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ASSESSMENT OF P-WAVE ANISOTROPY BY MEANS OF VELOCITY ELIPSOID

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ABSTRACT
A method for estimation of elastic wave velocity anisotropy based on ultrasonic sounding data during rock-sample loading was developed. The subject matter of the method is approximation of ultrasonic sounding data by triaxial velocity ellipsoid. The applicability of proposed method was verified on investigation of anisotropic rock samples.
Laboratory loading of migmatite samples was realized under various mutual orientations between acting force direction and rock foliation – perpendicular, parallel and under 45°. P-wave velocity of ultrasound waves was monitored by 8 sensors network. The velocity ellipsoid was computed and changes of sizes and orientation its main axes during loading were analyzed for separate experiments with regard to loading level. It was found, that independently to mutual orientation between rock foliation and loading direction, the minimum velocity vector turns to perpendicular direction to final rupture plane and maximum velocity vector turns to the plane of final rupture.

KEYWORDS: foliation, loading, ultrasound sounding, elastic wave velocity, anisotropy

1. INTRODUCTION

Rocks relatively often show a macroscopic anisotropy of mechanical properties (e.g. Tilman and Bennett, 1973). This anisotropy can be caused by crystal structure of single minerals, arrangement of mineral grains and orientation of cracks or microcrack systems. Anisotropy of mechanical properties depends also on the stress level and on the system of acting forces – uniaxial force, confining pressure etc. The rock anisotropy is especially characteristic for sedimentary and metamorphic rocks but it can be also observed at eruptive rocks. Anisotropy of mechanical properties has effect to rock behaviour during its loading, a way of final failure and it also causes anisotropy of elastic wave propagation in rocks.

Elastic wave velocity anisotropy complicates as velocity models of seismic wave propagation through rock medium. Otherwise, it can reveal valuable information about rock structure not accessible by direct observation. The relationship between velocity anisotropy and rock structure can therefore be used for interpretation of geophysical measurements. However, for successful application it is essential to know influence of stress-strain state of rock to velocity anisotropy.

A method for estimation of elastic wave velocity anisotropy based on ultrasonic sounding data during rock-sample loading was developed. The subject matter of the method is approximation of ultrasonic sounding data by triaxial velocity ellipsoid. The mutual relations between velocity anisotropy, rock structure, loading direction and rupture plane were further observed in dependence to the loading level.

Testing of anisotropy velocity ellipsoid method was demonstrated on the study of effect of the mutual orientation of rock structure and applied uniaxial loading on selected parameters of ultrasonic sounding. Migmatite (locality Skalka, Czech Republic) was chosen as a suitable rock material for our experiments. This migmatite has a macroscopic visible plane-parallel structure (foliation).

Anisotropy of ultrasonic wave propagation was experimentally investigated in dependence to mutual orientation between acting force direction and migmatite foliation. Tested migmatite was also subjected to ultrasonic sounding under confining pressure condition. The purpose of this experiment was to investigate the course of anisotropy changes in dependence to increasing hydrostatic pressure.

2. PRESENT STATE OF LABORATORY INVESTIGATION OF ULTRASOUND VELOCITY ANISOTROPY

Investigation of elastic wave velocity anisotropy relates to determination of rock elastic parameters. Rock elastic parameters can be determined by means of static or dynamic methods. The static methods are based on the Hook's law, which propose linear relation between acting force and deformation. In the case of presence of open cracks in rock, acting force can induce their closing and relation between force and strain is not linear.

Dynamic methods based on ultrasonic sounding consist in determination of elastic wave propagation time through the sample along the known path. Dynamic methods, in contrast to static methods, result from application of small deformations and short-time acting force. The overall magnitude of seismic velocities in rocks can be according to Ji et al. (2003) even calculated based on high-precision single crystal elastic constants and volume fractions of the constituent minerals of the rock and appropriate mixture rules. In this case, however, the orientation distribution function of rock forming minerals has to be considered.

Babuška (1984) studied velocity anisotropy of minerals and rocks especially with respect to its application for deep structure research of the lithosphere. Přikryl et al. (2007) demonstrated that hydrostatic pressure can cause closing the cracks in eruptive rocks, so the increasing hydrostatic pressure can produce changes of velocity anisotropy. If the eruptive rock structure is isotropic, the velocity anisotropy can disappear under the sufficiently high hydrostatic pressure. Babuška (1984) supposed the anisotropy caused by effect of cracks disappears at depths of 5-6 km. Hydrostatic pressure approximately 200 MPa (Owens and Bamford, 1976) corresponds to these depths. The anisotropy of intact rocks is caused only by their structure. Přikryl et al. (2007) showed the anisotropy of these rocks is almost constant and independent to the changes of hydrostatic pressure.

If the rock shows anisotropy caused by the combination of both effects of the mineral grains arrangement and the preferential crack orientation, then the anisotropy changes depend on the mutual orientation of both these structure elements. In the case of diverse orientation of mineral grains and cracks anisotropy, the changes of hydrostatic pressure don't induce only changes of anisotropy value, but they also cause changes in the anisotropy orientation.

3. PROCESSING OF ULTRASONIC SOUNDING MEASUREMENTS

The results of ultrasonic sounding were transformed to the velocity ellipsoid. The ellipsoid is constructed as a quadric passing through the end points of velocity vectors. The origin of each velocity vector is situated in the midpoint of the ellipsoid. Direction of velocity vector is determined by the source and receiver join (Fig. 2). The amplitude of velocity vector is the velocity value corresponding source - receiver join.

In the whole 56 different propagation times are obtained in one sounding cycle, however only 28 data are independent. The half of 56 propagation times is only the duplicate sounding in straight and reverse directions along the same propagation path. The average of straight and reversed propagation times is used for the velocity calculation and this velocity is assigned to the given path direction. For some sensor pairs waves can partially propagate through the steel press jaw. These measurement directions were excluded and the total number of velocity vectors was reduced to 22.

The ellipsoid of velocity anisotropy is a quadric with the mid point situated in beginning of coordinate system. Every point $[x_i \quad y_i \quad z_i]$ of this ellipsoid has to fulfill following equation:

$$[x_i \quad y_i \quad z_i]\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = 1\,, \tag{1}$$

where a_{ij} are the coefficients of symmetrical matrix **A**, which determines the parameters of ellipsoid. The equation (3) can be converted to the form:

$$\begin{aligned} &x_i^2 a_{11} + y_i^2 a_{22} + z_i^2 a_{33} + 2x_i y_i a_{12} + \\ &+ 2x_i z_i a_{13} + 2y_i z_i a_{23} = 1 \end{aligned}\,, \tag{2}$$

where $i = 1,2, \ldots 22$.

The six unknown parameters of matrix **A** can be found by means of Gauss generalised inversion method (Meju, 1994). This method is based on the minimisation of sum of squared differences between the measured velocity vectors and the velocity ellipsoid approximation.

The set of equations (4) can be written in matrix form:

$$\mathbf{Gm} = \mathbf{d}\,, \tag{3}$$

where

$$\mathbf{G} = \begin{bmatrix} x_1^2 & y_1^2 & z_1^2 & 2x_1y_1 & 2x_1z_1 & 2y_1z_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{22}^2 & y_{22}^2 & z_{22}^2 & 2x_{22}y_{22} & 2x_{22}z_{22} & 2y_{22}z_{22} \end{bmatrix},$$

$$\mathbf{m} = \begin{bmatrix} a_{11} \\ a_{22} \\ a_{33} \\ a_{12} \\ a_{13} \\ a_{23} \end{bmatrix}, \qquad \mathbf{d} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

The unknown vector **m** containing six independent components of matrix **A** can be expressed by means of generalised inversion:

$$\mathbf{m} = \left(\mathbf{GG}^T\right)^{-1}\mathbf{G}^T\mathbf{d}\,, \tag{4}$$

where G^T denotes transposed matrix G.

The amplitudes and the directions of velocity ellipsoid semi-axes are calculated as eigenvalues and eigenvectors of matrix A. The sizes and directions of velocity ellipsoid semi-axes correspond to the velocities v_{MAX}, v_{MEAN}, v_{MIN} and their directions. The presence of a single maximum velocity direction and a single minimum velocity direction is a disadvantage of this method. Moreover the directions of maximum, mean and minimum velocities are mutually perpendicular. However this model of velocity anisotropy is generally accepted (e.g. Wang et al., 2005 and others). The basic advantage of the proposed method of ultrasonic sounding processing is the possibility to use a sparse net of sensors for detailed study of elastic waves anisotropy. Resulting velocity ellipsoid reveals not only estimate of anisotropy coefficient, but also the main directions of velocities in the material.

The parameters of above velocity ellipsoid were found by optimization procedure of solving the overdetermined problem. The resulting quadric represents therefore smoothed approximation of directional velocity distribution. The differences between measured data and their approximation by ellipsoid can be caused for example by inner inhomogeneity of rock material, inaccuracies in determined propagation time and also by the simplified model of velocity anisotropy used.

For the quantitative evaluation of differences between measured velocity vectors and established velocity ellipsoid the root mean square value *RMS* was used for evaluating of approximation quality:

$$RMS = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\Delta v_i^2}\ , \tag{5}$$

where Δv_i is the difference between the measured velocity and the calculated velocity in the corresponding direction. The number n of velocity pairs was usually 22.

4. EXPERIMENT

4.1. CONFINING LOAD EXPERIMENT

Migmatite from the locality Skalka (Czech Republic) was chosen for the experiment as a material with distinct, macroscopic visible plane-parallel structure (Fig. 1). At first, the ultrasonic sounding was made to prove the effect of migmatite structure on the P-wave velocity anisotropy. P-wave velocity anisotropy measurement was carried out on the spherical migmatite sample under several levels of hydrostatic pressure. The spherical sample was sounded at 132 independent directions at each level of hydrostatic pressure in a range from 0 to 200 MPa (Pros and Podroužková, 1974; Pros, 1977; Pros et al., 1998; Pros et al., 2003). The results of velocity anisotropy measurement are showed on Fig. 1. The minimum velocity direction is perpendicular to the migmatite foliation and the maximum velocity direction lies in the foliation plane, as it follows from the Fig. 1.

Anisotropy can be quantitatively described by the value of anisotropy coefficient k, in this study defined by the relation:

$$k = 100\%\frac{v_{MAX} - v_{MIN}}{v_{MAX}}\ , \tag{6}$$

where v_{MAX} is maximum and v_{MIN} is minimum velocity value. This definition is a modification of anisotropy coefficient c introduced by Birch (1961):

$$c = 100\%\frac{v_{MAX} - v_{MIN}}{v_{MEAN}} \tag{7}$$

where v_{MEAN} is average value of velocity. In the case of sphere sample sounding, average velocity is calculated as a weighted average of 132 measured velocities (Pros, 1977).

The results of the velocity anisotropy measurement of migmatite spherical sample under confining pressure revealed that the anisotropy orientation doesn't change during loading up to 200 MPa. The coefficient of anisotropy k decreases with increasing hydrostatic pressure from 41.1% to 12.6%. The anisotropy coefficient and the velocity values return to its original values after unloading. Substantial decrease of anisotropy coefficient, together with fact that the orientation of velocity anisotropy doesn't change during loading, indicate the presence of crack system with orientation parallel to foliation plane. It was caused by a reversible process of closing this cracks system during the sample loading.

4.2. UNIAXIAL LOADING EXPERIMENTS

The migmatite cylindrical samples length of 100 mm and diameter 50 mm were used for this study. Experiments were carried out under different mutual orientations between the loading direction and the migmatite foliation (parallel, angular 45° and perpendicular). The computer controlled MTS loading system was used for samples loading. There was applied uniaxial loading regime with constant loading rate 0.5 MPa/min.

There were attached eight wideband (WD) sensors of ultrasonic emission (PAC, USA) on sample surface (Fig. 2).

All sensors were used for both, ultrasonic sounding, and monitoring of ultrasonic emission, arising during sample loading. The ultrasonic emission together with ultrasonic sounding signals was recorded by eight channel transient recorder Vallen Systeme, AMSY 5. This apparatus was set up in triggered regime, the sampling frequency was 10 MHz and the record length was 2048 points, so recorded signal duration was 204.8 μs and retriggering was 50 μs.

Ultrasonic sounding was realized in consequent sounding cycles at selected load levels (see Figs. 3 – 5). Every sounding cycle includes eight steps – every sensor act as a source of ultrasonic sounding and the others 7 sensors were receivers in the individual step. From the recorded signals of ultrasonic sounding the corresponding velocities were calculated. Due to large dimensions of sensors, special measurement on glass cylinder was realized and obtained data were used for corrections of measured times namely in dependence on direction of ray path.

5. RESULTS AND THEIR DISCUSSION

This study was focused on the research of the influence of mutual orientation between loading direction and rock structure orientation on the rock material (migmatite) anisotropy changes and the sample final failure process. Loading of samples used uniaxially acting stress with constant loading rate. Three types of experiments were performed with the mutual orientations close to 0°, 45° and 90° between the loading direction and the migmatite foliation. The ultrasonic sounding with the sparse net of sensors was used for the research of velocity anisotropy changes during the process of closing of open cracks at the beginning of loading and an increase of anisotropy when approaching the sample strength limit. Also the relation between orientations of main velocity directions and the sample final failure plane was studied. During the sample loading the ultrasonic emission was monitored, too.

The velocity ellipsoid parameters were used for interpretation of ultrasonic sounding measurement. They were: the size of its semi-axes (v_{MAX}, v_{MEAN}, v_{MIN}) and their directions (α_{MAX}, α_{MEAN}, α_{MIN}) and anisotropy coefficient k (defined according to formula (6)). The developed method of velocity ellipsoid calculation enables the evaluation of ultrasonic soundings realised on cylindrical samples under uniaxial stress by the similar way as the velocity anisotropy measurement on spherical sample under hydrostatic pressure. This makes possible to mutually compare results obtained during uniaxial loading of cylindrical samples with results from hydrostatically loaded spherical samples. This comparison can be used in the interpretation of sources of elastic waves anisotropy.

5.1. ACTING FORCES PERPENDICULAR TO THE FOLIATION

In this configuration the minimum velocity performs the maximum changes during the loading (Fig. 3A). The values of mean and maximum velocities don't significantly change. Then the value of minimum velocity has decisive influence on the coefficient of anisotropy. Anisotropy coefficient k decreases up to 70% of ultimate strength and from this loading *level* k increases with loading up to ultimate strength (Fig. 3B). Decrease of anisotropy coefficient in the first part of loading, up to 70% strength is probably caused by closing of primary microcracks. The increase of k in the final part of loading could be induced by formation of new microcrack system in direction of ultimate failure.

Orientation of maximum velocity vector is particularly given by the migmatite structure and it doesn't change during the whole loading experiment. Maximum velocity lies in the foliation plane. During loading the orientation of minimum velocity vector changes from the direction perpendicular to migmatite foliation to the direction perpendicular to the plane of final rupture (Fig. 3C).

The ultrasonic emission slowly increases up to 80% of ultimate strength and after reaching this value it increases rapidly (Fig. 3B). This is caused by great creating of new microcraks. Coefficient of anisotropy k reflects very well changes of rock failure state.

Up to 60% of strength, the deviations between measured velocity vectors and calculated velocity ellipsoid, characterized by value of *RMS*, decrease and it has similar trend as anisotropy coefficient. After reaching this loading level the *RMS* value slightly increases until sample rupture.

5.2. ACTING FORCES PARALLEL TO FOLIATION

In this configuration the maximum velocity is practically constant during the whole loading experiment. The minimum velocity doesn't change up to 70% of ultimate strength, and then it decreases up to sample failure (Fig. 4A). Difference between course of minimum velocities during perpendicular and parallel loading experiments can be explain by fact, that major part of primary cracks is oriented parallel to the foliation. Closing of this crack system takes place only in the beginning of perpendicular loading. After reaching approximately 70% strength the new cracks origin in both loading manners and this is reflected in course of minimum velocities and even in course of anisotropy k.

From the courses of maximum and minimum velocities follows that anisotropy coefficient is almost constant up to 70% strength and then it increases up to total sample failure (Fig. 4B).

During this experiment (force parallel to foliation) the minimum velocity direction is perpendicular to the migmatite foliation, and in this case is also perpendicular to the plane of final sample rupture (Fig. 4C). The maximum velocity direction lies in the foliation plane and it turns to the direction of uniaxial loading.

The ultrasonic emission number slowly increases up to 80% of ultimate strength. At higher loading level the UE increase is faster (Fig. 4B). That can be related to the sample fracturing process.

The *RMS* value increases during whole loading experiment and it has a similar trend as anisotropy coefficient and UE number.

5.3. ACTING FORCE UNDER 45° TO FOLIATION

Any substantial changes in velocity values and velocity vectors directions don't occur in this experimental configuration. Obtained results can be explained by this, that neither closing nor widening of primary microcrack system nor formation of new microcrack system occurs. The orientation of minimum and maximum velocity vectors correspond to the plane of final rupture parallel with migmatite foliation. In this case the sample deformation is realized by sliding along the predisposed foliation planes.

The ultrasonic emission only slowly increases up to 90% of ultimate strength. At higher loading level the UE increase is fast (Fig. 5B). The final UE number is much lower than in other experimental configurations.

6. CONCLUSIONS

Laboratory experiments were focused on determination of elastic waves anisotropy and its changes during rock samples uniaxial loading.

New method for processing of velocity anisotropy was developed on the base of velocity ellipsoid. This approach is enabled by ultrasonic sounding with monitoring by sensors network usually used for monitoring ultrasonic emission. It was proved, that for this method the sparse network with only eight sensors is sufficient for determination of velocity ellipsoid. The accuracy of velocity ellipsoid approximation was tested by root mean square method (*RMS*). The *RMS* values are smaller then 10% of P-wave velocity values for all experiments.

Applied method of velocity ellipsoid makes possible to study influence of mutual orientation of acting load and rock foliation on the changes of velocity vectors and anisotropy during loading.

Values of velocity vectors determined on the cylindrical samples by velocity ellipsoid method correspond to velocities measured on the spherical samples under atmospheric stress.

Method of velocity ellipsoid was successfully used for determination of P-wave velocity anisotropy during sample loading. This way of determination velocity anisotropy is shown as a suitable tool for characterization of rock fracturing process.

It was found, that the minimum velocity vector turns perpendicularly to final rupture plane. The maximum velocity vector lies in the plane of final rupture during the whole sample loading. This behavior is independent on mutual orientation between rock foliation and loading direction.

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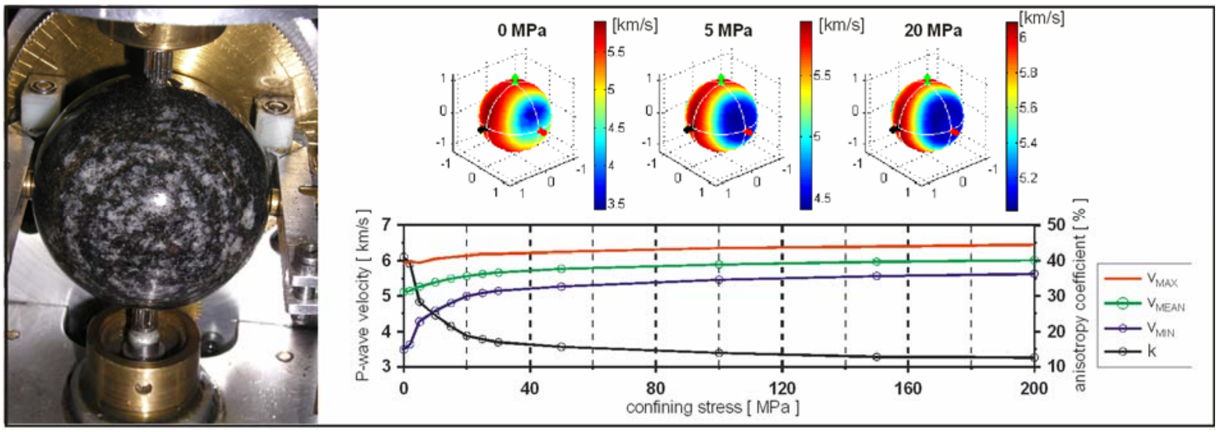

Fig. 1 Ultrasonic sounding of the spherical rock sample. Left: migmatite sample with the pronounced structure. Top right: projection of ultrasonic wave velocity distribution on the surface of spherical sample under confining pressure at levels 0, 5 and 20 MPa. Bottom right: course of maximum, mean and minimum velocities, and coefficient of anisotropy k in dependence to the confining pressure level.

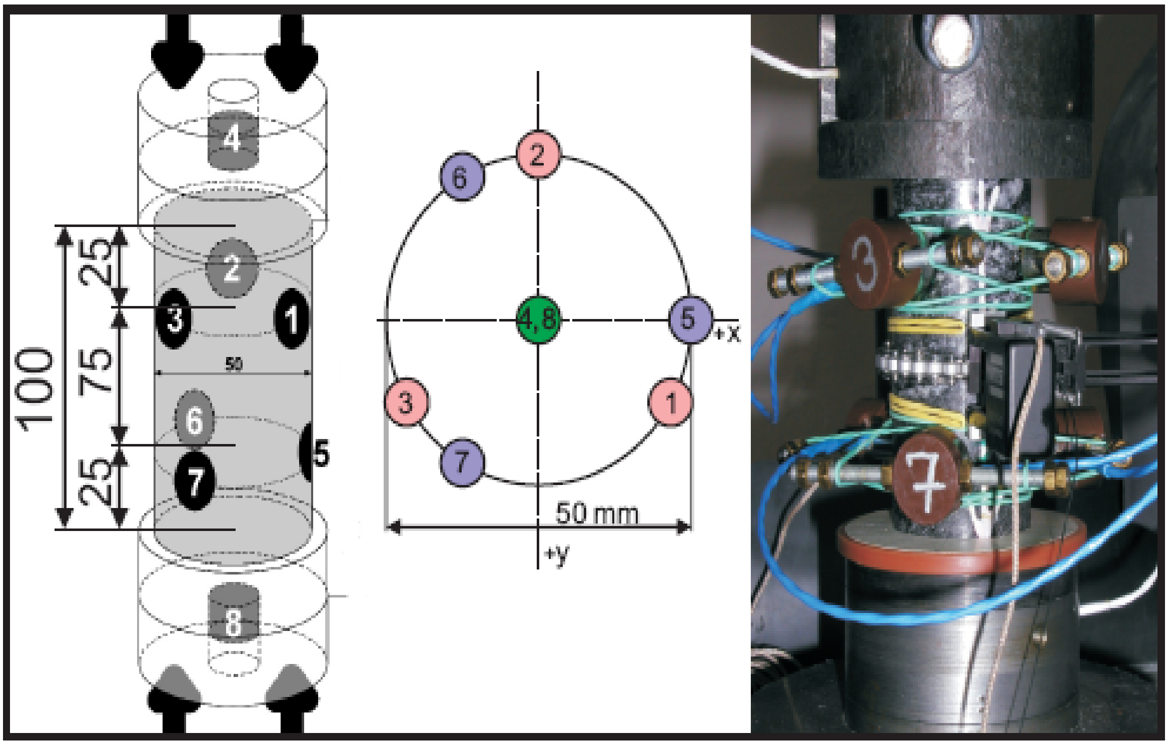

Fig. 2 Arrangement of the experiment.
Circles 1-8 denote the position of ultrasonic sensors.

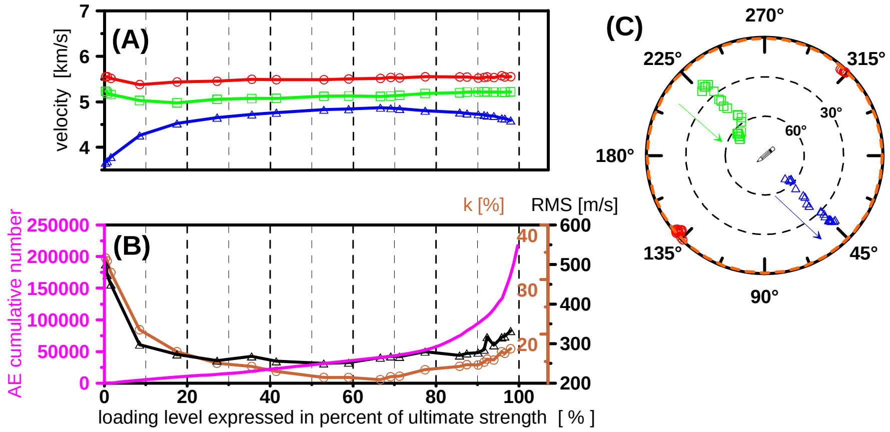

Fig. 3 Loading: direction perpendicular to foliation, ultimate strength: 111.8 MPa.

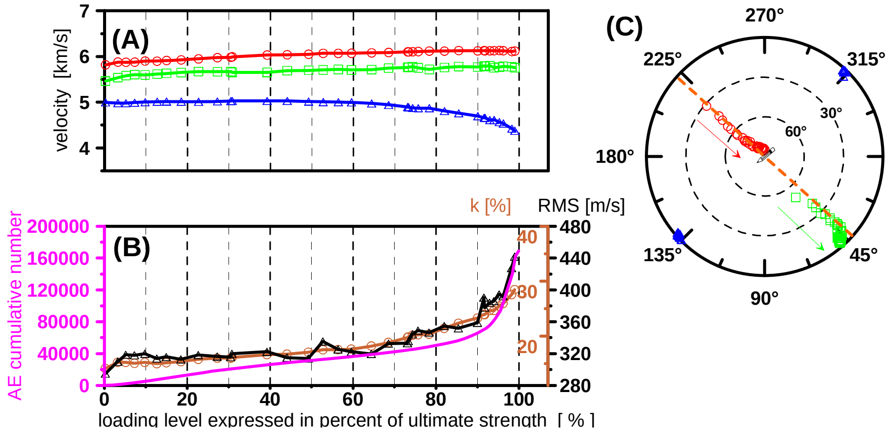

Fig. 4 Loading: direction parallel to foliation, ultimate strength: 139 MP.

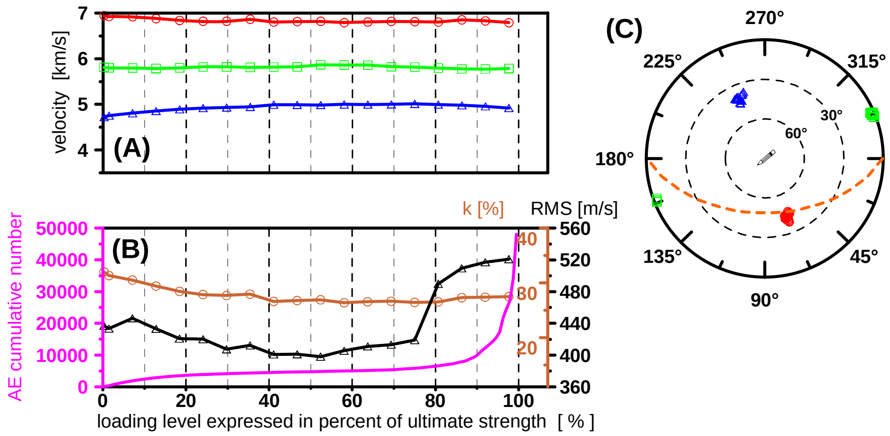

Fig. 5 Loading: direction under 45° loading direction, ultimate strength: 125.7 MPa.

A) v_{MAX}, v_{MEAN}, v_{MIN}: velocities in directions of velocity ellipsoid axes versus loading level.

v_{MAX} v_{MEAN} v_{MIN}

B) AE number: cumulative number of recorded AE events; k: coefficient of P-wave velocity anisotropy; RMS: approximation error of velocity ellipsoid.

AE number k RMS

C) Stereographic projection of the v_{MAX}, v_{MEAN}, v_{MIN} directions during loading up to failure.

v_{MAX} direction v_{MEAN} direction v_{MIN} direction loading direction foliation plane