INVESTIGATION ON INFLUENCE OF THE NUMBER AND LOCALIZATION OF STATIONS ON ESTIMATION ACCURACY OF THE PLATE MOTIONS

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ABSTRACT
This paper concerns an analysis of accuracy of estimated parameters (Φ, Λ, ω) which define the tectonic plate motions. The study is based on the velocities of station positions published by ITRF2000 for (Satellite Laser Ranging) SLR and (Global Positioning System) GPS technique, separately. The Eurasian, North-American, Australian and Pacific plates were analysed. Influence of the number and localization of stations on estimation accuracy of the tectonic plate motions for SLR and GPS technique were analysed. The discrepancies were discussed.

KEYWORDS: SLR and GPS techniques, the coordinate shifts, the tectonic plate motions

1. INTRODUCTION

The definition and maintenance of a geodetic system by the satellite technique requires systematic temporal changes of the station positions to be taken into account. The most of station velocities are accommodated by the tectonic plate motions. The space geodetic methods such as (SLR) and (GPS) offer determinations of the 3-dimensional station positions with accuracies of the order of 1cm or better and the coordinate velocities of 1mm/y from repeated observations in the time interval Δt. The kinematic plate parameters are described by the rotation vector $\vec{\Omega}$. We represent $\vec{\Omega}$ by the geographical position of the pole rotation (Φ, Λ) and the rotational velocity (ω) (Drewes, 1982).

The relations between the station velocities ($\Delta\varphi, \Delta\lambda$) and the plate parameters $\vec{\Omega}$ (Φ, Λ, ω) are given in the expressions (1) and (2) (Drewes, 1982).

$$\Delta\varphi = \omega \cdot \Delta t \cdot \cos\Phi \cdot \sin(\lambda - \Lambda) \quad (1)$$

$$\Delta\lambda = \omega \cdot \Delta t \cdot (\sin\Phi - \cos(\lambda - \Lambda)\tan\varphi \cdot \cos\Phi) \quad (2)$$

where:
φ, λ - the station position,
$\Delta\varphi$, $\Delta\lambda$ - the station velocity,
Φ, Λ, ω - the parameters of the plate motion.

For estimation of the three parameters of *n* plates we need *3n* observations, it means velocities for two stations on the each plate. In the case of the redundant observations a least squares adjustment is performed. The kinematic datum can be realized by fixing of the rotation parameters of one plate $\vec{\Omega} = const$. In the other case the plate kinematic datum is given with respect to the reference frame of the station coordinates (SLR or GPS reference system).

The accuracies of estimated plate rotation parameters depend on accuracies of the geodetic observations, length of the time interval Δt used for analysis, the number and configuration of stations.

The parameters of the tectonic plate motion are described by ($\omega_x, \omega_y, \omega_z$) or ($\Phi, \Lambda, \omega$). The angular velocities ($\omega_x, \omega_y, \omega_z$) can be transformed to (Φ, Λ, ω) by the following expressions.

$$\begin{aligned} \omega_x &= \omega \cos\Lambda \cos\Phi \\ \omega_y &= \omega \sin\Lambda \cos\Phi \\ \omega_z &= \omega \sin\Phi \end{aligned} \quad (3)$$

$$tg\Phi = \frac{\omega_z}{\sqrt{\omega_x^2 + \omega_y^2}}$$
$$tg\Lambda = \frac{\omega_y}{\omega_x} \quad (4)$$
$$\omega = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2}$$

The observation equations for a least squares adjustment are given by formulas.

$$v_{\varphi}=\left(\frac{\partial\Delta\varphi}{\partial\Phi}\right)d\Phi+\left(\frac{\partial\Delta\varphi}{\partial\Lambda}\right)d\Lambda+ +\left(\frac{\partial\Delta\varphi}{\partial\omega}\right)d\omega-\left(\Delta\varphi^{obs}-\Delta\varphi^{cal}\right) \quad (5)$$

$$v_{\lambda}=\left(\frac{\partial\Delta\lambda}{\partial\Phi}\right)d\Phi+\left(\frac{\partial\Delta\lambda}{\partial\Lambda}\right)d\Lambda+ +\left(\frac{\partial\Delta\lambda}{\partial\omega}\right)d\omega-\left(\Delta\lambda^{obs}-\Delta\lambda^{cal}\right) \quad (6)$$

where:

$\Delta\varphi^{obs}$, $\Delta\lambda^{obs}$ - observed station velocity in latitude and longitude (ITRF2000),

$\Delta\varphi^{cal}$, $\Delta\lambda^{cal}$ - calculated station velocity in latitude and longitude (NNR-NUVEL1A model).

The partial derivatives required in the observation equations (5) and (6) can be obtained by differentiation of the expressions (1) and (2).

The functions (1) and (2) are linearized by truncation of the Taylor's expression. It is described in detail by (Tapley, 1988).

$$\begin{aligned}
\frac{\partial\Delta\varphi}{\partial\Phi}&=-\omega\sin\Phi\sin(\lambda-\Lambda)\\
\frac{\partial\Delta\varphi}{\partial\Lambda}&=-\omega\cos\Phi\cos(\lambda-\Lambda)\\
\frac{\partial\Delta\varphi}{\partial\omega}&=\cos\Phi\sin(\lambda-\Lambda)\\
\frac{\partial\Delta\lambda}{\partial\Phi}&=\omega\cos\Phi+\omega\cos(\lambda-\Lambda)\tan\varphi\sin\Phi\\
\frac{\partial\Delta\lambda}{\partial\Lambda}&=-\omega\cos\Phi\sin(\lambda-\Lambda)\tan\varphi\\
\frac{\partial\Delta\lambda}{\partial\omega}&=\sin\Phi-\cos(\lambda-\Lambda)\tan\varphi\cos\Phi
\end{aligned} \quad (7)$$

The unknowns were calculated from the formula (8) in an iterative process.

The iteration formula given by the equation (8) solves the normal equation formed by minimizing the sum of squares of the weight residuals (O-C).

$$\Omega^{*}=\begin{bmatrix}\Phi\\ \Lambda\\ \omega\end{bmatrix}=(A^{T}WA)^{-1}A^{T}WL \quad (8)$$

where:

Ω^{*} - matrix of unknowns which describes $\vec{\Omega}$ vector,

A - matrix of the partial derivatives given by expressions (7),

W - the weighting matrix associated with the observations,

L - (O-C) matrix, O-observation , C- computed values for NNR-NUVEL1A model.

2. ANALYSIS METHOD AND RESULTS

The paper presents the parameters of the tectonic plate motion (Φ,Λ,ω) for the Eurasian, Pacific, Australian and North American plates which were adjusted using expressions (5) and (6). The computations were performed on the basis of the reference frame ITRF2000. The coordinate velocities caused by the tectonic plate motions for GPS and SLR techniques were taken from (ITRF2000_GPS.SSC) and (ITRF2000_SLR.SSC), separately.

The sequential method was adopted for analysis. In the first step, the tectonic plate parameters were adjusted for three stations localized on each plate. In the next steps, stations one after the other were included using the sequential method. The tectonic plate parameters were adjusted once again. The estimated unknowns (Φ,Λ,ω) and their errors were analysed. The results of analysis are shown in Tables 1-4. The stations localized near of boundary of the tectonic plates are not used in analysis. This problem is described in details by (Kraszewska, 2006).

For Eurasian plate, ten SLR and thirty GPS stations were investigated. The GPS stations were divided into three groups. The each group comprise the random distribution of ten stations. The estimated parameters and their errors ($\Phi, m_{\Phi}, \Lambda, m_{\Lambda}, \omega, m_{\omega}$) are given in Table 1 and shown in Figures 1-4. For GPS solutions, agreements of adjusted parameters (Φ , Λ) are equal to $0^{0}.4$ and $1^{0}.2$ respectively. Computed differences between the both techniques are smaller then $1^{0}.6$ for Φ and $4^{0}.0$ for Λ. The parameter ω is the same in all solutions. Stability of estimated parameters and their errors become visible for about eight stations (Figs. 1-4). The GPS station (SFER) is situated near of the plate boundary, inclusion of this station does not determinate the solution worse. Distribution of observing stations for this solution is shown in Fig. 5. The stations situated in Asia are omitted in this figure.

The three SLR and nine GPS stations are localized on the Australian plate, this means that we can not use sequential method of analysis for SLR technique. The adjusted parameters of the plate motion and their errors are given in Table 2 (for SLR and GPS techniques). The estimated parameters are equal to $\Phi=32.^{0}6\pm0.14$, $\Lambda=40^{0}.8\pm0.27$,

$\omega=0.^{0}600\pm0.0020$ and

$\Phi=32.^{0}2\pm0.14$, $\Lambda=39^{0}.2\pm0.27$,

$\omega=0.^{0}600\pm0.0020$ for SLR and GPS techniques respectively. The very good agreement of estimated parameters for both techniques can be seen in spite small number of stations.

For Pacific plate, seven SLR stations and six GPS stations were studied. The results are given in Table 3. The solution was limited by small number of stations adjusted in ITRF2000. The differences between both techniques are smaller than 0.4 degree

Table 1 The stations used in analysis, adjusted parameters of the plate motions and their errors. The SLR stations are denoted by numbers but GPS by abbreviations in ITRF2000.

EURASIAN PLATE								
EURASIAN PLATE	STATION POSITIONS		PARAMETERS OF THE TECTONIC PLATE MOTION					
NUMBER AND NAME	ϕ[°]	λ[°]	Φ[°]	m_Φ[°]	Λ[°]	m_Λ[°]	ω[°/mln lat]	m_ω[°/mln lat]
SLR TECHNIQUE								
3 (7835,	43	6	51.45	2.71	-109.30	3.82	0.2333	0.0013
7840,	50	0						
7836)	52	13						
4 (+7839)	46	15	55.30	1.99	-104.40	3.38	0.2500	0.0120
5 (+7811)	52	17	55.25	1.63	-103.25	2.80	0.2500	0.0100
6 (+7236)	30	114	57.53	1.30	-98.34	2.65	0.2667	0.0100
7 (+7810)	46	7	57.49	1.17	-98.39	2.40	0.2667	0.0090
8 (+7837)	30	121	58.33	1.01	-96.20	2.07	0.2667	0.0080
9 (+8834)	48	12	58.39	0.96	-96.12	1.98	0.2667	0.0080
10 (+7546)	44	11	58.47	0.94	-95.49	1.98	0.2667	0.0080
GPS TECHNIQUE, FIRST GROUP OF STATIONS								
3 (MADR,	40	-4	57.60	0.84	-99.43	1.53	0.2500	0.0065
GRAZ,	46	15						
KIRU)	67	20						
4 (+WSRT)	52	6	57.70	0.68	-99.37	1.25	0.2500	0.0053
5 (+KOSG)	51	5	57.10	0.69	-99.50	1.29	0.2500	0.0055
6 (+TIXI)	71	128	56.80	0.70	-101.60	1.16	0.2500	0.0049
7 (+MDVO)	55	37	56.11	0.63	-100.56	1.04	0.2500	0.0044
8 (+OBER)	47	11	56.80	0.59	-100.59	0.97	0.2500	0.0040
9 (+KARL)	48	8	56.80	0.55	-100.59	0.90	0.2500	0.0037
10 (+JOZE)	51	21	56.80	0.52	-101.60	0.84	0.2500	0.0035
GPS TECHNIQUE, SECOND GROUP OF STATIONS								
3 (WETT,	48	12	57.23	0.39	-99.14	0.60	0.2500	0.021
BOR1,	52	17						
KIT3)	38	66						
4 (+VILL)	40	-3	56.53	0.28	-99.47	0.48	0.2500	0.0016
5 (+POTS)	52	13	57.20	0.25	-99.33	0.43	0.2500	0.0015
6 (+BRUS)	50	4	57.23	0.31	-99.15	0.55	0.2500	0.0020
7 (+GRAS)	43	6	57.22	0.27	-99.20	0.50	0.2500	0.0017
8(+DELFT)	51	4	57.22	0.24	-99.20	0.46	0.2500	0.0016
9(+TOUL)	43	1	57.15	0.24	-99.27	0.46	0.2500	0.0016
10 (+RIGA)	56	24	57.20	0.27	-99.57	0.49	0.2500	0.0017
GPS TECHNIQUE, THIRD GROUP OF STATIONS								
3 (HERS,	50	0	55.25	1.37	-103.37	2.43	0.2500	0.0093
ZIMM,	46	7						
ZWEN)	55	36						
4 (+POL2)	42	74	56.43	0.84	-101.70	1.53	0.2500	0.0059
5 (+GINA)	43	5	56.39	0.73	-101.13	1.32	0.2500	0.0051
6 (+BRST)	48	-4	56.36	0.64	-101.16	1.17	0.2500	0.0045
7 (+YAKZ)	61	129	57.38	0.53	-99.30	0.99	0.2500	0.0039
8(+DENT)	50	3	57.37	0.50	-99.10	0.94	0.2500	0.0037
9 (+EBRE)	40	0	57.25	0.51	-99.19	0.96	0.2500	0.0038
10 (+SFER)	36	-6	57.15	0.51	-99.25	0.96	0.2500	0.0038

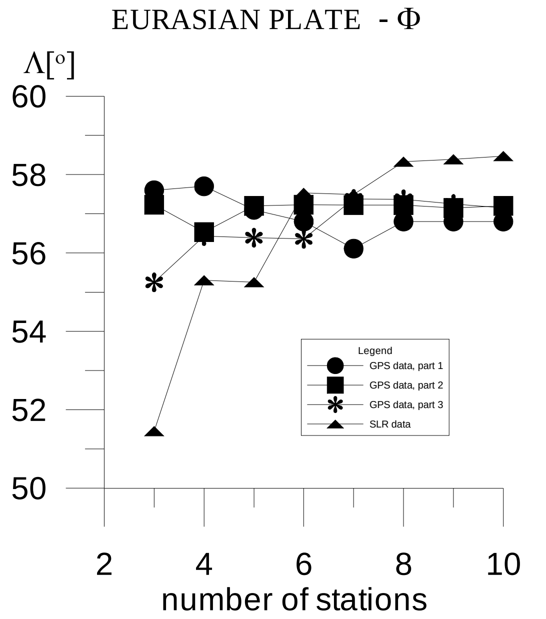

Fig. 1 Estimated parameter Φ of motion for SLR and GPS techniques.

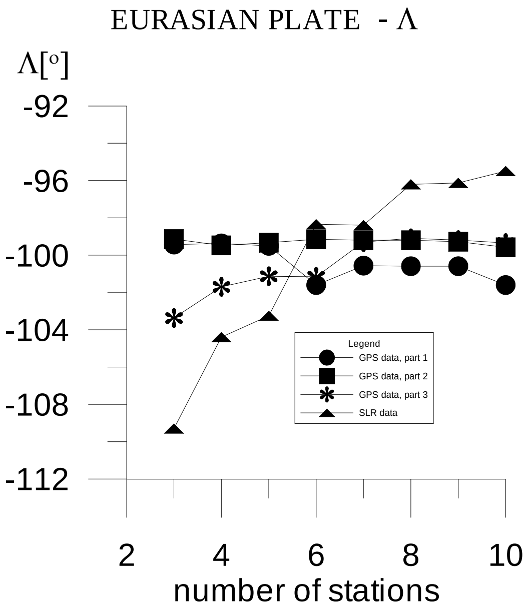

Fig. 2 Estimated parameter Λ of plate motion for SLR and GPS techniques.

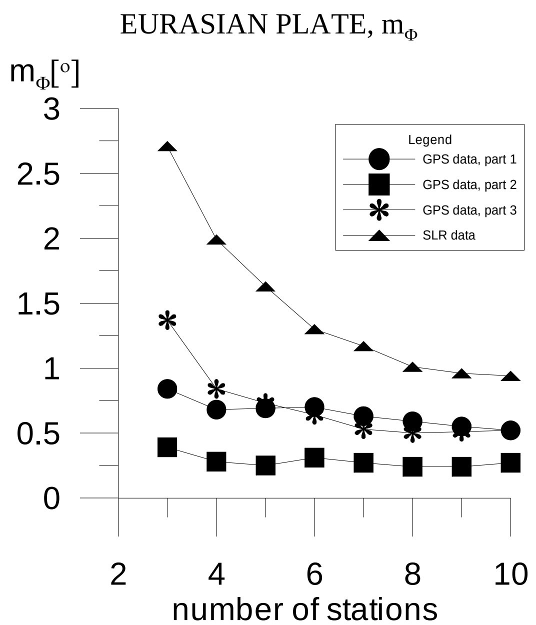

Fig. 3 Estimated errors m_{Φ} of plate motion for SLR and GPS techniques

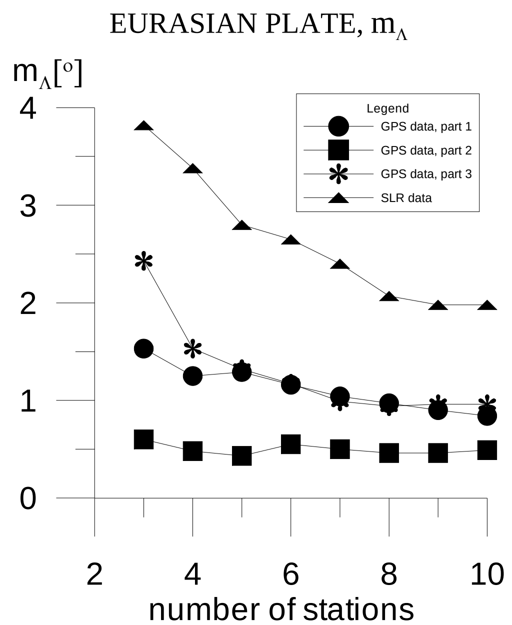

Fig. 4 Estimated errors m_{Λ} of motion for SLR and GPS techniques.

Fig. 5 Distribution of SLR and GPS stations in Europe used in solutions.
White circle denotes SLR station
Black circle denotes GPS station (I group)
Black triangle denotes GPS station (II group)
Black square denotes GPS station (III group)

for parameter Φ, but for parameter Λ are equal to 8 degrees. Estimated parameter ω was the same for both techniques. For this plate, the number of stations is not sufficient. Additionally, the stations do not cover the plate uniformly, particular it concerns GPS technique.

For North American plate, eight SLR and ten GPS stations were studied. The differences between both techniques are smaller a half degree for parameter Φ and Λ. The estimated parameters ω are the same. Stability of estimated parameters and their errors can be seen for about eight stations (Table 4).

3. CONCLUSIONS

On the base of performed analysis, the following conclusions can be drawn:

- Parameters of the tectonic plate motions (Φ, Λ, ω) estimated for GPS and SLR techniques differ smaller then one degree for all analysed plates, only for Eurasian plate $\Delta\Lambda$ is equal to 4^0 and for Pacific plate $\Delta\Lambda$ is equal to 8^0.
- For Pacific plate, number of stations used for analysis is not sufficient for estimation of high quality solution.
- Stabilization of estimated tectonic plate parameters an their errors for random distribution of about 8 stations situated on each plate can be seen. For European plate, convergence of solution is shown in Figs. 1-4. For Australian plate only, number of sufficient stations is significantly smaller (3-5).

Table 2 The stations used in analysis, adjusted parameters of the plate motions and their errors.

AUSTRALIAN PLATE									
AUSTRALIAN PLATE	STATION POSITIONS		PARAMETERS OF THE TECTONIC PLATE MOTION						
NUMBER AND NAME	ϕ[°]	λ[°]	Φ[°]	m_Φ[°]	Λ[°]	m_Λ[°]	ω[°/mln lat]	m_ω[°/mln lat]
SLR TECHNIQUE								
3(7090,	-28	115						
7843,	-35	148						
78490	-34	149	32.60	0.14	40.80	0.27	0.6000	0.0020
GPS TECHNIQUE								
3 (YAR1,	-28	115	32.12	0.41	39.70	0.73	0.6000	0.0050
HOB1,	-42	147						
TOW2)	-19	147						
4 (+ALIC)	-23	133	32.11	0.32	39.70	0.56	0.6000	0.0038
5 (+KARR)	-20	117	32.11	0.27	39.70	0.48	0.6000	0.0032
6 (+DARW)	-12	131	32.11	0.23	39.80	0.42	0.6000	0.0028
7 (+TIDB)	-35	148	32.20	0.17	39.22	0.32	0.6000	0.0026
8 (+CEDU)	-31	133	32.20	0.15	39.22	0.29	0.6000	0.0023
9 (+JAB1)	-12	132	32.20	0.14	39.20	0.27	0.6000	0.0022

Table 3 The stations used in analysis, adjusted parameters of the plate motions and their errors.

PACIFIC PLATE								
PACYFIC PLATE	STATION POSITIONS		PARAMETERS OF THE TECTONIC PLATE MOTION					
NUMBER AND NAME	ϕ[°]	λ[°]	Φ[°]	m_Φ[°]	Λ[°]	m_Λ[°]	ω[°/mln lat]	m_ω[°/mln lat]
SLR TECHNIQUE								
3 (7096,	-14	-170	-64.55	0.15	114.00	1.36	0.6667	0.0040
7123,	-16	-151						
7092)	9	167						
4 (+7210)	20	-156	-63.25	0.06	106.16	0.83	0.6667	0.0030
5 (+7882)	22	-109	-63.25	0.09	104.44	1.11	0.6667	0.0040
6 (+7883)	9	167	-62.24	0.10	103.52	1.21	0.6667	0.0050
7 (+7888)	31	-110	-63.24	0.09	103.53	1.09	0.6667	0.0040
GPS TECHNIQUE								
3 (KOKB,	21	-159	-63.54	0.29	112.10	1.22	0.6500	0.0060
TAHI,	-17	-149						
CHAT)	-43	-176						
4 (+MKEA)	19	-155	-63.58	0.20	112.29	0.96	0.6667	0.0044
5 (+KOK1)	21	-159	-63.58	0.17	112.30	0.81	0.6667	0.0037
6 (+UPO1)	20	-155	-63.58	0.15	112.30	0.72	0.6667	0.0032

Table 4 The stations used in analysis, adjusted parameters of the plate motions and their errors.

NORTH AMERICAN PLATE								
NORTH AMERICAN PLATE	STATION POSITIONS		PARAMETERS OF THE TECTONIC PLATE MOTION					
NUMBER AND NAME	ϕ[°]	λ[°]	Φ[°]	m_Φ[°]	Λ[°]	m_Λ[°]	ω[°/mln lat]	m_ω[°/mln lat]
SLR TECHNIQUE								
3 (7105,	38	-76	-1.70	1.96	-82.50	0.65	0.2000	0.0110
7295,	25	-80						
7410)	45	-78						
4 (+7122)	23	-106	-3.35	1.98	-83.39	0.55	0.1833	0.0130
5 (+7091)	42	-71	-3.49	1.60	-83.34	0.41	0.1833	0.0090
6 (+7067)	32	-64	-2.58	1.33	-83.41	0.39	0.1833	0.0080
7 (+7891)	35	-111	-2.59	1.21	-84.41	0.35	0.1833	0.0080
8 (+7894)	32	-114	-3.00	1.11	-83.41	0.32	0.1833	0.0070
GPS TECHNIQUE								
3 (GODE,	38	-76	-1.51	1.07	-84.30	0.25	0.1833	0.0077
NLIB,	41	-91						
RCM6)	25	-80						
4 (+KELY)	66	-50	-2.22	0.70	-84.22	0.24	0.1833	0.0045
5 (+CHUR)	58	-94	-2.35	0.66	-84.12	0.24	0.1833	0.0039
6 (+DUBO)	50	-95	-2.37	0.58	-84.13	0.21	0.1833	0.0034
7 (+SOL1)	38	-76	-2.36	0.52	-84.12	0.19	0.1833	0.0031
8 (+HKLO)	35	-95	-2.43	0.51	-84.14	0.18	0.1833	0.0030
9 (+WSMN)	32	-106	-2.49	0.49	-84.14	0.18	0.1833	0.0029
10 (+MIL1)	42	-87	-2.49	0.46	-84.15	0.17	0.1833	0.0027

- Errors of estimated parameters of the tectonic plate motions (Φ, Λ, ω) for all analysed plates are smaller then one degree.
- The parameters of the tectonic plate motions (Φ, Λ, ω) for Australian plate are estimated with highest accuracy, the final errors ($m_\Phi, m_\Lambda, m_\omega$) are equal to ($0.14^0, 0.27^0, 0.002^0 / M.y$).
- The result may not be representative for the entire interior of the plates, for example at plate boundaries. It is described in detail by (Kraszewska, 2006).

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