes of the stations and shot points. For details and numerical values, we refer the reader to the paper mentioned. In the present paper, we consider only the topographically corrected data.

d) Near-source measurements. The travel-time curves of refracted waves are frequently characterised by a large curvature at small epicentral distances. This is a consequence of the usual velocity-depth distribution, which is characterised by a relatively low velocity at the surface, but a high velocity gradient that gradually decreases with depth. Such velocity distributions were found in western Bohemia not only from refraction measurements, but also from structural studies using earthquake data, and laboratory measurements on rock samples; see the references in Málek et al. (2004a). Similar structural features have also been found in many other regions.

To give a true picture of the observed travel-time curve, it is therefore necessary to deploy several receivers at small epicentral distances. These requirements were not satisfied in older seismic measurements in the region. Only the recent refraction measurements, such as the CELEBRATION 2000 experiment, used sufficiently small offsets.

e*) Smoothing.* Since the WH method requires the travel-time curve of refracted wave to be continuous with decreasing derivatives, a smoothing of the observed travel-time values is practically always needed. We have tested several variants of polynomial approximation and the rational approximation, i.e. approximation by a quotient of two polynomials. We shall present here some results obtained by both the methods. According to our tests, both the methods are more stable than smoothing by approximative splines. Some details will be given in the next sections.

3. POLYNOMIAL APPROXIMATION OF A TRAVEL-TIME CURVE

We shall consider only simple approximating functions that will enable us to describe some specific features of the travel-time curves of refracted waves, such as a possible large curvature at small epicentral distances, and almost linear form at large distances. First, let us discuss the polynomial approximation.

As a basis, we use quadratic polynomials in the form

$$t(r) = a + br + cr^2 \quad . \tag{2}$$

The quadratic approximation is applied piece-wise in several sections of the travel-time curve. The first smoothed point of the *(j+1)*-th section must be equal to the last smoothed point of the *j*-th section. The coefficients of the approximation in each section are computed by the least square method, considering the approximation in the previous section. Coefficient *a* for the first section equals to zero.

The continuity of the travel time derivatives between the sections is not required, because this constraint could cause “oscillations” of the derivatives, which is not allowed in the WH method.

To smooth the pluton data from the Libá profile (squares in Fig. 2), the use of two sections seems to be sufficient as a consequence of the data gap in the middle part of the epicentral distances. The result of the travel time smoothing and, consequently, the velocity-depth distribution depend on the separation of the observation points into the individual sections. We tested several possibilities of the point separation. In the case that the first section is formed by all points from the Smrčiny-Fichtelgebirge pluton, we get an oscillation of derivatives of the smoothed travel times, which must be than removed prior to the WH computations. If the first section is shorter, formed only by 6 points from the Smrčiny-Fichtelgebirge pluton, and the second section contains all the remaining points (the sixth point belongs to both sections), the oscillation of derivatives is suppressed. The coefficients of the obtained approximation, using L2 norm, are as follows: a_1=0, b_1=0.2380, c_1=-0.005113, a_2=0.1714, b_2=0.1789 and c_2=-0.0001213. The corresponding smoothed travel-time curve is shown in Fig. 2, and the corresponding velocity-depth distribution in Fig. 3 (dashed lines). The velocity at the surface is equal to 4.20 km/sec. We have also tested other divisions of the points into sections with similar results.

4. RATIONAL APPROXIMATION OF A TRAVEL-TIME CURVE

Rational functions, i.e. quotients of two polynomials, are used to approximate complicated functions, e.g., functions in vicinities of singularities. In seismology, rational functions are used routinely to approximate transfer functions of seismographs.

The rational approximation extends the possibilities of the polynomial approximation, as each polynomial is a special rational function with the unit denominator. Rational functions can describe rather well the specific features of the travel-time curves, mentioned above.

Consider a general rational approximation of a travel-time curve,

$$t(r) = \frac{P(r)}{Q(r)} \ , \tag{3}$$

where the polynomials *P* and *Q* are of the form

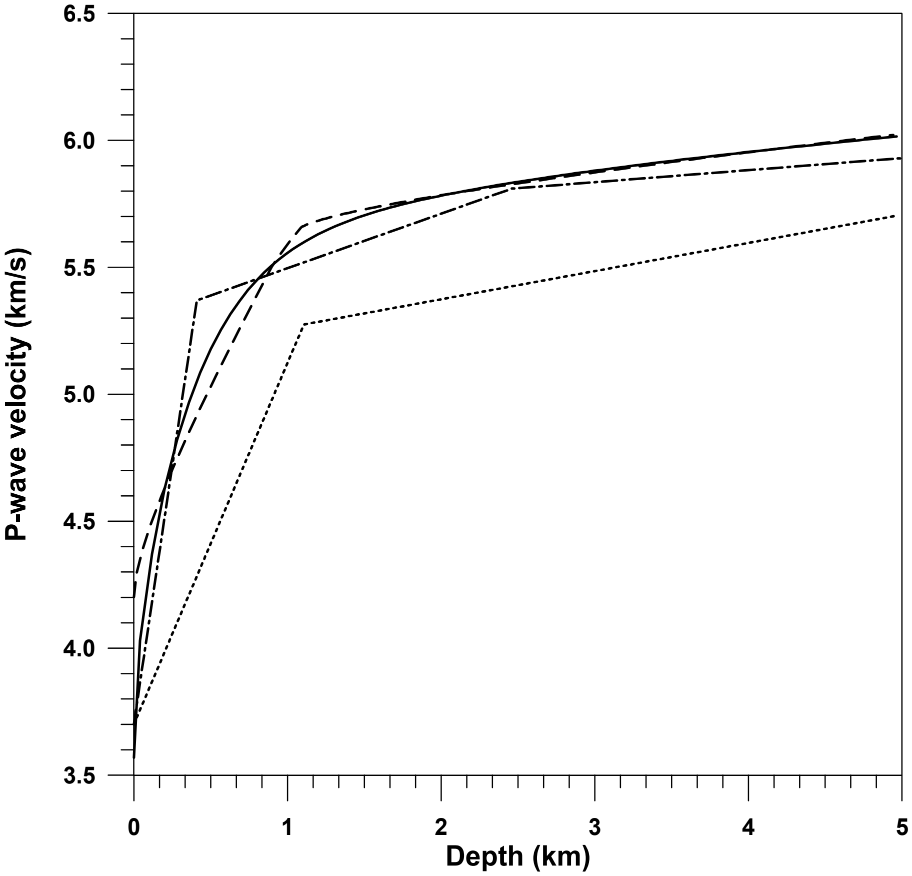

Fig. 3 Velocity-depth distributions obtained by the WH method from the pluton travel times using the two-section polynomial approximation (dashed line) and rational approximation (solid line). For comparison, two piecewise linear models are shown by the dotted line (Málek et al., 2000) and by the dot-and-dash line (Málek et al., 2004b).

$$P(r) = \sum_{m=0}^{M} a_m r^m \;, \quad Q(r) = \sum_{n=0}^{N} b_n r^n \;. \tag{4}$$

To get $t = 0$ for $r = 0$, we always set $a_0 = 0$. Moreover, as the numerator and denominator in formula (3) can be divided by an identical constant, we usually choose $b_N = 1$ (or $b_0 = 1$).

The determination of the unknown coefficients a_i and b_j represents a solution of the corresponding inverse problem. The problem is linear in coefficients a_i, but non-linear in b_j. Consequently, a method for solving non-linear inverse problems must be used.

We have chosen a variant of the method of conjugate gradients (Tarantola, 1987; Press et al., 1992). The partial derivatives with respect to coefficients a_m and b_n, which are also needed in the method, can be computed analytically. Differentiating formula (3), one gets easily

$$\frac{\partial t}{\partial a_m} = \frac{r^m}{Q} \;, \quad \frac{\partial t}{\partial b_n} = -\frac{r^n P}{Q^2} = -t \frac{\partial t}{\partial a_n} \;. \tag{5}$$

We use a computer code based on the general formulae (3) to (5). Moreover, our generalisation of the method of conjugate gradients makes it possible to control the variability of the individual coefficients during the inversion. In addition to an initial value of the coefficient, its variability (usually between 0 and 1) is also given in the input data. For example, if the variability of a coefficient equals zero, the respective coefficient remains fixed. This modification makes it possible to simulate various special approximations using one general computer code.

The simplest rational approximation of a travel-time curve, which we have used frequently, is of the form

$$t(r) = \frac{a_1 r + a_2 r^2}{b_0 + r} \;. \tag{6}$$

This approximation is described by three coefficients only. The surface velocity is then approximated by b_0/a_1, and the value of $1/a_2$ approximates the velocity at a large depth (depth of the deepest turning point).

Formula (3) was also used to approximate the measurements on the plutons along the Libá profile. However, since the travel-time curve is rather long, a better fit was obtained for a more general approximation in the form

$$t(r) = \frac{a_1 r + a_2 r^2 + a_3 r^3}{b_0 + r} , \qquad (7)$$

which is characterised by four coefficients. We shall describe the results for this approximation.

Consider the travel time data shown in Fig. 2 by squares. After several preliminary tests using formula (6), we chose the initial values of the coefficients in (7) as follows:

$$a_1 = 0.7,\ a_2 = 0.17,\ a_3 = 0,\ b_0 = 2.5 , \qquad (8)$$

still assuming the distance given in kilometres and time in seconds. In several iterations, we arrived at the final values

$$a_1 = 0.7005,\ a_2 = 0.1753,\ a_3 = -0.00008467,\ b_0 = 2.50 \qquad (9)$$

The smoothed travel-time curve (7) with coefficients (9) is shown in Fig. 2 by the solid line. This curve is close to the curve obtained by the polynomial approximation.

Using the smoothed travel-time curve (the solid line in Fig. 2), and the integration from zero to a distance of 56 km with a step of 0.5 km, the WH method yielded the velocity-depth distribution shown in Fig. 3 by the solid line. The velocity distribution starts at a surface velocity of 3.57 km/sec, and terminates at a depth of nearly 5.0 km with a velocity of 6.02 km/sec.

Figure 3 also shows two piecewise linear velocity models derived by Málek et al. (2000) using data from earthquake swarms only (dotted line), and by Málek at al. (2004b) using jointly both the data from earthquake swarms and from older refraction measurements (dot-and-dash line). The latter velocity-depth distribution is very close to the velocity-depth distributions obtained by the WH method, showing so the importance of surficial shots in studies of shallow structures.

5. LAYERED VELOCITY MODELS

Vertically inhomogeneous structures are frequently approximated by velocity models composed of homogeneous layers. To get a comparison with WH approximation, let us apply such an approximation to the Libá data.

According to the form of the Libá reduced travel times, three homogeneous layers should be appropriate for this approximation. The resulting reduced travel time, obtained again using the L2 norm, is shown in Fig. 4 as dotted line. The layer velocities are 4.60, 5.68 and 5.97 km/s for the first, second and third layer, respectivelly. The upper boundary of the second layer is at a depth of 0.74 km and the upper boundary of the third layer at a depth of 2.83 km.

The model is, similarly as the WH models, again characterised by low velocity in the upper layer. For comparison, the dot-and-dashed line in Fig. 4 shows the reduced travel time for the model (Málek et al., 2004b) from Fig. 3. This model was derived using data from broader area of the West Bohemia/Vogtland region, which explains the deviations from the special Libá data.

6. CONCLUSIONS

Geologically complicated regions, such as the seismoactive region of western Bohemia, exhibit significant three-dimensional distributions of seismic velocities. However, the individual geological units seem to be more laterally homogeneous, with velocity variations dependent predominantly on depth. This substantiates the approximation of the individual geological units with vertically inhomogeneous models (1-D models). In our opinion, the classical Wiechert-Herglotz (WH) method can still be used to determine such models if an adequate smoothing of the travel-time curves is applied. In the present paper, two methods of smoothing have been proposed and described in detail, namely a variant of the polynomial approximation and rational approximation.

In spite of using two different analytical forms to smooth the observed travel times from the plutons along the Libá refraction profile, both the velocity-depth distributions obtained by the WH method are close to each other. The difference is more pronounced only for the surface velocity value.

The most prominent feature of the derived velocity-depth distribution is the existence of a superficial zone of low *P*-wave velocities. This velocity distribution is in agreement with laboratory measurements of seismic velocities on rock samples (Martínková et al., 2000), and with structural studies using a combination of earthquake and refraction data (Málek et al., 2003b). On the other hand, this result improves the models of the upper crust obtained by the methods of deep seismic sounding. It is surprising that such a prominent low-velocity layer has not been recognised in these previous studies. This emphasises the importance of the WH method as a powerful method of studying the geological units with significant vertical inhomogeneities. The method can especially be recommended for studying shallow crustal structures, where prominent velocity variations with depth are frequently observed.

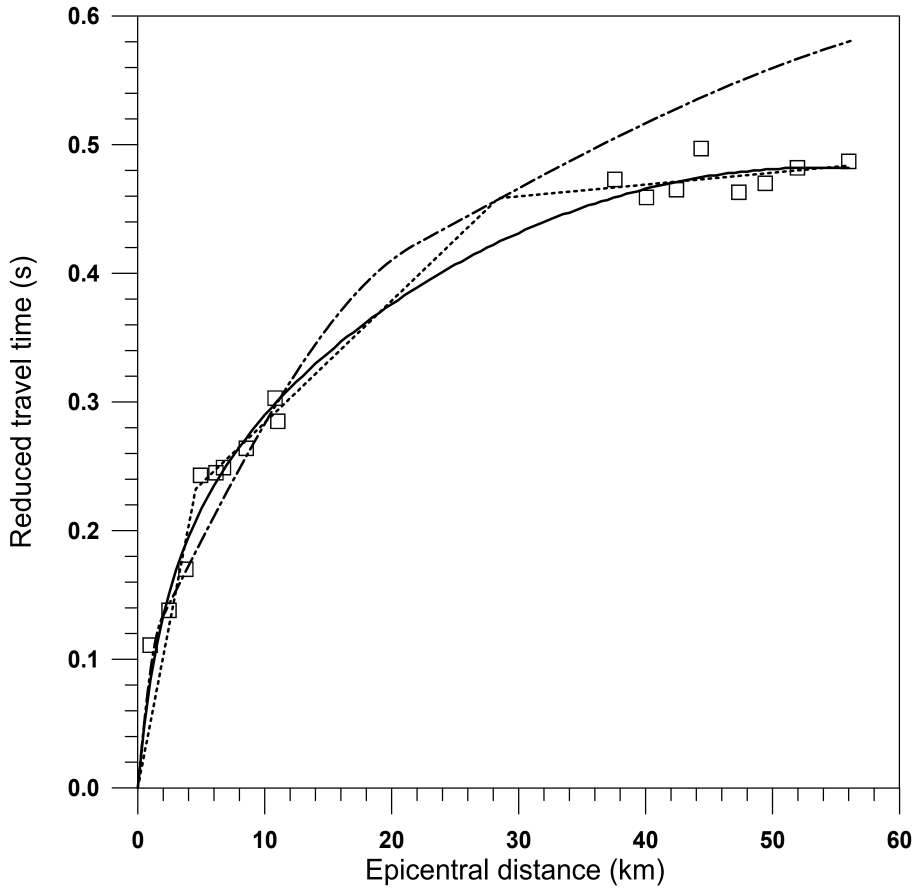

Fig. 4 *P*-wave travel times for the Libá profile and Skalná station, reduced with a velocity of 6.0 km/s (squares); their rational approximation (solid line) and approximation by three homogeneous layers (dotted line). For comparison, the reduced travel time for the Málek et al., 2004b model from Fig. 3 is shown by the dot-and-dash line. For details, see the text.

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