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QUASI-DIURNAL ATMOSPHERIC AND OCEANIC EXCITATION OF NUTATION

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ABSTRACT
The periodic motion of the Earth's spin axis in space (nutation) is dominantly forced by external torques exerted by the Moon, Sun and planets. On the other hand, long-periodic geophysical forces (with periods longer than several days), mostly caused by the changes in the atmosphere and oceans, have dominant effects in polar motion (in terrestrial frame) and Earth's speed of rotation. However, even relatively small short-periodic (near-diurnal) motions of the atmosphere and oceans can also have a non-negligible influence on nutation, thanks to the resonance that is due to the existence of a flattened outer fluid core. The retrograde period, corresponding to this resonance, is roughly equal to 430 days in non-rotating quasi-inertial celestial reference frame, or 23h 53min (mean solar time) in the terrestrial frame rotating with the Earth. The aim of the present study is to use the geophysical excitations in the vicinity of this resonance to estimate their influence on nutation, based on recent models of atmospheric and oceanic motions. To this end, we use the numerical integration of Brzezinski's broad-band Liouville equations and compare the results with VLBI observations. Our study shows that the atmospheric plus oceanic effects (both matter and motion terms) are capable of exciting free core nutation; both its amplitude and phase are compatible with the observed motion. Annual and semi-annual geophysical contributions of nutation are of the order of 100 microarcseconds. They are slightly different for different atmospheric/oceanic models used, and they also differ from the values observed by VLBI – the differences exceed several times their formal uncertainties.

KEYWORDS: Earth orientation, nutation, geophysical excitation, VLBI

1. INTRODUCTION

It is well known that the periodic motion of the Earth's spin axis in terrestrial frame (polar motion) is dominantly dictated by geophysical excitations (mainly caused by the motions of the atmosphere and oceans). These motions have mostly seasonal character, i.e., with annual and semi-annual period. On the other hand, precession and nutation (i.e., the motion of the Earth's axis in celestial frame) are dominantly forced by external torques, exerted by the Moon, Sun and planets. There are only weak geophysical excitations in short-periodic (near-diurnal) range, if counted in terrestrial frame. Due to the existence of a flattened fluid and rigid inner cores, there are however strong resonances in this part of the spectrum. This leads to a non-negligible influence on the celestial motion of the Earth's spin axis in space – nutation. The influence of these resonances is twofold: the amplitudes of forced nutations computed for the rigid Earth model are modified, and any geophysical excitations with frequencies close to resonance frequencies are highly amplified. In addition, free motion corresponding to these resonances is possible. The motion of the axis of rotation of the mantle in space is observed by modern space techniques, such as very long-baseline interferometry (VLBI). This motion is described by offsets of the instantaneous celestial pole from its position defined by conventional IAU precession-nutation model, so called celestial pole offsets.

The strongest resonance in this part of the spectrum is due to the flattened outer fluid core. It is accompanied by the retrograde free core nutation (RFCN), a small periodic motion of the Earth's axis of rotation in space. The resonance period of RFCN is dominantly given by the shape of the core; the period is inversely proportional to its dynamical flattening, but electromagnetic coupling between the mantle and the core also plays a role. If the core were in hydrostatic equilibrium, the period would be approximately 460 days – this is the value used by Wahr (1981) in his model of nutation IAU1980. Later observations by VLBI nevertheless revealed that the core's flattening is slightly larger (by almost 4 per cent), leading to the motion with the period of RFCN (in celestial frame) of about 430 solar days (corresponding to retrograde 23h 53min in terrestrial frame). In spite of the fact that the direct analysis of VLBI-based celestial pole offsets yields an apparent change of this period, indirect methods demonstrate that the period is rather stable in time and is always around 430 days (see, e.g., Roosbeek et al., 1999;

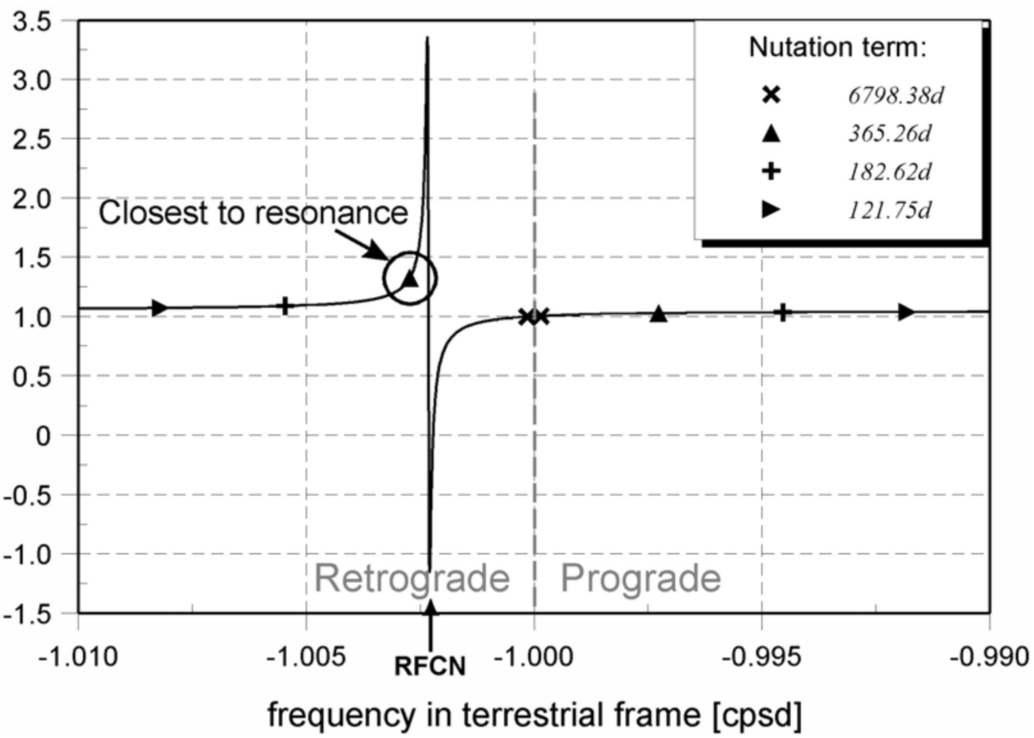

Fig. 1 Real part of MHB transfer function $T(\sigma)$ around frequency –1cpsd and some closest forced nutation terms.

Hinderer et al., 2000; Vondrák and Ron, 2005 or Vondrák et al., 2005). The value 430.21 was chosen to calculate the recently adopted model of nutation IAU2000 (Mathews et al., 2002, further referred to as MHB). Our own most recent value (Vondrák and Ron, 2006) yielded 430.32±0.07 days. Dehant et al. (2003) proposed that the difference between the direct and indirect determination of RFCN period may be due to additional excitation produced by external parts of the Earth (oceans, atmosphere).

The atmospheric effects in nutation were recently studied, e.g. by Bizouard et al. (1998), Yseboodt et al. (2002) or Lambert (2006). These authors conclude that the atmospheric excitation is not large enough to explain the observed free core nutation, and that these effects are variable in time. Here we use a combination of atmospheric and oceanic excitations, and an alternative 'integration' approach to study their effects in nutation in detail.

2. RESONANCES IN EARTH ORIENTATION

All resonances, influencing Earth orientation, are given by MHB transfer function used to derive the IAU2000 model of nutation,

$$T(\sigma) = \frac{e_R - \sigma}{e_R + 1} N_0 \left[1 + (1+\sigma)\left(Q_0 + \sum_{j=1}^{4} \frac{Q_j}{\sigma - s_j} \right) \right]. \quad (1)$$

The complex function $T(\sigma)$ expresses the ratio of the non-rigid amplitude of a forced nutation term with terrestrial frequency σ (in cycles per sidereal day – cpsd) to its rigid Earth value. Here e_R denotes the dynamical ellipticity of the rigid Earth used to compute the rigid solution, N_0, Q_j are complex constants, and s_j are four complex resonance frequencies corresponding to Chandler Wobble (CW, with terrestrial period of about 435 days), RFCN, with celestial period of about 430 days, Prograde Free Core Nutation (PFCN, with celestial period of about 1020 days) and Inner Core Wobble (ICW, with terrestrial period of about 2400 days) respectively. In our case, only $s_2 \approx -1.0023$ cpsd (RFCN frequency) is interesting since it is close to the frequencies of nutation and, at the same time, the corresponding coefficient Q_2 (~4.89×10^{-2}) is almost two hundred times larger than Q_3 (~2.96×10^{-4}), corresponding to PFCN. The real part of the MHB transfer function (only its small section close to –1cpsd) is depicted in Fig. 1, together with several closest forced nutation terms. All nutation terms appear in pairs (describing prograde and retrograde circular motions in celestial frame); they are all retrograde in terrestrial frame, placed symmetrically with respect to –1 cpsd. Closest to obvious RFCN resonance, and therefore the most sensitive to any change of resonance frequency, is evidently the retrograde annual term. PFCN resonance with $s_3 \approx -0.9990$ cpsd is so small that it is invisible in the plot.

3. DATA USED IN THIS ANALYSIS

The following data are used in this study:

1. Nutation series as observed by VLBI, with irregular distribution (1 – 7 days):

(a) Combined IVS solution (Schlueter et al., 2002) ivsM6q4X.eops in interval 1979.6 – 2007.0. Celestial pole offsets d*X*, d*Y* from IAU2000A model of precession-nutation are given. Combined IVS solution is assumed to be more robust than the VLBI results of individual VLBI networks and analysis centers, as it is less vulnerable to failure than any individual member of the assembly of VLBI systems/VLBI networks/VLBI analysis centers used to define the combined solution (Cannon, 2001).

2. Geophysical excitations at strictly 6-hour intervals:

(a) Atmospheric Angular Momentum (AAM) functions:
   - i) NCEP/NCAR re-analysis in interval 1948.0 – 2006.7 (Salstein, 2005);
   - ii) ERA40 in interval 1979.0 – 2001.0 (Thomas et al., 2007, Dobslaw and Thomas, 2007).

(b) Oceanic Angular Momentum (OAM) functions:
   - i) ECCO model in interval 1993.0 – 2006.2 (Gross et al., 2005);
   - ii) OMCT model in interval 1979.0 – 2001.0 (Thomas et al., 2007, Dobslaw and Thomas, 2007);
   - iii) Rui Ponte model in interval 1993.0 – 2000.5 (Ponte and Ali, 2002).

AAM and OAM equatorial excitation functions (in complex notation) $\chi = \chi_1 + \mathrm{i}\,\chi_2$ are given in rotating terrestrial frame, so they must be first transformed into celestial (non-rotating) one. To this end, we use a simple formula

$$\chi' = -\chi\, \mathrm{e}^{\mathrm{i}\phi}, \tag{2}$$

where ϕ denotes Greenwich sidereal time. Before doing so, the mean value was removed from the series in order to get rid of large diurnal wave of the result. Because we are concentrated on long-periodic nutation terms close to FCN, the transformed series was filtered by using the method proposed by Vondrák (1977) with coefficient of smoothing $\varepsilon = 1.3\times10^{-4}\,\mathrm{day}^{-6}$ that suppresses the signal with periods shorter than 60 days.

4. NUMERICAL INTEGRATION OF BROAD-BAND LIOUVILLE EQUATIONS

In order to solve theoretically the rotation of non-rigid Earth model, Brzezinski (1994) proposed so called broad-band Liouville equations, that take into consideration two main resonances – Chandler wobble and Free core nutation. In celestial frame and complex notation it reads

$$\ddot{P} - \mathrm{i}(\sigma'_C + \sigma'_f)\dot{P} - \sigma'_C\sigma'_f P = \\ = -\sigma_C \begin{Bmatrix} \sigma'_f(\chi'_p + \chi'_w) + \sigma'_C(a_p\chi'_p + a_w\chi'_w) + \\ + \mathrm{i}[(1+a_p)\dot{\chi}'_p + (1+a_w)\dot{\chi}'_w] \end{Bmatrix}, \tag{3}$$

in which

$P = \mathrm{d}X + \mathrm{i}\,\mathrm{d}Y$ is the motion of Earth's spin axis in celestial frame;

$\sigma'_C = 6.32000 + 0.00237\,\mathrm{i}$,

$\sigma'_f = -0.0146011 + 0.0001533\,\mathrm{i}$ rad/day

are respectively the complex Chandler and FCN frequencies in celestial frame, whose imaginary parts are closely related to the quality factors;

$\sigma_C = \sigma'_C - \Omega$ is the Chandler frequency in terrestrial frame, where $\Omega = 6.30038$ rad/day is the angular speed of Earth's rotation;

χ'_p, χ'_w are excitations (matter and motion term) in celestial frame, and

$a_p = 9.2\times10^{-2}, a_w = 5.5\times10^{-4}$ are dimensionless numerical constants, expressing the response to a matter and motion excitation, respectively.

Solution of Eqs (3) generally yields two free circular motions: prograde Chandler wobble and retrograde FCN with celestial frequencies σ'_C and σ'_f, respectively.

Some investigators (e.g., Chao, 1985) use Liouville equations to calculate 'geodetic' excitations from the observed values of celestial pole offsets, and compare these with the directly observed geophysical excitations. Their argument against 'integration' method is that there are difficulties with free terms that are dependent on the choice of initial conditions. Here we shall follow namely this alternative integration approach, in which we numerically integrate celestial pole offsets, using the geophysical excitations, and compare them with the observed celestial pole offsets. The reasons why we prefer integration to differentiation approach are the following:

- To calculate geodetic excitations, time derivatives of the observed celestial pole offsets are needed. The observed data are rather noisy, and the differentiation even enhances the noise if no preliminary smoothing is applied. When using integration, the short-periodic noise in geophysical excitation is smoothed out automatically.
- In calculating the geodetic excitations, any free term completely disappears, so it is not possible, when comparing them with the geophysical excitations, to estimate how well both series agree near the FCN frequency. This is not the case for integration approach, but its disadvantage is that we need to choose optimal initial values for the integration which in fact define the initial amplitudes and phases of the free terms. This

problem can however be overcome if a thorough selection procedure is applied (see below).

The numerical integration is made by fourth-order Runge-Kutta method with 6-hour step. Namely we use the procedure rk4 from Numerical Recipes (Press et al., 1992) that we have adapted to our purpose by rewriting it into complex form. To obtain two first-order equations instead of a second-order one, we use the substitutions $y_1 = P$, $y_2 = \dot{P} - \mathrm{i}\sigma'_C P$, leading to differential equations for two complex functions y_1, y_2

$$\dot{y}_1 = \mathrm{i}\sigma'_C y_1 + y_2$$
$$\dot{y}_2 = \mathrm{i}\sigma'_f y_2 - \sigma_C \left\{ \begin{array}{l} \sigma'_f(\chi'_p + \chi'_w) + \sigma'_C(a_p\chi'_p + a_w\chi'_w) + \\ + \mathrm{i}[(1+a_p)\dot{\chi}'_p + (1+a_w)\dot{\chi}'_w] \end{array} \right\}. \quad (4)$$

Now we need to choose the initial values – in general two complex constants. Their choice defines the amplitudes and phases of both free motions. We are not interested in rapid (nearly diurnal) Chandlerian motion, so we choose only one, pole position at initial epoch P_0; its first derivative is constrained so that the Chandlerian amplitude disappears. It can be easily demonstrated that this is assured by choosing the values $y_1(0) = P_0$, $y_2(0) = \mathrm{i}(\sigma'_f - \sigma'_C)P_0$. The final choice of P_0 is made by repeating the integration with different values P_0 until the fit of the integrated motion to VLBI observations attains a minimum.

Five different combinations of atmosphere/ocean are used to integrate Eqs (3):

1. NCEP/NCAR AAM (pressure + wind), 1983.0 – 2006.2;
2. NCEP/NCAR AAM (pressure with IB correction + wind), 1983.0 – 2006.2;
3. NCEP/NCAR AAM + ECCO OAM, 1993.0 – 2006.2;
4. ERA40 AAM + OMCT OAM, 1983.0 – 2001.0;
5. NCEP/NCAR AAM + Ponte OAM, 1993.0 – 2000.5.

We use both variants of pressure term, without and with inverted barometer (IB) correction, when only atmospheric effects are studied, in order to test which one yields a better agreement with observations. However, IB correction of NCEP atmospheric pressure term is used in combination with the oceanic models ECCO, PONTE, as recommended by their authors. OMCT model is forced by the atmospheric model ERA40, so we use the combination of both. Originally we used only matter terms, but after an anonymous referee's recommendation to add also wind effects we found that the inclusion of motion terms of both atmosphere and ocean improved the agreement with VLBI observations significantly (by about 20 per cent in all cases studied).

5. THE RESULTS AND THEIR ANALYSIS

The results are displayed in Figures 2 – 6, in which the annual and semi-annual terms were removed from both observed and integrated series. Thus the plots show more or less only the free motion. Integrated motion is depicted as full line, the VLBI observations as gray triangles.

All these figures demonstrate that the integrated amplitudes agree well with the observed ones, the best

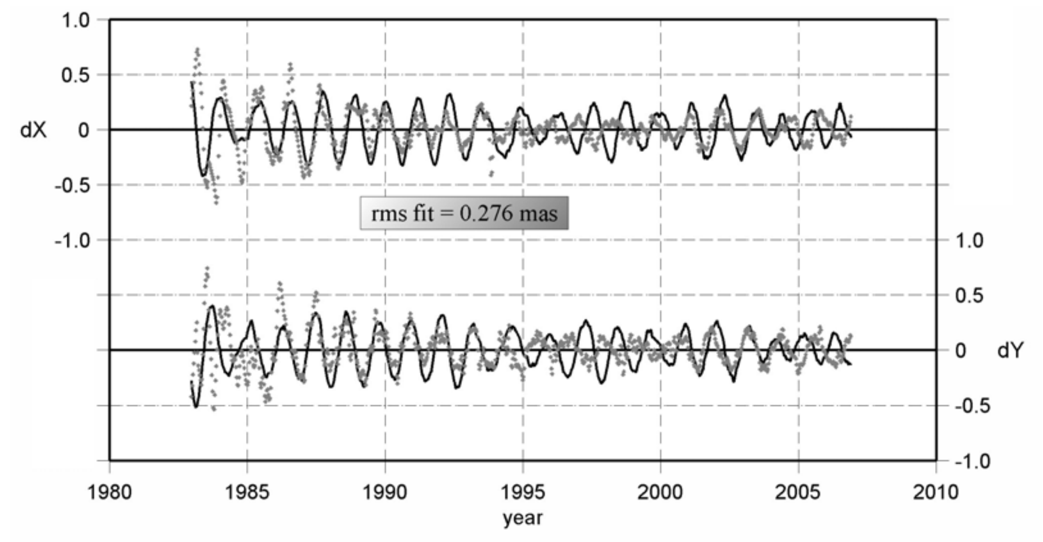

Fig. 2 Observed and integrated celestial pole offsets with NCEP AAM (pressure + wind).

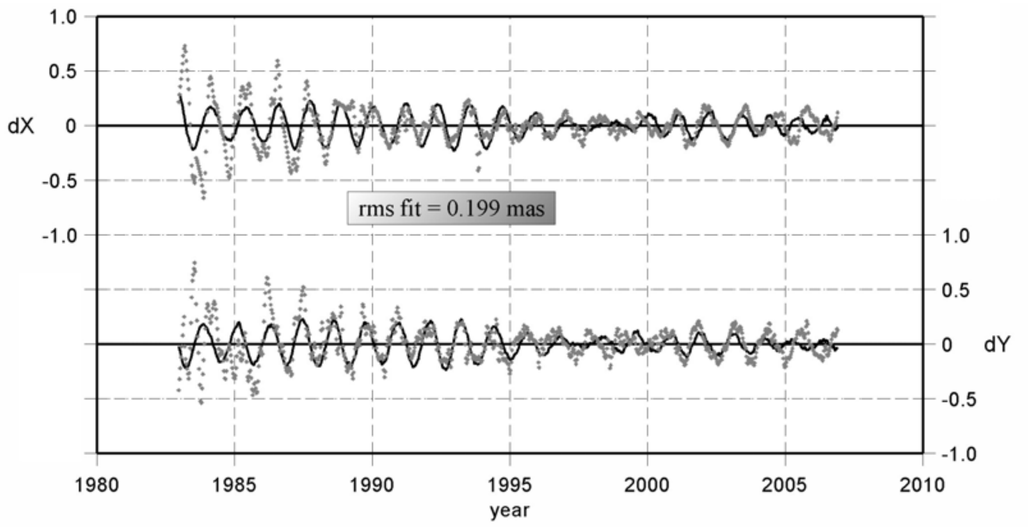

Fig. 3 Observed and integrated celestial pole offsets with NCEP AAM (pressure with IB + wind).

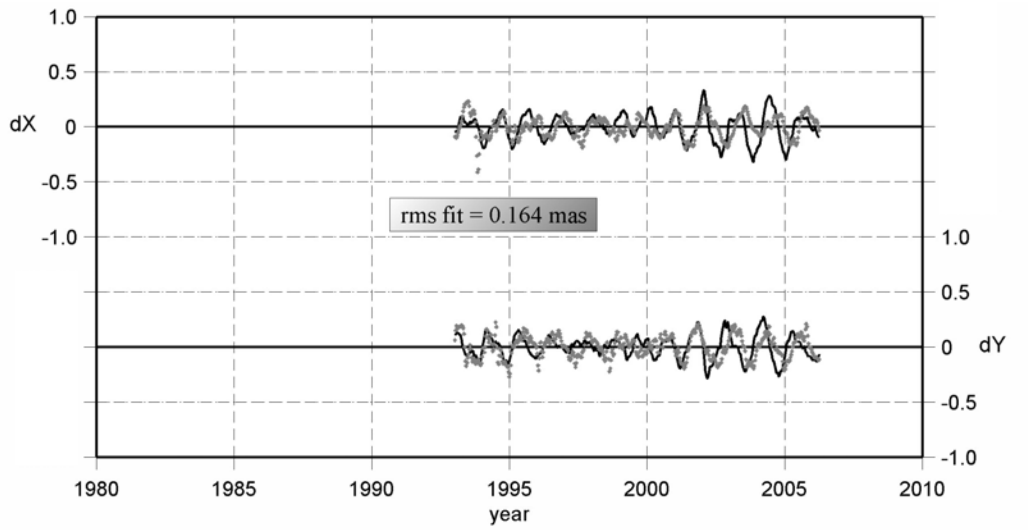

Fig. 4 Observed and integrated celestial pole offsets with NCEP AAM + ECCO OAM.

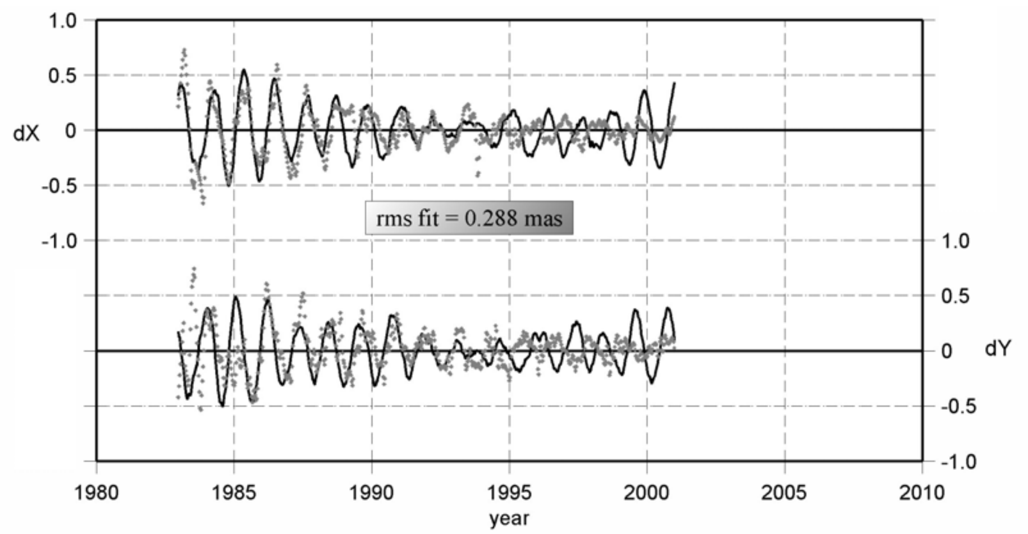

Fig. 5 Observed and integrated celestial pole offsets with ERA40 AAM + OMCT OAM.

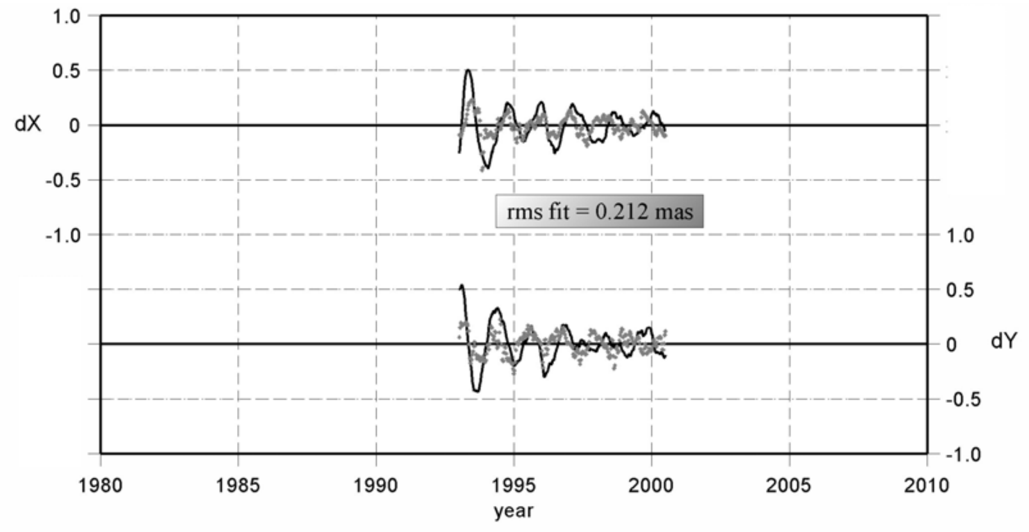

Fig. 6 Observed and integrated celestial pole offsets with NCEP AAM + Ponte OAM.

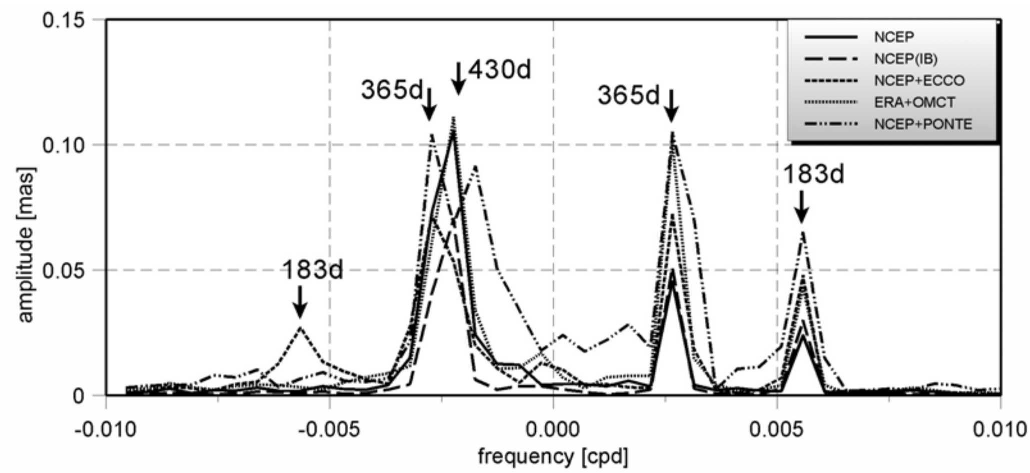

Fig. 7 Spectra of integrated celestial pole offsets from different AAM and OAM models.

fit (0.164 mas) is obtained for combination NCEP/NCAR AAM + ECCO OAM presented in Fig. 4. Even the simplest oceanic model (i.e., the AAM with inverted barometer correction, in which it is assumed that the ocean reacts inversely on atmospheric pressure changes at all frequencies), that has a longest history, yields very satisfactory results, with a fit to VLBI equal to 0.199 mas. The fit, if calculated for the same interval as ECCO (i.e., 1993.0 – 2006.2), is equal to only 0.128 mas. On the other hand, the fit for atmospheric model without IB correction is substantially worse than for the one with IB correction (0.276 versus 0.199 mas).

All integrated series were then subject to FFT spectral analysis. The obtained amplitude spectra (long-periodic part only) are depicted in Fig. 7. It is clear that all series display a dominant retrograde free term, with period of about 430 days. In addition to this, there are significant peaks at annual and semi-annual frequencies that are however slightly different for different atmospheric/oceanic models used. It is necessary to say that only the amplitude of dominant FCN peak depends on the choice of initial values for the integration – the rest of the spectra remain untouched.

The least-squares estimation of geophysical contribution to the annual and semi-annual terms was then made in the form of sine/cosine terms, for each component X and Y separately. The results (in microarcseconds) are given in Table 1.

Table 1 Geophysical contribution to nutation and comparison with VLBI observations

Excitation AAM+OAM	dX [μas]				dY [μas]			
	annual		semi-annual		annual		semi-annual	
	sin	cos	sin	cos	sin	cos	sin	cos
NCEP (NIB)	+90.8 ±4.8	+39.6 ±4.8	−19.0 ±4.8	−53.4 ±4.8	+143.3 ±4.9	+63.1 ±4.9	−51.5 ±4.9	+32.0 ±4.9
NCEP (IB)	+66.1 ±2.5	+90.9 ±2.5	−42.5 ±2.5	−54.8 ±2.5	+128.4 ±2.4	+72.9 ±2.4	−56.6 ±2.4	+46.0 ±2.4
NCEP+ECCO	+31.7 ±4.7	+42.4 ±4.7	−40.3 ±4.7	−75.7 ±4.7	+178.9 ±4.1	+33.1 ±4.2	−32.1 ±4.2	+52.9 ±4.2
ERA+OMCT	−1.6 ±6.8	+186.2 ±6.8	−18.3 ±6.7	−73.8 ±6.7	+174.5 ±6.7	+128.3 ±6.7	−80.7 ±6.6	+17.6 ±6.6
NCEP+PONTE	−7.4 ±8.3	−27.6 ±8.3	−35.6 ±8.2	−71.7 ±8.2	+242.5 ±8.7	−167.7 ±8.7	−52.5 ±8.6	+16.9 ±8.6
IVS	−17.8 ±4.0	+125.2 ±4.0	−1.0 ±3.9	+11.5 ±4.1	+85.6 ±4.7	+22.8 ±4.9	−16.4 ±4.7	+13.3 ±4.8

Standard errors were obtained as formal least-squares estimation from the individual residuals at 6-hour steps, under the assumption that they are mutually de-correlated after 4 days. For comparison, the last row shows the VLBI results, computed as a sum of sine/cosine terms estimated from IVS solution and the MHB Sun-synchronous correction. The latter empirical correction, concerning only prograde annual term, was included by MHB into their nutation model to make the annual term consistent with the observations. There is a relation between the prograde/retrograde components A^+, A^- of a term of nutation with angular velocity ω and pole position $P = X + \mathrm{i}\,Y$

$$X + \mathrm{i}\,Y = -\mathrm{i}(A^+\,\mathrm{e}^{\mathrm{i}\omega} + A^-\,\mathrm{e}^{-\mathrm{i}\omega}) \tag{5}$$

that leads to the relations enabling to calculate the sine/cosine terms in X, Y

$$S_X = \mathrm{Re}\,A^+ - \mathrm{Re}\,A^-, \quad C_X = \mathrm{Im}\,A^+ + \mathrm{Im}\,A^-,$$
$$S_Y = \mathrm{Im}\,A^+ - \mathrm{Im}\,A^-, \quad C_Y = -\mathrm{Re}\,A^+ - \mathrm{Re}\,A^-. \tag{6}$$

We used these relations to re-calculate the MHB Sun-synchronous values to sine/cosine terms. All individual terms of Table 1 are relatively small (only exceptionally exceeding 200 μas), however not fully consistent for different models used.

Bizouard et al. (1998) and Yseboodt et al. (2002) used NCEP(NIB), in a much shorter time interval than this study, to estimate the atmospheric contribution in nutation. Therefore we can compare only the first row of our Table 1, recalculated to prograde/retrograde terms by using eqs. (6), with their results. The comparison of prograde and retrograde amplitudes is given in Table 2. The agreement is relatively good.

Table 2 Comparison of prograde/retrograde amplitudes of NCEP(NIB) atmospheric contribution in nutation [μas]

Author	annual		Semi-annual	
	A^+	A^-	A^+	A^-
Yseboodt	82	56	45	5
Bizouard	77	53	45	0
this study	93	93	58	6

6. CONCLUSIONS

The observed celestial pole offsets around RFCN contain not only free, but also forced motions due to excitations by outer parts (atmosphere, oceans) of the Earth. Significant are annual and semi-annual terms. Atmospheric pressure term with inverted barometer correction (a simple oceanic model), combined with wind effects is sufficiently large to excite the observed free core nutation. The numerical integration yields both amplitudes and phases of the free motion comparable to the observations, even in the interval almost 25 years long. If no IB correction is applied, the fit with the observations is substantially worse – the amplitudes are too large and the phases are often not consistent with the VLBI observations. The best fit to observation is obtained with the combination of NCEP/NCAR AAM with ECCO OAM model, the oceanic model OMCT gives somewhat worse results. Geophysical contributions to nutation (annual and semi-annual term) are rather small, typically not exceeding 100μas, different models yield slightly different results (exceeding their formal standard errors), and they are also different from VLBI-based corrections of IAU2000A model.

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