IMPLEMENTATION OF THE VONDRAK'S SMOOTHING IN THE COMBINATION OF RESULTS OF DIFFERENT SPACE GEODESY TECHNIQUES

Vojtěch ŠTEFKA [1)] * and Ivan PEŠEK [2)]

[1)] *Center for Earth Dynamics Research (CEDR), Astronomical Institute of the Czech Academy of Sciences of the Czech Republic, Boční II, 141 31 Prague 4, Czech Republic*
[2)] *Center for Earth Dynamics Research (CEDR), Department of Advanced Geodesy, Faculty of Civil Engineering, Czech Technical University in Prague, Thákurova 7, 166 29 Prague 6, Czech Republic*
**Corresponding author's e-mail*: stefka@ig.cas.cz

(*Received* May 2007*, accepted* August 2007)

ABSTRACT
Four space geodesy techniques, namely VLBI, GPS, SLR and Doris, produce Earth orientation parameters (EOP) and station coordinates independently of each other. A method to combine them in a non-rigorous way (as described elsewhere) was modified by implementing Vondrak's smoothing. It replaces a simple constraint to tie EOP at adjacent epochs by a more general expression defining smoothness of the resulting curves. This gives the method better stability of derived EOP.
The new method was tested on GPS, VLBI and SLR data covering a one-year interval. The results were compared with the results of the original method and with the IERS c04 solution. The former indicates the effect of modification while the latter shows differences of this particular solution from the official IERS series.

KEYWORDS: Earth orientation parameters, combination of space geodesy techniques, Vondrak's smoothing

1. INTRODUCTION

Orientation of the Earth's body in space is described by five angles, called Earth orientation parameters, EOP, which tie the Earth-fixed coordinate system ITRF to the celestial reference frame. The EOP are two coordinates of the intermediate pole with respect to the ITRF, x_p, y_p, a time correction UT1–UTC, which characterizes irregularity of the Earth's proper rotation and, finally, two components of the celestial pole offset, dX, dY, which denote the observed corrections to the adopted precession-nutation model (not used in the present paper). International reference frame ITRF is realized by geocentric rectangular coordinates of reference points of a set of stations (observatories) equipped with one or more high precision observation techniques.

The space geodesy techniques used to produce the EOP and station coordinates are Global Position System (GPS), Very Long-Baseline Interferometry (VLBI), Satellite Laser Ranging (SLR), and recently also Doris, all of them working with a high internal accuracy. The individual techniques, though, are referred to different standards and constants, and use different mathematical models, so their results suffer form mutual systematic differences and biases.

There are basically two ways to derive one representative set of EOP and station coordinates from all the contributing techniques:

- The rigorous approach needs either to process the original data at the level of observation equations or to solve a new system of normal equations created from the normal equations and covariance matrices of the individual techniques (*e.g.* Gambis et al., 2006). Efforts are made by several groups to develop the necessary algorithms, but: The first approach is extremely complicated and, up to now, it has only been tested on a very limited network. The second approach is apparently simpler but even in this case the problem of applying properly all specific constraints to the new system remains still open.
- It is also possible to derive an approximate solution by combining *results* of the individual techniques omitting covariances. We use this non-rigorous approach because it yields a stable solution, provided some simple constraints are applied.

For deriving a function from scattered data, the Vondrak's smoothing (Vondrák, 1977) is widely used. The method is designed to find the most probable function values as a compromise between the least squares fit and the demanded function's smoothness.

We implemented the Vondrak's smoothing to the non-rigorous combination as a more sophisticated approach to tie EOP at the adjacent epochs. The first comparisons with the simple linear constraints are presented in this paper.

2. NON-RIGOROUS COMBINATION

The basic idea of the method is to combine station position vectors, $\mathbf{x}_C$, in the celestial reference frame (e.g. Pešek and Kostelecký, 2006), where they are functions of both the Earth orientation parameters and the station coordinates $\boldsymbol{x}$,

$$\boldsymbol{x}_C = \mathbf{PN}(t)\mathbf{R}_3(-\Theta)\mathbf{R}_1(y_P)\mathbf{R}_2(x_P)\boldsymbol{x} \,. \tag{1}$$

$\boldsymbol{PN}(t)$ is precession-nutation matrix, Θ the Earth rotation angle, and $\boldsymbol{R}_i$ are matrices of rotation around the axis i.

Input data for the combination consists of M sets of Earth orientation parameters (x_p, y_p, UT1-UTC, dX, dY)$_m$ and corresponding sets of station coordinates $(\boldsymbol{x})_m$, m = 1, … M, as derived by analysis centres for individual techniques.

From all stations/instruments only those, which are collocated with other techniques, are selected to enter the combination. Local ties of the collocated instruments have to be known.

To make the combination more stable, parameters of a seven-parametric transformation formula are derived for each technique instead of corrections to the station coordinates themselves. This allows, moreover, the transformation of coordinates of stations not entering the combination.

The aim of combination is to produce a „representative“ set of the Earth orientation parameters (x_p, y_p, UT1-UTC, dX, dY) for some epochs T_i, and parameters $\boldsymbol{p} = p_1, \ldots\, p_7$ of the seven-parametric transformation for each input set of station coordinates.

The transformation (1) yields observation equations of the form

$$\sum_j \frac{\partial \mathbf{x}_C}{\partial U_j} \mathrm{d}U_j \quad = \quad \mathbf{x}_C\,|_{obs} - \mathbf{x}_C\,|_0 \quad + \quad v\,, \tag{2}$$

where the „observed“ vectors $\boldsymbol{x}_C|_{obs}$ are calculated from the respective input solution, $\boldsymbol{x}_C|_0$ are functions of adopted *a priori* values of the unknowns, and U stands for any EOP and parameter $\boldsymbol{p}$.

To remove singularity of the system (2), a no net-rotation constraint, minimizing mutual shifts and preserving the system as a whole unchanged, has to be introduced,

$$\sum \boldsymbol{p}^{\mathrm{T}} \boldsymbol{p} \quad = \quad \min\,, \tag{3}$$

which stabilizes the station coordinates. On the other hand, orientation parameters are calculated for each individual epoch independently of the others. In result, errors in the input data, including station coordinates, are transferred to the EOP and increase their scatter substantially. The effect can be reduced by including constraints, in the form of additional observation equations (pseudo-observations), that tie values of the respective EOP, E, at adjacent epochs, i.e.

$$\mathrm{d}E_i - \mathrm{d}E_{i-1} \quad = \quad -\left(E_i - E_{i-1}\right) \quad + \quad v. \tag{4}$$

By weighting, these constraints control smoothness of the combined orientation parameters.

3. VONDRAK'S SMOOTHING AND ITS IMPLEMENTATION

The basic idea of smoothing (Vondrák, 1969, 1977) consists of finding a compromise between two contradictory requirements. The derived function should be as close to the observed data as possible on one hand (denoted here as “fidelity”) and, at the same time, as smooth as possible, on the other.

The smoothness S is expressed by the integral of the third derivative squared of a third order Lagrange interpolation polynomial, which yields a formula

$$S = \textstyle\sum_{i=1}^{N-3} \left(a_i y_i + b_i y_{i+1} + c_i y_{i+2} + d_i y_{i+3}\right)^2 \,. \tag{5}$$

Here y denotes the observed quantities, as observed at N points, t, and

$$a_i = 6P_{i,0}^{-1}\sqrt{(t_{i+2} - t_{i+1})(t_N - t_1)}$$
$$b_i = 6P_{i,1}^{-1}\sqrt{(t_{i+2} - t_{i+1})(t_N - t_1)}$$
$$\ldots$$
$$P_{i,m} = \prod_{\substack{j=0.3 \\ j\neq m}} (t_{i+m} - t_{i+j})$$

The fidelity is

$$F = \frac{1}{N-3} \textstyle\sum_1^N pvv \,.$$

Compromise between the two requirements leads to generalized least squares condition by minimizing their combination

$Q = S + \varepsilon.F = \min$,

or, equivalently

$Q' = w.S + F = \min$,

where $w = 1/\varepsilon$ and $\varepsilon > 0$. Magnitude of ε controls smoothness of the result within the limits of a second order parabola ($\varepsilon \to 0$) and a curve running through all the points ($\varepsilon \to \infty$).

Vondrak's approach is superior when compared to that used in the combinations so far because it utilizes up to seven consecutive points while the constraint (4) ties only two nearest points. Thus, it would make the combined EOP more comprehensive.

Implementation of Vondrak's smoothing to the non-rigorous combination consists of replacing constraints (4) by equations (5), because fidelity F becomes an integral part of the least-squares minimum constraint of the whole system. The weight w of constraints (5) controls the smoothness. Bigger weight yields a smoother solution.

The original Vondrak’s method makes it possible to derive the level of smoothing as a function of ε analytically (Huang and Zhou, 1982). Here, the effect of the constraints (5) as applied to the respective EOP is propagated into other parts of the equation system, i.e. the other EOP and transformation parameters are also affected. Thus, it is better to estimate the weight empirically.

Note that the two constraints differ not only in the form, but also in units they represent. While the unit of weight w of constraint (5) is basically a reciprocal of the third derivative squared of the searched function, the weights of (4) are simply reciprocals of the unit of the respective EOP, to make the constraints dimensionless. Consequently, the weights would differ numerically to produce the same result.

4. DATA AND NUMERICAL SOLUTION

The method was tested on the following data: GPS and VLBI data was taken from the IERS Combination Pilot Project database (*see* Data-I). For SLR, the constrained 'ilrsb' solution was used, as published by ILRS analysis centre (Data-II). Both GPS and SLR are weekly Sinex solutions, from which the EOP and station coordinates were extracted. VLBI data consists of *per seance* singular normal equation matrices. They were regularized by constraining the station coordinates to the VTRF 2005 frame (Nothnagel, 2005) with the a priori precision of 5 mm. As none of the techniques currently provides the database with the celestial pole offset, only the x_p, y_p, and UT1‑UTC were solved for.

To obtain effect of introduction of the new smoothing, the optimum weight was derived for both combination modes from fit to the IERS c04 series (Data-III), which was used as an independent reference. The best fits were achieved with the weights 200 for the constraints (5) of Vondrak's smoothing and 0.01 for the constraints (4). Then the two results are very close to each other. As the same data and processing was used, but the smoothing constraints, the mutual differences can be treated as the effect of implementation of the Vondrak's smoothing. Rms of the differences is 0.006 mas, 0.011 mas and 0.008 ms, for x_p, y_p, and UT1‑UTC, respectively.

In both cases, differences from the IERS c04 series do not exceed 0.3 mas, 0.3 mas and 0.2 ms, with the rms 0.112 mas, 0.098 mas and 0.055 ms, for x_p, y_p, and UT1‑UTC, respectively, in the case of y_p after removing the bias of ‑0.224 mas. This can be considered as a good agreement, taking into account the different data and processing used. Also the bias in y_p is due most likely to different data used, because a similar bias appears in the input data (Fig. 1).

5. CONCLUSIONS

A method for the non-rigorous combination of results of different space geodesy techniques to obtain representative sets of the Earth orientation parameters and station coordinates was modified by implementing the Vondrak's smoothing. This is a more complex approach to smoothing the data than a simple formula (4) used so far. The original and modified methods were compared with the IERS c04 series with the result, that the best fit is achieved with weighting the smoothing constraints, respectively, by 0.01 and 200. Then rms of the differences between the original and modified method are 0.006 mas, 0.011 mas and 0.008 ms, for x_p, y_p, and UT1‑UTC, respectively. They are treated as the effect of implementing the Vondrak's formula, because both the data and the algorithms, except the smoothing constraints, were kept unchanged in both cases.

Also comparison with the IERS c04 series, with differences not exceeding 0.3 mas and 0.2 ms for pole coordinates and time correction, respectively, can be considered as a good agreement, taking into account different data and processing used.

As it is easier to stabilize this method than the rigorous methods, it could be used for checking the rigorous combination methods at least at the initial stages of their testing.

ACKNOWLEDGEMENTS

The authors greatly appreciate the support of grant LC506 awarded by the Ministry of Education, Youth and Sports of the Czech Republic.

REFERENCES:

Data-I: iers1.bkg.bund.de/projects/combination/intra-technique/

Data-II: ftp://ftp.dgfi.badw-muenchen/de/slr/ilrsac/products/ops/pos+eop

Data-III: http://hpiers.obspm.fr/eoppc/eop/eopc04/eopc04.62-now

Gambis, D., Biancale, R., Lemoine, J.-M., Marty, J.-C., Loyer, S., Soudarin, L., Carlucci, T., Capitain, N., Bério, PH., Coulot, D., Exertier, P., Charlot, P. and Altamimi, Z.: 2006, Global combination from space geodetic techniques. In: Proccedings of the "Journées 2005 de Référence Spatio-Temporels", A. Brzezinski, N. Capitaine, B.Kolaczek (eds.), SpaceResearch Centre PAS, Warsaw, Poland, 62−65.

Huang, K.Y. and Zhou, X.: 1982, A new definition in the design of digital filter and the variational solution about extended Whittaker problem. Acta Astron. Sinica 23, 280.

Nothnagel, A., 2005: VTRF2005 − A combined VLBI Terrestrial Reference Frame. http://miro.geod.uni-bonn.de/vlbi/IVS-AC

Pešek, I. and Kostelecký, J.: 2006, Simultaneous determination of Earth orientation parameters and station coordinates from combination of results of different observation techniques. Stud. Geophys. Geod., 50 (2006), 537−548.

Vondrák, J.: 1969, A contribution to the problem of smoothing observational data. Bull. Astron. Inst. Czechosl., 20, 349−355.

Vondrák, J.: 1977, Problem of smoothing observational data II. Bull. Astron. Inst. Czechosl., 28, 84−89.

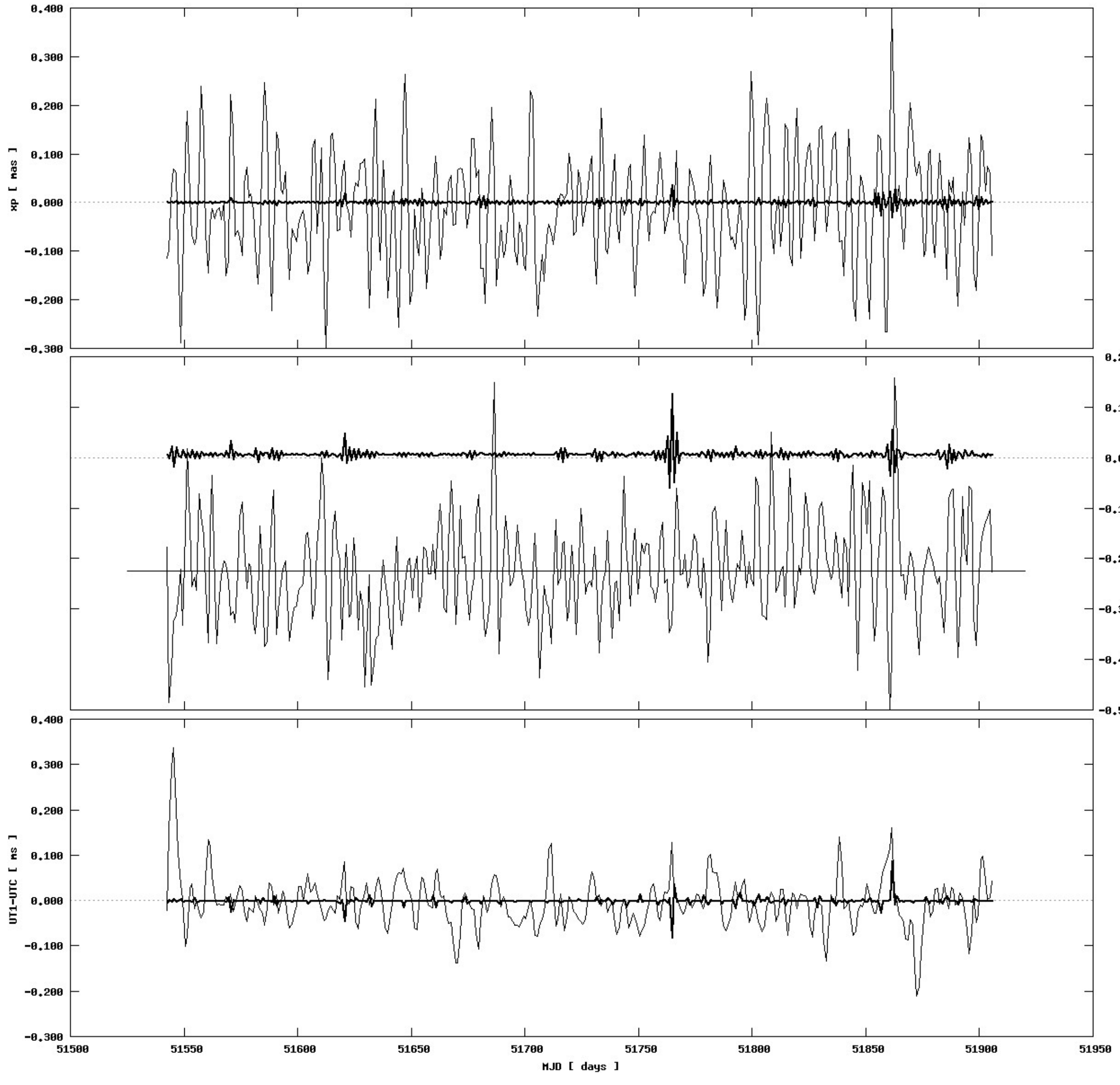

Fig. 1 Differences of original combination using the linear smoothing constraint (4) from the present solution (thick lines) show the effect of implementation of the Vondrak's smoothing (5) on the polar motion components x_p (above), y_p (middle), and the time correction UT1-UTC (below). Bigger differences and bias -0.224 mas in y_p of the IERS c04 series (thin lines) are due mainly to different data and processing used.