# INTEGRATION OF EPOCH-WISE GPS MEASUREMENT CAMPAIGNS INTO A PERMANENT REFERENCE FRAME

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#### ABSTRACT

Precise GPS geodetic observations are frequently used for identification and quantification of recent movements of Earth's crust. The GPS technique is preferred because of its precision, availability of observing instruments, automated observations, and high coordinates repeatability. Two kinds of GPS observations applied for geodynamic investigations are performed, namely the epoch-wise and permanent. The epoch networks allow to monitor the selected territory in detail, however the short observation period is influenced by systematic phenomena, which complicate the interpretation of obtained crustal movements. The permanent monitoring yields a more realistic insight into the crustal kinematics, however is restricted to limited number of stations. The paper presents the mathematical model for complex analysis of long-term observations and consecutively the method of inclusion of epoch campaign coordinates and covariance matrices into the permanent network adjustment.

KEYWORDS: processing and analysis of GPS time series, geodynamic monitoring, GPS networks

#### 1. INTRODUCTION

GPS monitoring applied to geodynamic investigations according to the observation strategy could be separated into two categories:

- Permanent measurements of the GPS satellite signals at stations especially equipped for continuous observations of long-term character. Usually stable sites with suitable infrastructure are selected as permanent GPS stations with the aim to serve as a reliable reference for precise GPS positioning. The number and distribution of permanent GPS stations is steadily increasing, however it is limited because of practical and economical reasons.
- Epoch GPS observations at temporarily occupied stations placed usually directly at sites where the recent movements are expected. The epoch observations are periodically repeated at identical points with the aim to detect slight changes of geometric position. The possibilities of establishing and systematic re-occupation of epoch stations in the area of interest are much larger when compared to permanent stations.

Detection of slight movements of monitored points at the Earth's surface is enabled by comparison of geocentric coordinates obtained at different epochs. Two or more repeated observing campaigns have to be analysed in a common reference frame to obtain mean geocentric position and velocity of the monitored point. However the experience from continuously observed sites indicates that the behaviour of coordinates obtained from repeated GPS observations is more complex and besides the linear trend (estimated as velocity) reflects also periodic, irregular and sudden changes of various origins. The network of permanent stations usually serves as the reference frame for velocity estimates from epoch campaigns (Becker et al., 2002, Davies & Blewitt, 2000).

In the paper we firstly present the mathematical model for complex analysis of long-term permanent network observations. Statistical methods taking into account the covariance matrices of daily network solutions are applied. The adjustment procedure includes stations mean positions, velocities, seasonal variation amplitudes, and systematic jumps causing discontinuities in the observation series.

In the second step we introduce the method of inclusion the epoch campaign coordinates and covariance matrices into the permanent network adjustment. This combination yields coordinates and velocities of epoch stations in a frame, which is homogeneous with the frame of permanent network. This procedure minimises the effects of influence of individual permanent stations on the estimated velocities of epoch sites. The outlined approach will be demonstrated on the Central Europe network of permanent stations analysed within the international project CERGOP-2/Environment (Fejes & Pesec, 2003, Hefty, 2003). The project is designated as a multipurpose and interdisciplinary sensor array for environmental research in Central Europe with the dominant role of GPS technique. The Central European GPS geodynamic reference network (CEGRN), which is a combination of permanent and epoch GPS sites, serves as the main source of data for regional and local investigations.

#### 2. LONG-TERM BEHAVIOUR OF GPS PERMANENT MONITORING

The systematic worldwide tracking of GPS satellites at permanent sites and its processing started in 1992 when the foundations of International GPS Service for Geodynamics (IGS, later renamed to International GPS Service) were defined (International GPS Service, 2003). In 1996 the EUREF Permanent Network (EPN) concentrating on the European continent started its activity. Since the establishing of these networks a more than 10-years experience has been gained. It is evident nowadays that the long-term GPS monitoring at permanent sites is a complex phenomenon influenced by variety effects. They are manifested as:

- Linear trend resulting from global plate motion and intraplate motions.
- Non-linear variations due to seismic, slip, technogene phenomena and slowly changed obstacles in signal reception.
- Seasonal variations due to thermic effects, snow, ice and ground water changes, as well as due to monumentation instability.
- Long-term periodic phenomena of non-seasonal nature.
- Sudden jumps and pulses due to variety sources like reference frame issues, changes of equipment receivers, antennae, radomes, permanent station environment, and sudden obstacles in signal reception.

For illustration of above-mentioned phenomena we reproduce some time series from EPN analysis (Bruyninx and Carpentier, 2004). The plots show 8year series of weekly coordinates of four Central European sites. In the graphs the global Eurasian plate motion has been removed. The reproduced series are represented by variations in north-south, east-west and up components.

The time series of Borowiec station (BOR1) on Fig. 1 shows steady behaviour with only one jump due to instrumental change. Remarkable is the systematic height decrease accompanied with significant seasonal variations in the last part of the series.

The effects of instrumental changes, namely the antenna radome manipulations are typical for the first

period of Modra-Piesok (MOPI) station observations on Fig. 2. After fixing the radome the coordinates became quite stable, with small linear trend in north and east components. The occasional height outliers are due to snow and ice layers on the antenna radome (Hefty, 2001).

Effect of the antenna malfunction is demonstrated on Lamkowko (LAMA) station (Fig.3). The stable linear behaviour was interrupted by nonlinear episode reported as an consequence of antennae failure. After replacement of antenna the linear trend is continuing.

Finally, the very significant seasonal effects are typical for Innsbruck – Hafelekar (HFLK) station. The possible reasons of nearly 10 mm amplitude of north-south variations (Fig. 4) are discussed in (Cristea et al., 2004).

#### 3. MATHEMATICAL MODEL FOR COMBINATION OF DAILY SOLUTIONS OF PERMANENT NETWORK

According to examples in Figs 1-4 a model for combined processing of long-term permanent data has to include a variety of phenomena. The general observation equation for geocentric coordinate  $x_t$  determined at epoch t as a result of daily processing (observation interval 0 - 24 hour) reads

$$\begin{aligned} x_{t} &= x_{0} + v_{x} \cdot (t - t_{0}) + b_{x} \sin\left(\frac{2\pi \cdot (t - t_{0})}{365}\right) + \\ &+ c_{x} \cos\left(\frac{2\pi \cdot (t - t_{0})}{365}\right) + d_{x} \sin\left(\frac{4\pi \cdot (t - t_{0})}{365}\right) + \\ &+ e_{x} \cos\left(\frac{4\pi \cdot (t - t_{0})}{365}\right) + \sum_{k=1}^{r} z_{xk} \delta(t - t_{k}) + \varepsilon_{x} \end{aligned}$$
(1)

where

 $\mathcal{E}_{x}$ 

 $x_0$  mean coordinate at reference epoch  $t_0$  $v_x$  velocity

- $b_x, c_x, d_x, e_x$ , amplitudes of seasonal variations (the harmonic development is usually applied also if variations are cyclic)
- $z_{xk}$  amplitudes of sudden changes and pulses  $\delta(t-t_k)$  function describing the time dependence of phenomenon occurred at epoch  $t_k$ 
  - observation error

The symbolic name  $x_t$  is used for geocentric coordinates  $X_j$ ,  $Y_j$ , and  $Z_j$  of *j*-th site (j = 1, 2, ..., n), observed at epoch  $t_i$  where *n* denotes number of processed sites in the permanent network. In this paper we do not deal with sub-daily coordinate variations, these issues are analysed e.g. in (Hefty et al., 2004). However we emphasise their possible effect on long-term cyclic variations due to aliasing effects (Penna & Stewart, 2003)

If the network coordinates obtained from processing of observations related to epoch  $t_i$  are grouped in vector  $\mathbf{x}_{ti}$  then model of common



Fig. 1 Time series of site coordinates transformed to north, east and up components at the permanent site BOR1 (Bruyininx and Carpentier, 2004)



Fig. 2 Time series of site coordinates transformed to north, east and up components at the permanent site MOPI (Bruyininx and Carpentier, 2004)



Fig. 3 Time series of site coordinates transformed to north, east and up components at the permanent site LAMA (Bruyininx and Carpentier, 2004)



Fig. 4 Time series of site coordinates transformed to north, east and up components at the permanent site HFLK (Bruyininx and Carpentier, 2004)

processing of *m* daily network solutions (i = 1, 2, ..., m) will be

$$\begin{bmatrix} \mathbf{x}_{t_1} \\ \mathbf{x}_{t_2} \\ \vdots \\ \mathbf{x}_{t_m} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1 & \mathbf{D}_1 & \mathbf{H}_1 & \mathbf{S}_1 \\ \mathbf{J}_2 & \mathbf{D}_2 & \mathbf{H}_2 & \mathbf{S}_2 \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{J}_m & \mathbf{D}_m & \mathbf{H}_m & \mathbf{S}_m \end{bmatrix} \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{v}_x \\ \mathbf{a} \\ \mathbf{z} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_m \end{bmatrix},$$

$$\Sigma_x = \begin{bmatrix} \Sigma_{x_1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \Sigma_{x_2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \Sigma_{x_m} \end{bmatrix}$$
(2)

where the unknown parameters are

 $\mathbf{x}_0$  permanent network coordinates at epoch  $t_0$ 

- $\mathbf{v}_x$  velocities
- a amplitudes of seasonal terms
- **z** amplitudes of jumps and pulses

The design matrices **J**, **D**, **H** and **S** relate the coordinates obtained from the daily adjustments to parameters common for whole estimated period. The covariance matrix  $\Sigma_x$  is of block diagonal type, assuming that the daily solutions are mutually uncorrelated. The treatment of observations as uncorrelated does not strictly correspond to reality. However it will not corrupt the estimates of unknown parameters but tends to result in too optimistic measures of uncertainties.

The model (2) assumes that all the daily solutions  $\mathbf{x}_{ti}$  are referred to one common reference frame. Such requirement is usually hard to fulfil, especially for longer series of observations. The alternative model respecting the possible alterations of reference frames is given below.

To deal with reference frame problem the enlargement of the model (2) is necessary. Each daily solution (except the reference one) is completed by set of transformation parameters **r** relating the daily coordinates to the reference set. To ensure a slovability of the problem a set of reference velocities  $\mathbf{v}_0$  with covariance matrix  $\boldsymbol{\Sigma}_{\nu}$  have to be introduced as fictive observations. Then the model reads

$$\begin{bmatrix} \mathbf{x}_{t1} \\ \mathbf{x}_{t2} \\ \vdots \\ \mathbf{x}_{tm} \\ \mathbf{v}_{0} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{1} & \mathbf{0} & \mathbf{D}_{1} & \mathbf{H}_{1} & \mathbf{S}_{1} \\ \mathbf{J}_{2} & \mathbf{T}_{2} & \mathbf{D}_{2} & \mathbf{H}_{2} & \mathbf{S}_{2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{J}_{m} & \mathbf{T}_{m} & \mathbf{D}_{m} & \mathbf{H}_{m} & \mathbf{S}_{m} \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_{v} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{0} \\ \mathbf{r} \\ \mathbf{v}_{x} \\ \mathbf{a} \\ \mathbf{z} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1} \\ \varepsilon_{2} \\ \vdots \\ \mathbf{a} \\ \mathbf{z} \end{bmatrix},$$

$$\mathbf{\Sigma}_{x} = \begin{bmatrix} \Sigma_{x_{1}} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Sigma_{x_{2}} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \Sigma_{x_{m}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \Sigma_{v} \end{bmatrix}$$
(3)

Design matrices  $T_i$  comprise of coefficients for translation and rotation parameters. In the above equation for clarity the first epoch is chosen as the reference. Generally, any other epoch could be taken as the reference.

Reference velocities  $v_0$  define the character of estimated velocities  $v_x$ . If the ITRF velocities of reference sites are chosen, the estimated velocities of all sites will be of absolute character (including the global plate motion). If the intraplate velocities serve as the reference, the estimated motions will reflect the relative movements of monitored sites.

The models (2) and (3) reflect all the systematic changes mentioned in section 2. It is worth to mention that it is necessary to define a-priori the type of function  $\delta(t - t_k)$  and the epochs  $t_k$  of appearance of changes, like antennae or receiver replacements, station environment changes etc. The most frequent type of function  $\delta(t - t_k)$  is to set it to zero for  $t < t_k$  and set to one if  $t \ge t_k$ . Other possibilities are to set  $\delta(t - t_k)$ as linear or polynomial function of t. The check on an appropriate parameterisation is that all the n residual time series of  $\{X_j\}$ ,  $\{Y_j\}$ ,  $\{Z_j\}$  coordinates reduced for estimated parameters have to be continuous.

#### 4. MODEL FOR INTEGRATION OF EPOCH CAMPAIGNS INTO THE PERMANENT NETWORK SOLUTION

The solution of eqs. (2) or (3) yields to a consistent set of coordinates and velocities of permanent network regarding all systematic variations. If in these equations p epoch solutions  $\mathbf{x}_{ek}$  (k = 1, 2, ..., p) are introduced, the compatibility of epoch networks with permanent network will be ensured.

For the sake of simplicity let's consider model (2) assuming that all permanent and epoch solutions are related to a common reference frame. Then the enlarged model will become

$$\begin{bmatrix} \mathbf{x}_{t_{1}} \\ \vdots \\ \mathbf{x}_{t_{m}} \\ \mathbf{x}_{e_{1}} \\ \vdots \\ \mathbf{x}_{e_{p}} \\ \mathbf{v}_{0} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{1} & \mathbf{D}_{1} & \mathbf{H}_{1} & \mathbf{S}_{1} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{J}_{m} & \mathbf{D}_{m} & \mathbf{H}_{m} & \mathbf{S}_{m} \\ \mathbf{J}_{e^{1}} & \mathbf{D}_{e^{1}} & \mathbf{0} & \mathbf{S}_{e^{1}} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{J}_{ep} & \mathbf{D}_{ep} & \mathbf{0} & \mathbf{S}_{ep} \\ \mathbf{0} & \mathbf{D}_{v} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{0} \\ \mathbf{v}_{x} \\ \mathbf{a} \\ \mathbf{z} \end{bmatrix} + \begin{bmatrix} \varepsilon_{n} \\ \varepsilon_{e^{1}} \\ \varepsilon_{e^{1}} \\ \vdots \\ \varepsilon_{e^{p}} \\ \varepsilon_{v} \end{bmatrix},$$

$$\Sigma_{x} = \begin{bmatrix} \Sigma_{x_{1}} & \cdots & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots & & \vdots \\ \mathbf{0} & \cdots & \Sigma_{x_{e^{1}}} & \cdots & \mathbf{0} \\ \vdots & & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \cdots & \Sigma_{v} \end{bmatrix}$$

$$(4)$$

where the symbols with indices e relate to the epoch measurement campaigns. Note that the epoch observations are used to estimate the coordinates and velocities only (very exceptionally the model could



Fig. 5 Central European permanent stations analysed at SUT in Bratislava within the CERGOP-2/Environment.

introduce also jumps and pulses amplitudes for epoch stations) but no for seasonal parameters. Enlargement of the model (4) by addition of transformation parameters (similarly to transition from (2) to (3)) is straightforward.

## 5. APPLICATION: CENTRAL EUROPE PERMANENT NETWORK

The models (2) - (4) are very complex and are demanding appropriate software package development. The permanent network comprising 30 sites leads to nearly 40 000 observing equations with 2000 unknowns per year (note that transformation parameters are estimated for each set of daily coordinates). At the Slovak University of Technology (SUT) in Bratislava within the CERGOP-2/Environment project (Fejes and Pešec, 2003) permanent network comprising of more than 30 sites is analysed since March 2003 (Hefty, 2003). This activity is continuation of analysis of regional GPS network at SUT which started in 2000 (Hefty and Kártiková, 2000). Distribution of stations included in the network from 2003 is given in Fig. 5. The set of stations is

progressively increased when a new appropriate station in region is available. The running processing of observed permanent and epoch data follows the strategy outlined above.

Software package for simultaneous analysis of permanent and epoch observations is now under the completion at SUT. It is intended to include any relevant local or regional epoch networks into the permanent network frame if the solutions are available in form of geocentric coordinates and their covariance matrices. First experience is gained with CERGOP2 epoch campaign performed in June 2003 and with long-term GPS epoch observations performed in High Tatra Mountains.

#### 6. CONCLUSIONS

An uniform reference frame for geokinematic investigation related to an area of interest is of essential importance for relevant interpretation of long-term GPS monitoring. The principles of common simultaneous processing of permanent and epoch GPS observation data are formulated. The permanent observations enable to estimate mean coordinates, velocities, seasonal variations of monitored sites as well as jumps in time series due to various phenomena of instrumental and environmental origin. Moreover the permanent network could serve as a reference frame for any GPS activities in the region. The epoch observations are integrated into the permanent processing and enable to adjust mean coordinates and velocities of epoch stations in a common reference frame. The realisation of outlined procedure concentrating on stations and epoch projects in Central Europe is ongoing at the SUT in Bratislava.

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## REFERENCES

- Becker et al.: 2004, A Regional ITRF Densification by Blending Permanent and Campaign Data – The CEGRN campaigns and the Central European Velocity Field. In: Adam and Schwarz (eds.) Vistas for Geodesy in the new Millennium. IAG Symposia, Vol. 125. Springer. p. 53-58.
- Bruyninx, C. and Carpentier, G.: 2004, Plots of EPN time series. www.epncb.oma.be.
- Cristea, E., Haslinger, C. and Stangl, G.: 2004, Hafelekar GPS permanent station seasonal monitoring. Presented at: 10 years IGS, Celebrating a Decade of the International GPS Service. Berne, March 01-05, 2004.
- Davies, P. and Blewitt, G.: 2000, Methodology for global geodetic time series estimation: A new tool for geodynamcs. Journal of Geophysical Research, 105, No. B5, p. 11083 – 11100.

- Fejes, I. and Pešec, P.: 2003, CERGOP-2/Environment – a challenge for the next 3 years. Proc. of EGS-AGU-EUG G17 Symposium, Nice 2003. Reports on Geodesy No. 1 (64), p. 13-22.
- Hefty, J.: 2001, The permanent Modra-Piesok GPS station and its long-term and short-term stability. Slovak Journal of Civil Engineering 1-2, Vol. 9, p. 31-37.
- Hefty, J.: 2003, Work package 5 of CERGOP-2/environment: GPS data analysis and the definition of reference frames. Reports on Geodesy No. 3 (66), p. 55-57.
- Hefty, J. and Kártiková, H.: 2000, Network of GPS permanent stations in central Europe and its significance for geodynamics research and geocentric reference frame realisation. Proc. of conference at 50 anniversary of Institute of Geodesy and cartography,Bratislava. p. 65-75. (in Slovak)
- Hefty et al.: 2004. Sub-daily site coordinates variations in EUREF permanent network. Presented at: 10 years IGS, Celebrating a Decade of the International GPS Service. Berne, March 01-05, 2004.
- International GPS Service: 2003, IGS Information and Resources. Pasadena: IGS Central Bureau, 45 p.
- Penna, N. T. and Steward, M. P.: 2003, Aliased tidal signatures in continuous GPS height time series. Geophysical research letters, 30, No. 23, 2184, p. 2184.