Acta Geodyn. Geomater., Vol.2, No.3 (139), 87-94, 2005

COMBINING GPS AND VLBI MEASUREMENTS OF CELESTIAL MOTION OF THE EARTH'S SPIN AXIS AND UNIVERSAL TIME

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(*Received* January 2005*, accepted* February 2005)

ABSTRACT
GPS and VLBI are used to measure global parameters of Earth's orientation in space – polar motion, celestial motion of the spin axis, and universal time (defining the Earth's angle of rotation around the spin axis). Polar motion (i.e., the motion of the spin axis with respect to terrestrial frame) is observed by both methods. However, only VLBI is capable to observe the latter two components directly (i.e., with respect to celestial frame); GPS, because of simultaneously determined orientation of the satellite orbits, can observe only their time derivatives. The method of "combined smoothing", recently developed at the Astronomical Institute in Prague, is used to provide a unique combined solution that fits well both to the values observed by VLBI and their rates observed by GPS. Two solutions are presented here:
Celestial pole offsets (observed by VLBI) combined with their rates (observed by GPS);
Universal time (observed by VLBI) combined with length-of-day changes (observed by GPS).
The results of these combinations, covering the last decade, are analyzed. It is demonstrated that the combination improves both resolution and accuracy of the results with respect to VLBI-only solution.

KEYWORDS: geodynamics, Earth orientation, space techniques, combinations

1. INTRODUCTION

There are five Earth Orientation Parameters (EOP), characterizing the full orientation of the Earth in space:

- Precession-nutation (given by two angles with respect to celestial frame) describing the motion of the Earth spin axis (or better the Celestial Intermediate Pole – CIP) in space. In practice, their relatively small offsets from an adopted model (celestial pole offsets) are measured and published. The CIP moves around the pole of ecliptic once per about 26 thousand years (precession), and at the same time it exhibits, around the mean position, a great number of small elliptic motions (nutation). The largest, so called principal term of nutation, has a semi-major axis of about 9.2" and period 18.6 y. By definition, all these motions have periods longer that 2 days; all motions with shorter periods are interpreted as polar motion. The exact definition of CIP and other discussions concerning the most recent IAU model of precession-nutation can be found in Capitaine et al. (2002). Celestial pole offsets are measured directly by Very Long-Baseline Interferometry (VLBI), Global Positioning System (GPS) is capable of measuring only its time rate.
- Proper rotation that expresses the diurnal rotation of the Earth around the CIP. Strictly speaking, the rotation is not uniform; it is measured by means of Universal Time UT1, and expressed as its difference from a uniform time scale, TAI or UTC. Roughly, UT1 is the angle between the zero meridian and the mean equatorial Sun, expressed in time units. Its negative daily variation, length-of-day (LOD), expresses the excess of the length of instantaneous day over the nominal 24 hours. UT1 is measured by VLBI, GPS can measure only LOD.
- Polar motion (given by two angles with respect to terrestrial frame) that describes the motion of the CIP within the Earth, around the *z*-axis of the International Terrestrial Reference Frame (ITRF). The amplitude of this motion never exceeds 0.5", and the periods cover the whole spectrum from sub-diurnal to several decades. This motion is measured directly by all space techniques, such as VLBI, GPS, Satellite Laser Ranging (SLR) or DORIS (Doppler Orbit determination and Radiopositioning Integrated on Satellite).

In the following, we shall concentrate on only the first two items, i.e., the celestial pole offsets and Universal Time. To combine the independent observations by two main techniques, VLBI and GPS, we use our recently developed method of so called 'combined smoothing' (Vondrák and Čepek, 2000).

Generally, we have two independently measured series:

- values of a function (celestial pole offsets, universal time) measured by VLBI. These observations are characterized by low time resolution (typical interval between two measurements is several days), unequally spaced data and the direct link to celestial frame, from which follows their long-term stability.
- values of the time derivative of the same function (celestial pole offset rates, length-of-day) measured by GPS. These observations have high time resolution (they are made in strictly daily intervals), but they have only indirect link to ICRF (via satellite orbits and their modeled motions with respect to celestial frame), from which follows their long-term instability.

We combine these two series to obtain the solution that is sufficiently smooth, whose function values fit well to the first measured series, and whose time derivatives fit well to the second measured series. To this end, we minimize the weighted combination of three constraints:

$$Q = S + \varepsilon F + \bar{\varepsilon}\bar{F} \ , \qquad (1)$$

where S is the measure of smoothness (smaller value means smoother solution), $F, \bar{F}$ are measures of 'fidelity' of the results to both series of observations in a least-squares sense (smaller value means a better fit). The weighting factors $\varepsilon, \bar{\varepsilon}$ are so called 'coefficients of smoothing', defining to which extent the solution is to be smoothed – smaller values lead to smoother solution whose values are more deviated from the observations, and vice versa. These coefficients have dimensions (time unit)$^{-6}$ and (time unit)$^{-4}$ respectively, where (time unit) is the unit of time used in the argument. It has been demonstrated that the solutions obtained by this method take over the long-term stability of the first series and preserve the short-periodic signal of the second one (Vondrák et al., 2002).

2. DATA USED IN THIS STUDY

In this study we use the data provided by the International VLBI Service (IVS) and International GPS Service (IGS):

1. IVS solutions, covering the interval 1979.6-2004.6, provided at unequally spaced intervals 1-7 days long:
   - Series ivs04q3x.eops, containing all EOP, celestial pole offsets dX, dY are referred to IAU2000A model of precession-nutation (Mathews et al., 2002);
   - Series ivs04q3e.eops, with the same content, but the celestial pole offsets dε, dψ are referred to older model IAU1980 (Wahr, 1981).
2. European IGS analysis center (CODE) solution, covering the interval 1994.3-2004.6 and containing GPS-based celestial pole offset rates dε/dt, dψ/dt provided at equally spaced 1-day intervals (referred to IAU1980 model);
3. IGS solution igs00p02.erp, covering the interval 1996.5-2004.6, from which we use length of day changes, provided at equally spaced 1-day intervals.

3. COMBINATION OF CELESTIAL POLE OFFSETS

Before using the data for combination, let us have a look at the VLBI data alone, covering a longer time span and therefore more adequate to determine offsets for longest periods. The values dX, dY, referred to the IAU2000A model of precession-nutation, are depicted in Fig. 1. It is clear that the precision of the observations gradually improved. The smoothed values display a quasi-periodic character. Next we interpolated these values for equidistant arguments (step equal to 3 days) and the interpolated values, in a complex form dX + id$Y$, were subject to spectral analysis. To obtain the time evolution of the spectrum we used Fast Fourier Transform (FFT) in complex form, with a sliding Parzen window 7 years wide. Thus we arrived to the spectrum that is shown in Fig. 2. Only the part for positive frequencies is depicted, containing the dominant peak around 0.84 cpy that corresponds to Free Core Nutation (FCN). This motion is caused by the existence of a flattened fluid core of the Earth that can rotate, in principle, around an axis that is not identical with the axis of rotation of the mantle. The period of FCN is dominantly given by the flattening of the core, its amplitude is very small and depends on the excitation. The figure demonstrates that this period was apparently not constant in time – it was more or less constant for most of the period studied, but it increased substantially during the last years.

There are also smaller and less visible peaks in the spectrum; if the values dX, dY are analyzed in more detail, we find long periods of 18.6, 9.3 and 1 year that can be identified with principal nutation periods. Table 1 shows the result of fitting these terms, together with dominant FCN of 435 days, by least-squares method. They represent the corrections of IAU2000A nutation model. Arguments Ω and l' are the mean longitude of the lunar ascending node and mean anomaly of the Sun, respectively.

However, a simple sinusoidal model of FCN is not adequate – both its amplitude and phase change substantially, which can be seen in Fig. 3. Therefore, in order to find a 'real' FCN, we subtract the latter

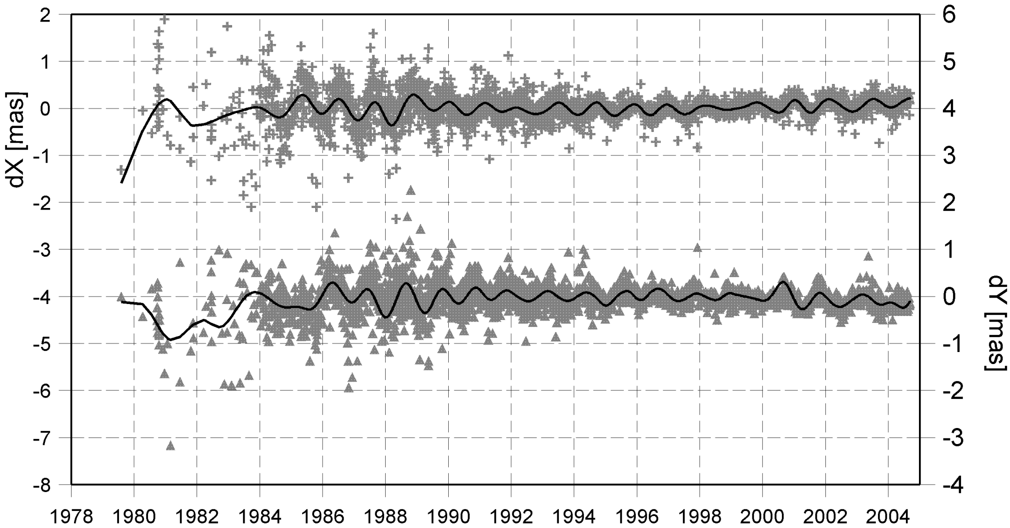

Fig. 1 Differences between the VLBI observations and IAU2000A model. Gray crosses in upper plot (left axis) depict the values d*X*, gray triangles in lower plot (right axis) depict the values d*Y*. Full lines are smoothed values.

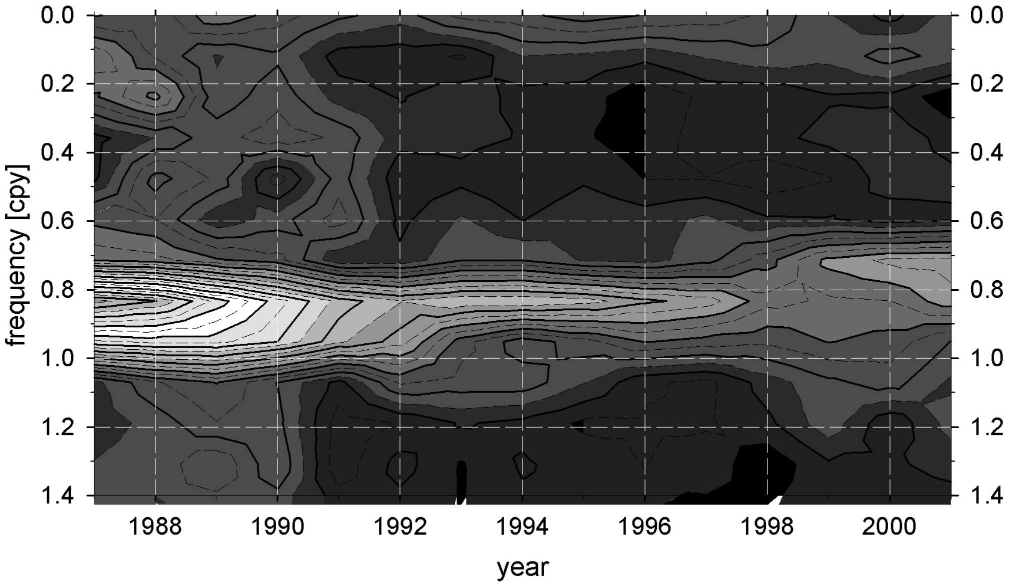

Fig. 2 Time evolution of the spectrum of celestial pole offsets d*X*,d*Y* measured by VLBI.

Table 1 Least-squares fit (FCN + corrections of 3 long-periodic nutation terms) to VLBI celestial pole offsets 1979.6-2004.6

Term, argument	Period [days]	d*X* [μas]		d*Y* [μas]	
		cos	sin	cos	sin
FCN	435	-49.3±4.5	-58.5±4.5	-67.6±5.2	76.3±5.1
Ω	-6798	12.3±5.0	37.8±4.3	-41.9±5.5	-24.4±5.1
2Ω	-3399	-7.3±5.1	3.9±4.3	-7.6±5.7	-46.9±4.9
l'	365.26	11.2±4.5	-15.0±4.6	-17.2±5.1	-44.0±5.2

three long-periodic forced nutation terms from the data and smooth the residuals. The result is depicted, in three dimensions, in Fig. 3, in which the time runs from bottom upwards.

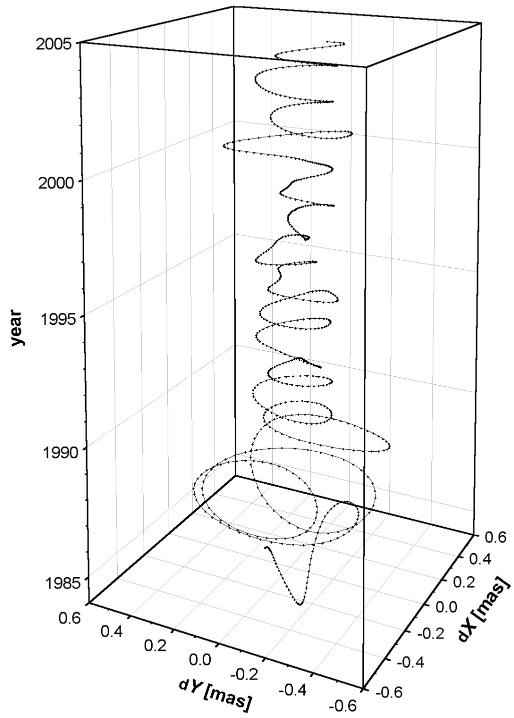

Fig. 3 Free Core Nutation observed by VLBI.

Now we use the method of ‘combined smoothing’ to combine the VLBI and GPS observations of celestial pole offsets, applied to:

- VLBI data (IVS combined solution ivs04qe.eops, in the interval 1994.3-2004.6, referred to IAU1980 model);
- GPS data (CODE, in the interval 1994.3-2004.6, referred to the same model).

Before the combination, the GPS-based offset rates had to be calibrated against the VLBI results, or, in other words, fitted in a long-periodic sense to the VLBI solution. The reason for doing this is to remove the long-periodic part of orientation of GPS orbits that is due to their mismodeling (for more details see Vondrák et al., 2002). The coefficients of smoothing used are $\varepsilon = 1\,\mathrm{day}^{-6}, \bar{\varepsilon} = 0.2\,\mathrm{day}^{-4}$, ensuring that all periodic changes of 3 days and shorter are completely smoothed out.

The combined results in dε, dψ were then corrected for the difference between the new and old model of nutation IAU2000A-IAU1980, and transformed to dX, dY, using simplified formulas, assuring the transformation accuracy of 1 μas in the interval 1900-2100:

$$\begin{aligned} \mathrm{d}X &= 0.39778\,\mathrm{d}\psi + (0.02236\,\mathrm{d}\varepsilon - 0.00021\,\mathrm{d}\psi)T \\ \mathrm{d}Y &= \mathrm{d}\varepsilon - 0.00889\,\mathrm{d}\psi\,T, \end{aligned} \tag{2}$$

where T is the time in centuries counted from the epoch J2000.0. The result is shown in Fig. 4, where the gray crosses in upper part (left scale) and gray triangles in lower part (right scale) denote the VLBI-based values of dX, dY, respectively, and the full lines the values obtained as their combination with GPS observations. It is evident that the combination contains more signal in short-periodic range of the spectrum than the VLBI observations alone.

The spectrum of the combined solution is depicted in Fig. 5 – the full one in upper part, its zoomed central part (long periods) in lower part. The dashed line represents the 95% significance level. It is clear that the periodic changes with periods 460 days (FCN), 365, 182, 30, 14 and 5.8 days (nutation terms) are significant. Therefore we removed the longest periods (18.6 and 9.3 years) found earlier from 20-year long VLBI observations, and made a least-squares fit of the terms with these periods to the corrected series. The results are shown in Table 2.

In addition to the arguments Ω, l' defined above, F denotes the mean argument of the latitude of the Moon, D is the mean elongation of the Moon from the Sun and l is the mean anomaly of the Moon.

Table 2 Least-squares fit (FCN + corrections of 7 short-periodic nutation terms) to combined VLBI/GPS celestial pole offsets 1994.3-2004.6

Term, argument	Period [days]	dX [μas] cos	dX [μas] sin	dY [μas] cos	dY [μas] sin
FCN	460	-23.4±3.5	-85.7±3.6	-93.5±4.3	37.4±4.4
l'	365.26	42.0±3.6	-5.7±3.6	5.7±4.3	-35.1±4.4
$2F$-$2D$+2Ω	182.62	-18.7±3.6	7.6±3.6	-10.8±4.4	38.8±4.4
l-$2D$	-31.81	-12.2±3.6	1.5±3.6	-14.3±4.4	6.1±4.4
l	27.55	-3.2±3.6	-4.9±3.6	-9.5±4.4	6.7±4.4
$2F$+2Ω	13.66	-24.7±4.3	-8.7±4.3	18.4±5.3	-3.3±5.3
$2F$+Ω	13.63	-14.9±4.3	1.7±4.3	-5.4±5.3	14.8±5.3
l+$2F$+$2D$+2Ω	5.64	-5.3±3.7	-14.4±3.6	-32.8±4.4	6.5±4.4

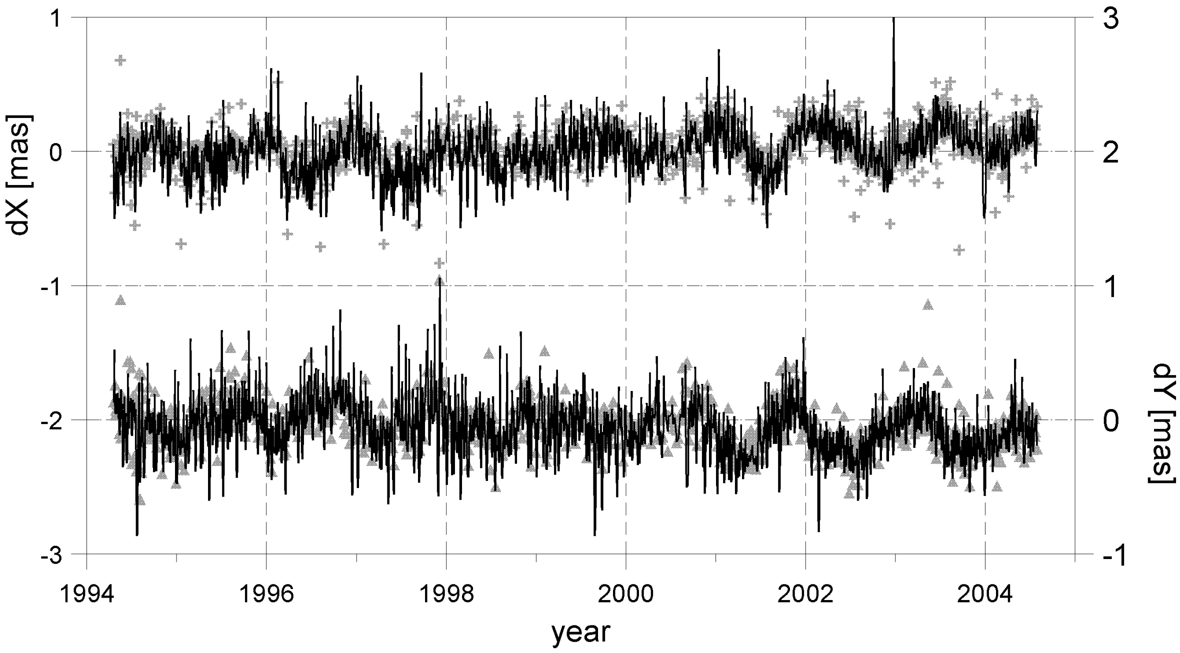

Fig. 4 Celestial pole offsets with respect to IAU2000A model of precession-nutation observed by VLBI (crosses, triangles) and combined with their rates observed by GPS (full line).

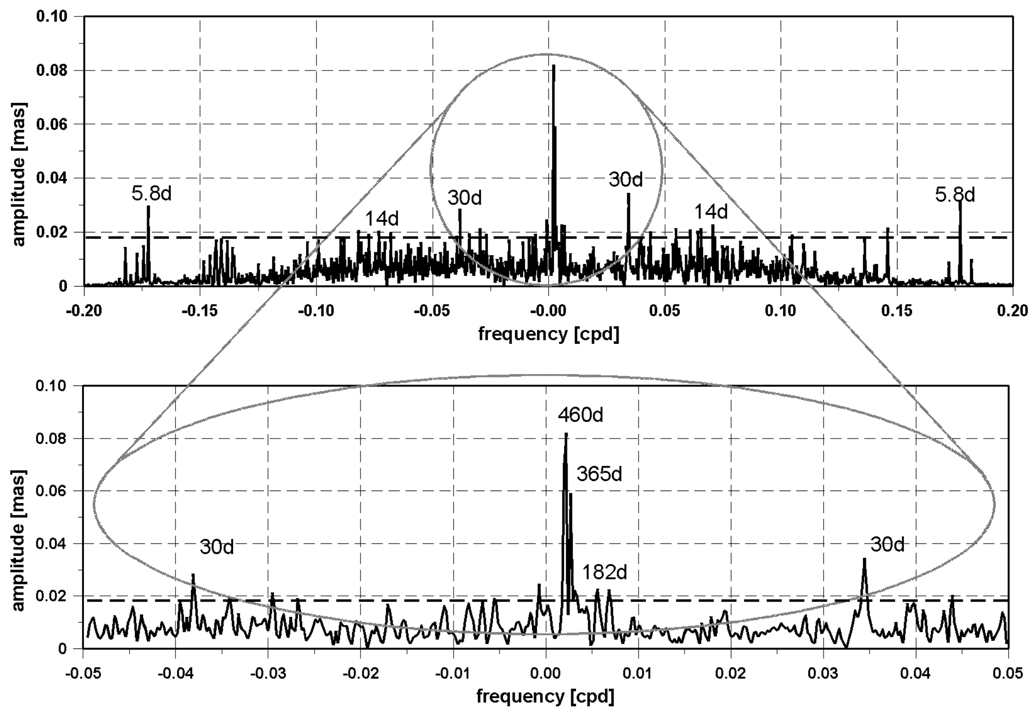

Fig. 5 Spectrum of VLBI/GPS combined celestial pole offsets.

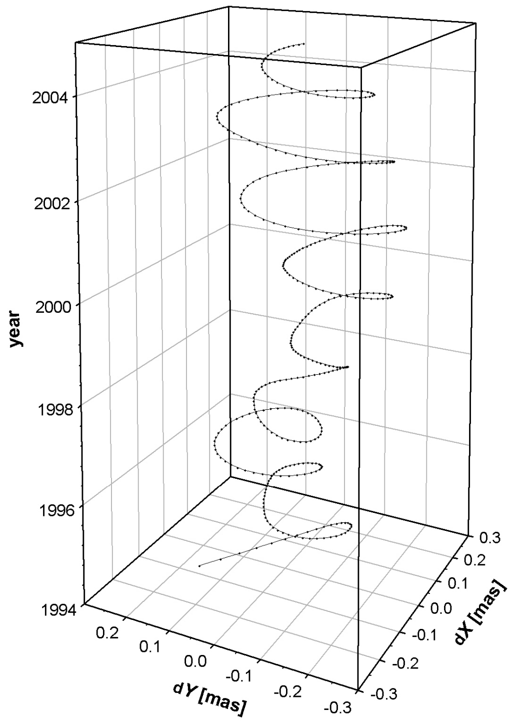

Fig. 6 Free Core Nutation from the combination of VLBI and GPS observations.

Again, a simple sinusoidal representation of FCN is not adequate. Therefore we subtract the seven short-periodic terms of Tab. 2 plus two long-periodic terms of Tab.1 from the data, and smooth the residuals in order to find a 'real' FCN for the interval 1994.3-2004.6, displayed in Fig. 6.

4. COMBINATION OF UNIVERSAL TIME WITH LENGTH OF DAY

To combine universal time measured by VLBI with length-of-day measured by GPS, the method of combined smoothing is used again, this time applied to:

- VLBI observations of UT1-TAI (IVS combined solution ivs04qx.eops, in the interval 1979.6-2004.6);
- GPS observations of length-of-day (IGS combined solution igs00p02.erp, in the interval 1996.5-2004.6),

from which we obtain the combination in the common interval covered by both techniques (and containing UT1-TAI in the first series and LOD in the second one), i.e. 1997.1-2004.6. The coefficients of smoothing used are $\varepsilon = 34\,\text{day}^{-6}, \bar{\varepsilon} = 34\,\text{day}^{-4}$, ensuring that all periodic changes with periods 1.5 days and shorter are completely smoothed out. In this case, the preliminary calibration of GPS-based LOD to VLBI observations of UT1-TAI was not necessary since it has already been done by IGS.

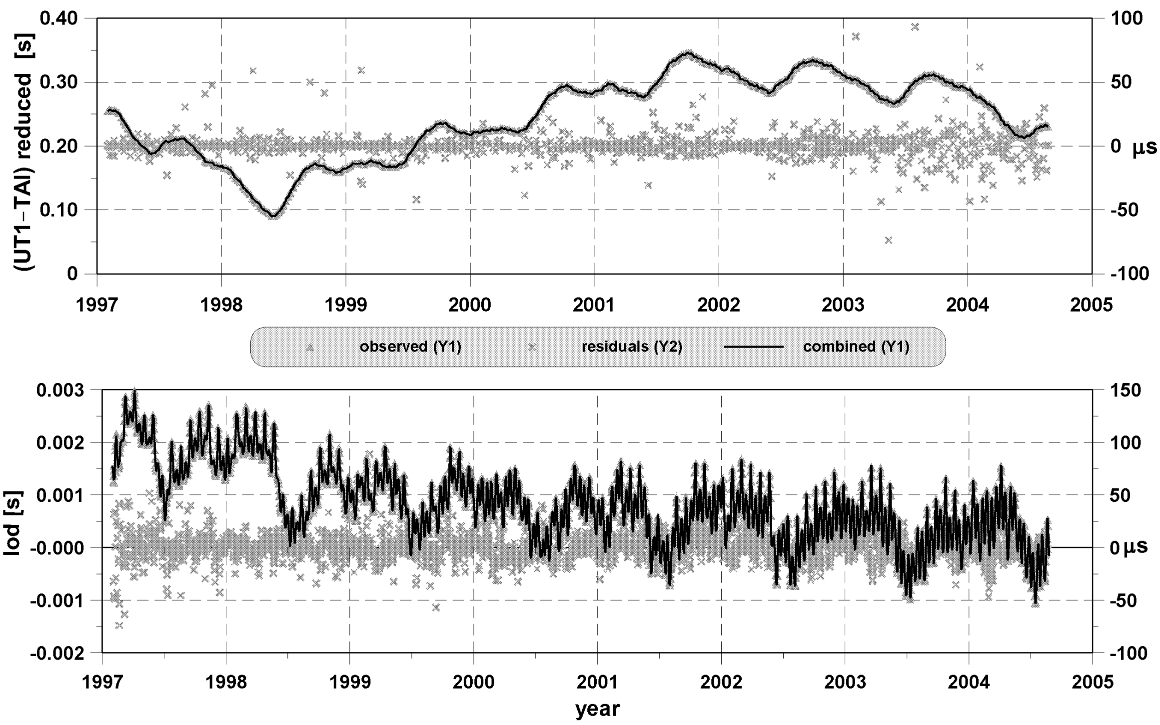

Fig. 7 Combination of UT1-TAI observed by VLBI with LOD observed by GPS.

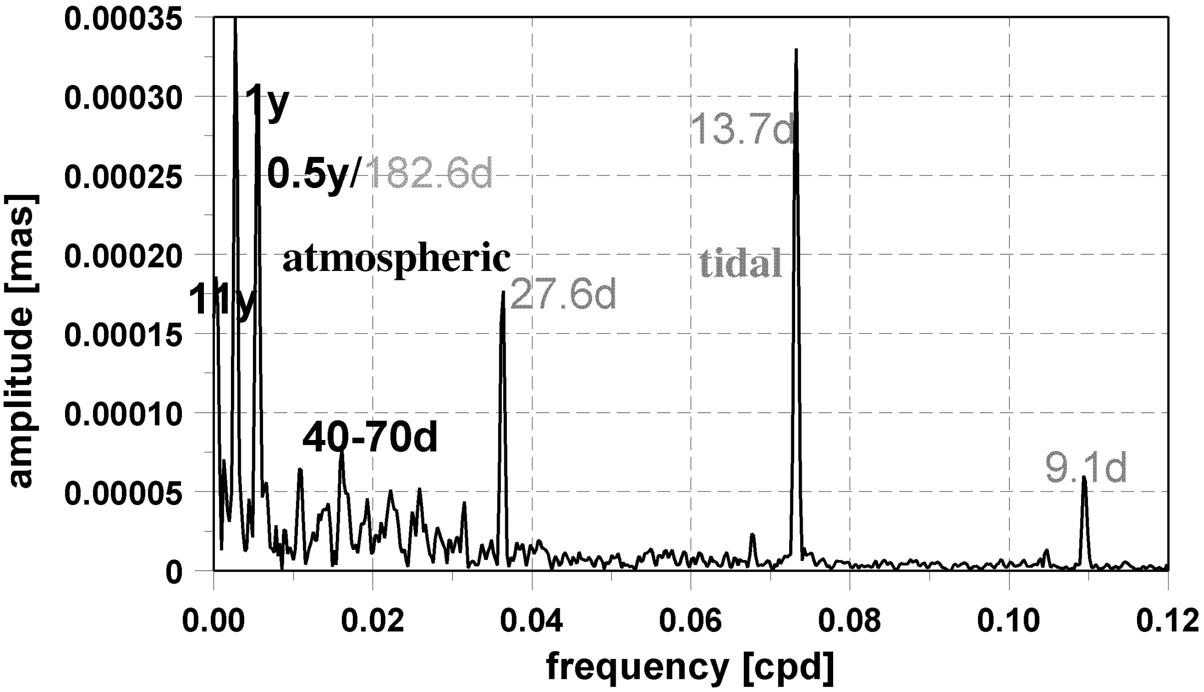

Fig. 8 Spectrum of VLBI/GPS combined length of day changes.

The results of the combination are shown in Fig. 7, in which the upper plot depicts the reduced (i.e., with a parabolic approximation removed) values of UT1-TAI, and the lower plot displays the values of LOD. In both plots the gray triangles and crosses represent the observations and residuals, respectively. The combination (full lines) is so close to observations (triangles) that they are hardly distinguishable; therefore the residuals (crosses) are depicted in enlarged scales (right axes). The periodic character of the results is evident. If we make a spectral analysis of the combined length-of-day changes, we obtain the periodogram depicted in Fig. 8. All peaks present there can be easily identified with either theoretical tidal effects after Yoder et al. (1981) – periods given in gray, or with the influence of the atmosphere – periods given in black. These two effects are clearly separated: tidal effects cause the dominant periodic changes with periods 9.1, 13.7, 27.6 and 182.6 days, while the atmosphere plays dominant role for periods in the range 40-70 days, and 0.5, 1 and 11 years. The only period for which both effects have approximately the same magnitude is a semi-annual variation.

5. CONCLUSIONS

It is demonstrated that the combination with GPS data helps to improve the VLBI-alone solution, both in resolution and accuracy. It is especially the short-periodic part that is improved. Statistically significant corrections of 9 nutation terms (with periods 18.6y, 9.3y, 365.26d, 182.6d, 31.81d, 27.55d, 13.66d, 13.63d and 5.64d) are found from the combination of celestial pole offsets from IAU2000A model. The results can be found in Tab. 1 (periods 6798 and 3399 days) and Tab. 2 (periods 365.26, 182.6, 31.81, 27.55, 13.66, 13.63 and 5.64 days). Period of FCN of 435 days (based on VLBI observations in 1984-2004) is not confirmed by more recent data (combination of VLBI with GPS in the interval 1994.6- 2004.6); the period of 460 days yields much better fit to the combined solution in this interval. Combined length-of day changes are studied, and the prominent spectral peaks are identified with atmospheric excitation (periods 11y, 1y, 0.5y, 40-70d) and tidal influences (periods equal to182.6d, 27.56d, 13.66d and 9.13d).

The combination of EOP (measured by VLBI) and EOP rates (measured by GPS) has been implemented as an interaction tool on the website of the IERS EOP Product Center at Paris Observatory (http://hpiers.obspm.fr/eop-pc/).

ACKNOWLEDGMENT

This work was supported by Ministry of Education, Youth and Sports of the Czech Republic in the frame of the project LN00A005.

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