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# SOME PRACTICAL PROCEDURES FOR USE WHEN PREPARING DATA FOR GPS VECTOR SOLUTIONS TIME SERIES ANALYSIS

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**ABSTRACT**
In this work, the set of procedures to prepare the GPS vector solutions time series (VSTSGPS) to spectrum analyses is presented. This preparation is shared on two stages. In the first stage, the breaks filling was processed. This stage was achieved in two steps. Firstly, the breaks filling was computed on the base of time series of other vectors. Secondly, the breaks filling was computed using the interpolation or extrapolation methods. The next stage of VSTSGPS preparing implicated the time series smoothing to remove the impulse noises. After breaks filling and smoothing the VSTSGPS were tested for admission to further analyses.

**KEYWORDS:** GPS permanent observations, GPS solutions, time series

## 1. INTRODUCTION

In the article (Góral and Jasiurkowski, 2006) the authors showed the research results of GPS vector solutions time series (VSTSGPS). Time series mentioned were the series of GPS vector solutions in ASG-PL+KRAW network, obtained from three-hours sessions; more in mentioned work. VSTSGPS are the discrete representation of continuous process of GPS vectors coordinates changes in the time domain. To detect main trends of this process, the spectral mathematical techniques were used. These techniques require stationary and equidistant data. Because of equipment failures and atmospheric influences, the VSTSGPS were interrupted and degraded with impulse noises. In this work, the mathematical methods useful to eliminate described perturbations from the time series mentioned were presented. Every VSTSGPS was processed according to scheme in the Figure 1 below.

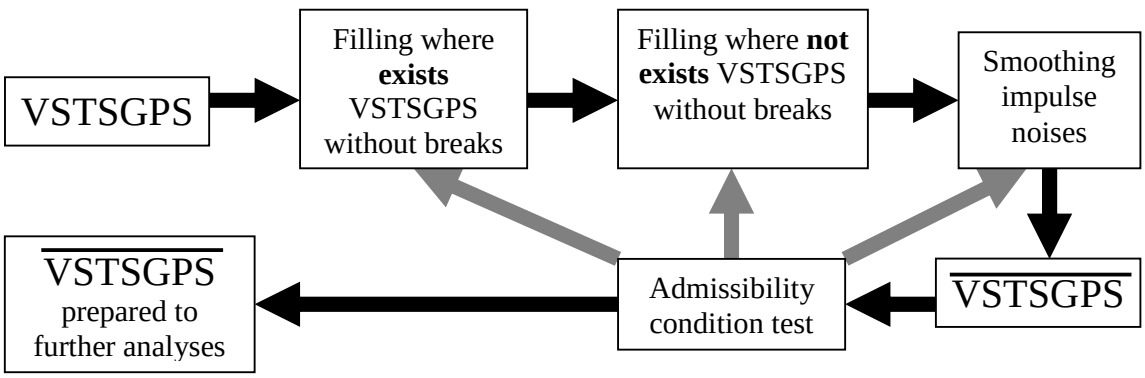


**Fig. 1** Scheme of VSTSGPS preparing.

## 2. VSTSGPS FILLING

The VSTSGPS were frequently interrupted. Average number of breaks occurred in the time series was presented in Table 1 in (Góral and Jasiurkowski, 2006). Indifferently, about 2.5% samples of all VSTSGPS samples required filling. Author introduced two steps of VSTSGPS filling. The first step was used in the case where, at the break moment $t_i$ of some vector time series, there existed others vectors time series in considered network, without breaks at $t_i$ time moment. In that case, the break at $t_i$ time moment was filled with using others vectors solutions. That approach was preferred to reach the most trendy values in breaks places. The second step was using when at $t_i$ time moment no vectors was solved. Then, the breaks was filled up using interpolation or extrapolation methods.

### *2.1. VSTSGPS FILLING – THE FIRST STEP*

To complete the VSTSGPS on the base of others vectors VSTSGPS, author suggested two methods, called triangle and vectors methods. Both of them are based on the assumption that the break value at $t_i$ time moment can be obtained from known polynomial surface of others vectors solutions. Assumed, that the mean error of any computed vector $V_{nk}$ component $\varepsilon v$ ($\varepsilon\varphi$, $\varepsilon\lambda$, $\varepsilon h$) is approximately proportional to vector length (Kostelecký, 2002). Hence, the mean error for meridian vectors is proportional to $\Delta\varphi_{nk}=\varphi_k-\varphi_n$ and for parallel vectors to $\Delta\lambda_{nk}=\lambda_k-\lambda_n$. For any azimuth vectors, the mean error proportionality to coordinate differences can be expressed as (1):

$$\varepsilon v_{nk} = \alpha\Delta\varphi_{nk} + \beta\Delta\lambda_{nk} \tag{1}$$

where $\varepsilon v$ is one of the mean errors ($\varepsilon\varphi$, $\varepsilon\lambda$, $\varepsilon h$), $\alpha$ and $\beta$ are the plane equation parameters which model $\varepsilon v$ errors in depends on coordinates differences $\Delta\varphi$ and $\Delta\lambda$. If we have the catalogue coordinates of marked vectors, for every $t_i$ time moment, the empirical value of mean errors $\varepsilon v$ in form of discrepancy $dv$ ($d\varphi$, $d\lambda$, $dh$) can be computed. Therefore, for discrepancies $dv$ the dependency (1) is implemented too.

#### *2.1.1. TRIANGLE METHOD OF VSTSGPS FILLING*

Let take a single sample of VSTSGPS at $t_i$ time moment. For the pair of vectors $V_{1\text{-}2}$, $V_{1\text{-}3}$ coming from one common point and possess solutions at $t_i$ time moment, for VSTSGPS of every component of those vectors, a set of two equations on the base of equation (1) can be formulated as (2). For simplicity, only the $h$ component is considered.

$$\begin{cases} dh_{12} = \alpha\Delta\varphi_{12} + \beta\Delta\lambda_{12} \\ dh_{13} = \alpha\Delta\varphi_{13} + \beta\Delta\lambda_{13} \end{cases} \tag{2}$$

The set of equations (2) presents some plane $P(\Delta\varphi,\Delta\lambda)$ of discrepancies $dh$ of $h$ component. The plane mentioned above is spread on $V_{12}$ and $V_{13}$ vectors. The parameters $\alpha$ and $\beta$ are the components of plane $P(\Delta\varphi,\Delta\lambda)$ normal vector $k(\alpha,\beta,\text{-}1)$. It is assumed that for any vector $V_{nk}$ located inside of triangle 1-2-3, the discrepancy $dh_{nk}$ can be computed from the established plane $P(\Delta\varphi,\Delta\lambda)$. Vector $k(\alpha,\beta,\text{-}1)$ is normal vector for any surface $S(\Delta\varphi,\Delta\lambda)$ tangential to the $P(\Delta\varphi,\Delta\lambda)$ plane in chosen point $K(\varphi_m,\lambda_m)$ too. According to (Bronsztejn and Siemiendiajew, 1988) the equation of plane $P(\Delta\varphi,\Delta\lambda)$ tangential to surface $S(\Delta\varphi,\Delta\lambda)$ in the point $K(\varphi_m,\lambda_m)$ is:

$$P(\varphi,\lambda) = \frac{\partial S}{\partial\varphi}(\varphi-\varphi_m) + \frac{\partial S}{\partial\lambda}(\lambda-\lambda_m) \tag{3}$$

The partial derivatives $\partial S/\partial\varphi$ and $\partial S/\partial\lambda$ from equation (3) are respectively $\alpha$ and $\beta$ parameters we computed from (2). Therefore, we know the partial derivatives of surface $S(\Delta\varphi,\Delta\lambda)$ in point $K(\varphi_m,\lambda_m)$. For each $i$ pairs of vectors coming from common point, a sets of equations (4) for chosen surface $S(\Delta\varphi,\Delta\lambda)$ can be written as:

$$\begin{cases} \dfrac{\partial S}{\partial\lambda_m} = \alpha_m \\ \dfrac{\partial S}{\partial\varphi_m} = \beta_m \end{cases}, m \in <1,..N> \tag{4}$$

where $N$ is the number of found vectors pairs. The least squares method (*LSM*) as $AX\text{-}L=0$ matrix equation was employed to find solution surface. On the left side of (4), the partial derivatives of surface $S(\Delta\varphi,\Delta\lambda)$ in points $K(\varphi_m,\lambda_m)$ are situated. Ones compose project matrix $A$ in *LSM*. For example, for some second degree surface $S(\Delta\varphi,\Delta\lambda) = a\Delta\varphi^2 + b\Delta\lambda^2 + c\Delta\varphi\Delta\lambda$ the project matrix $A$ for one vector pair is presented below:

$$A = \begin{vmatrix} 2a\varphi_m & 0 & c\lambda_m \\ 0 & 2b\lambda_m & c\varphi_m \end{vmatrix} \tag{5}$$

The free terms matrix $L$ is composed by $\alpha$ and $\beta$ coefficients. The result matrix $X$ contains ($a$, $b$, $c$,…) coefficients of surface $S(\Delta\varphi,\Delta\lambda)$ equation. Localization of $K(\varphi_m,\lambda_m)$ point can be obtained from several ways. For every vector pair the $K_m$ point should be different. The position of that point was adopted according to (6):

$$\varphi_m = \frac{p_1\varphi_1 + p_2\varphi_2 + p_3\varphi_3}{p_1 + p_2 + p_3},$$
$$\lambda_m = \frac{p_1\lambda_1 + p_2\lambda_2 + p_3\lambda_3}{p_1 + p_2 + p_3} \tag{6}$$

where coefficients: $p_1$=3, $p_2$=1 and $p_3$=1. This approach is motivated following. In Figures 2a and 2b the two another (extreme) localization of $K_m$ point in $K_m$' points is showed.

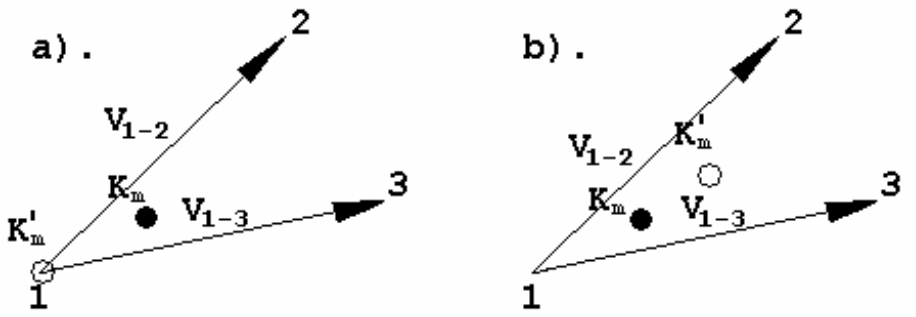


**Fig. 2** Localizations of $K_m$ point for $V_{1\text{-}2}$ and $V_{1\text{-}3}$ vectors pair.

The localization of $K_m$ point in $K_m$' point from Figure 2a is unusable where the others vectors are coming from the point 1. In that situation, for every pair of vectors which comes out from point 1, the $K_m$ point is constant. The localization of point $K_m$ in $K_m$' from Figure 2b is unusable too, because when we consider triangle of vectors $V_{12}$, $V_{13}$ and $V_{23}$, for every of three possible combinations of vectors pairs: ($V_{12}$, $V_{13}$), ($V_{12}$, $V_{23}$), ($V_{13}$, $V_{23}$), the $K_m$ point is constant. The coordinates of $K_m$ point computed form formula (6) are mediate between cases mentioned above. The *LSM* gave parameters of surface $S(\Delta\varphi,\Delta\lambda)$. The computed surface is approximately tangential to every planes $P_m(\Delta\varphi,\Delta\lambda)$ based on finding vectors pair. Surface $S(\Delta\varphi,\Delta\lambda)$ models discrepancies of solution for any vector in network at $t_i$ time moment.

#### *2.1.2. VECTOR METHOD OF VSTSGPS FILLING*

In the vector method, the sets of as many equations of type (1) was created as existed the vectors having solution at $t_i$ time moment. The choice of modeling surface $S(\Delta\varphi,\Delta\lambda)$ enforced the equation (1) developing. For example for second degree surface the equation (1) can be:

$$dv_{nk} = a\Delta\varphi_{nk} + b\Delta\lambda_{nk} + c(\Delta\varphi_{nk})^2 + d(\Delta\lambda_{nk})^2 + e\Delta\varphi_{nk}\Delta\lambda_{nk} \tag{7}$$

The complicity and degree of (7) depends of the number of vectors in network considered, the number of assumed degrees of freedom for approximation method (*LSM*) and others factor. The project matrix $A$ for equations of type (7) is presented in (8) below:

$$A = \begin{vmatrix} \Delta\varphi_{12} & \Delta\lambda_{12} & \Delta\varphi_{12}{}^2 & \Delta\lambda_{12}{}^2 & \Delta\varphi_{12}\Delta\lambda_{12} \\ \Delta\varphi_{13} & \Delta\lambda_{13} & \Delta\varphi_{13}{}^2 & \Delta\lambda_{13}{}^2 & \Delta\varphi_{13}\Delta\lambda_{13} \\ . & . & . & . & . \\ \Delta\varphi_{nk} & \Delta\lambda_{nk} & \Delta\varphi_{nk}{}^2 & \Delta\lambda_{nk}{}^2 & \Delta\varphi_{nk}\Delta\lambda_{nk} \end{vmatrix} \tag{8}$$

The free terms matrix L consisted of discrepancies dv. The parameters of surface $S(\Delta\varphi,\Delta\lambda)$ were computed using *LSM*.

### *1.3. WEIGHTING OF TRIANGLE AND VECTOR METHODS*

To improve efficiency of breaks filling, the weight matrixes $W$ were introduced to least square equations for both methods mentioned. There are several ways to find the $W$ matrixes. For triangle method a $W$ matrix finding was quite easy because every equations pair from formula (4) was valid for one point $K(\varphi_m,\lambda_m)$. The diagonal weight matrix values can be obtained for example as inverse of distance between point localized in 1/3 length of vector which VSTSGPS was filling and $K(\varphi_m,\lambda_m)$ point. That way is order to $K(\varphi_m,\lambda_m)$ point method computing. Unfortunately, in vector method, there were assumed that every equation (7) is valid on the entire length of $V_{nk}$ vector. To resolve that problem the author used the conclusions about azimuth vectors similarity from work by Góral and Jasiurkowski (2006). On that base, the matrix azimuth similarity (*MAS*) was introduced as:

$$MAS_{nk} = \cos(|Az_k - Az_n|) * |\cos(|Az_k - Az_n|)| \tag{9}$$

In Figure 3 the graphical representation of *MAS* is presented in dependency on vectors azimuth in ASG-PL+KRAW network.

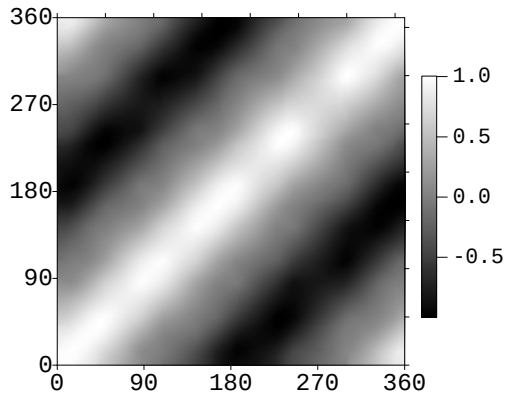


**Fig. 3** Graphical representation of *MAS* in vectors azimuth domain.

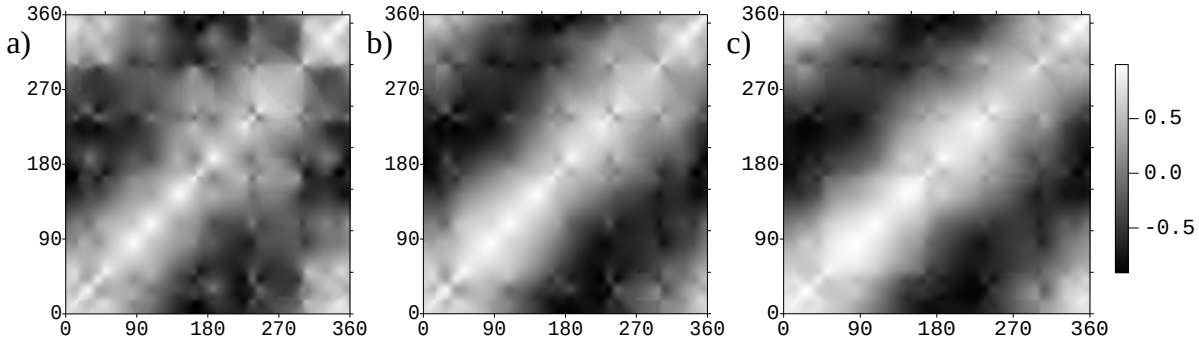


**Fig. 4** Graphical representation of *CM* matrixes for VSTSGPS of $d\varphi$, $d\lambda$, $dh$.

Form the other side, in Figures 4a, 4b and 4c the correlations matrixes (*CM*) for $d\varphi$, $d\lambda$, $dh$ components are presented according to (10) below:

$$CM_{nkn'k'} = \sum_{i=1}^{N} V_{nk}{}^{s}(t_i) V_{n'k'}{}^{s}(t_i) \tag{10}$$

where $V_{nk}{}^{s}(t_i)$ and $V_{n'k'}{}^{s}(t_i)$ are standardized VSTSGPS of *nk* and *n'k'* vectors, *N* is the number of time series samples.

For known *CM* of the all vectors VSTSGPS, it is possible to compute the empirical diagonal values of *W* matrix for every vector as the inverse of *CM* coefficients. Hence, for filling time series of vector $V_{nk}$ using the vector method, the weight matrix can be:

$$P = \begin{vmatrix} \frac{1}{CM_{nk12}} & 0 & 0 & 0 \\ 0 & \frac{1}{CM_{nk13}} & 0 & 0 \\ 0 & 0 & \frac{1}{CM_{nk..}} & 0 \\ 0 & 0 & 0 & \frac{1}{CM_{nkn'k'}} \end{vmatrix} \tag{11}$$

Comparing Figure 3 and Figure 4 there is easily visible the similarity of those graphical representations. The replacing the *CM* matrix with the *MAS* matrix is possible and gives satisfactory results. The using of weight matrixes for both methods gave even 15% filling efficiency improvement.

### *2.2. VSTSGPS FILLING – SECOND STEP*

In the case where at the $t_i$ time moment the breaks existed in VSTSGPS of every vector, the break filling was made with the interpolation or extrapolation methods. When the break was found inside the time series the Lagrange'a interpolation was used (Fig. 5).

The others interpolations like Newton or linear interpolation were proper to breaks filling in considered situation.

For breaks occurred on the edge of time series, author suggested the "mirror reflecting" method illustrated in Figure 6.

In "mirror reflecting" method, the as many mirror reflecting samples had to be added to beginning or end of time series as the samples we want to filled up. The mirror is situated on the $t_i$=0 or


**Fig. 5** The example of Lagrange'a interpolation using to VSTSGPS breaks filling.

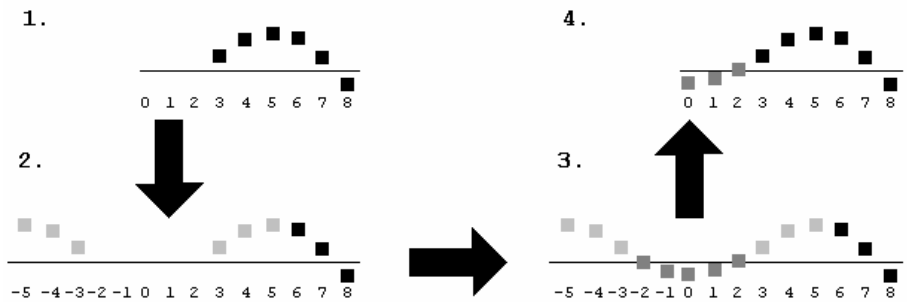


**Fig. 6** The example of "mirror reflecting" method employing to VSTSGPS breaks filling

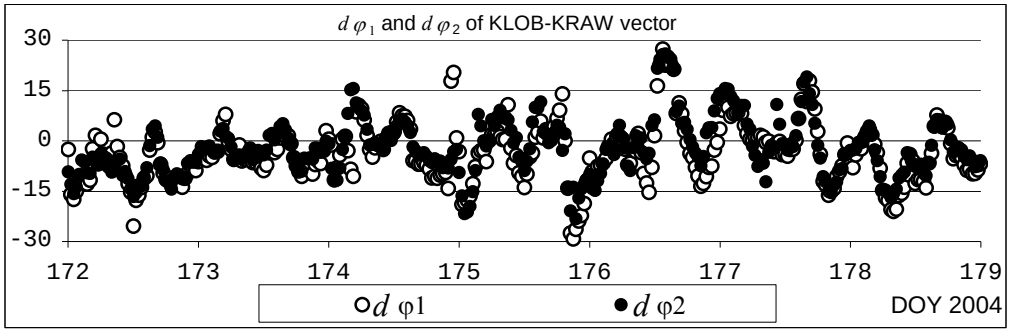


**Fig. 7** The example of VSTSGPS reconstruction of $d\varphi$ component using the vector method, vector KLOB-KRAW, 172-179 DOY2007.

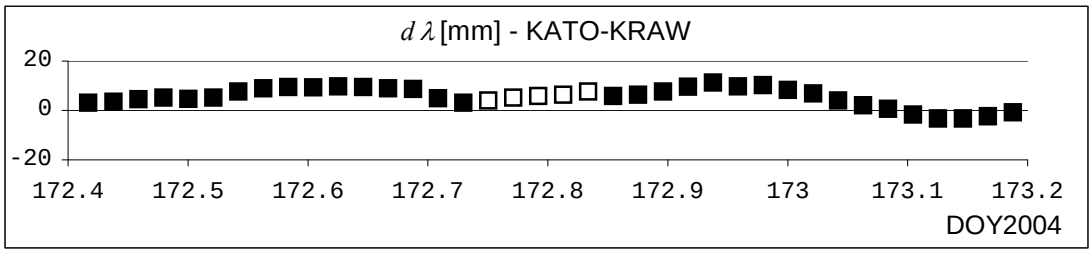


**Fig. 8** The example of VSTSGPS filling for $d\lambda$ component of KATO-KRAW vector, 172.4-173.2 DOY 2004 (- filled samples).

$t_i$=N. After samples attaching, the interpolation is computed to find breaks values. For VSTSGPS type of time series the "mirror reflecting" method were efficient for not more than 4 breaks.

### *2.3. EFFICIENCY OF VSTSGPS FILLING METHODS*

The best way to check efficiency of proposed methods is the reconstruction of all VSTSGPS samples for a vector we VSTSGPS generated empirically from GPS observations. In Figure 7 the example of that VSTSGPS reconstruction of $d\varphi$ component for vector KLOB-KRAW is presented. The $d\varphi_1$ is a time series obtained from GPS observations and $d\varphi_2$ is reconstructed time series using the vector method.

The discrepancies between the empirical values of VSTSGPS generated from GPS observations and reconstructed time series were middling on the 8mm level for the horizontal components and 25mm level for elevation. In the example presented above the discrepancies are respectively 5mm and 13mm. In Figure 8 the example of five breaks filling for $d\lambda$ component is presented. There is bold shown that the interpolation would give different results than proposed filling methods.

## 3. VSTSGPS SMOOTHING

In the VSTSGPS the impulse noises were found frequently. That phenomenon could disturb the main trends detecting. A simple and efficiency way to remove the impulse noises from the time series is the median filter (Stranneby, 2004). The median filter works with using the moving window along to time series. On the window samples the median filtration is performed. The median filtration consists in sorting samples according to their values and choosing the middle value. The chosen value is inserted in place of impulse noise instead of original value. In Figure 9 the example of impulse noises eliminating and breaks filling for VSTSGPS of ZYWI-KLOB vector for 175-176 DOY 2004 is presented.

## 4. ADMISSIBILITY CONDITIONS

After using the procedures of time series breaks filling and impulse noises elimination, author verified whether the adapted procedures gave the expected effects. To reach this goal the admissibility tests were introduced. For example, to admit the time series for a specific frequency detecting, the test should check if the series contain the much higher and much lower frequencies. In that case the continuity and

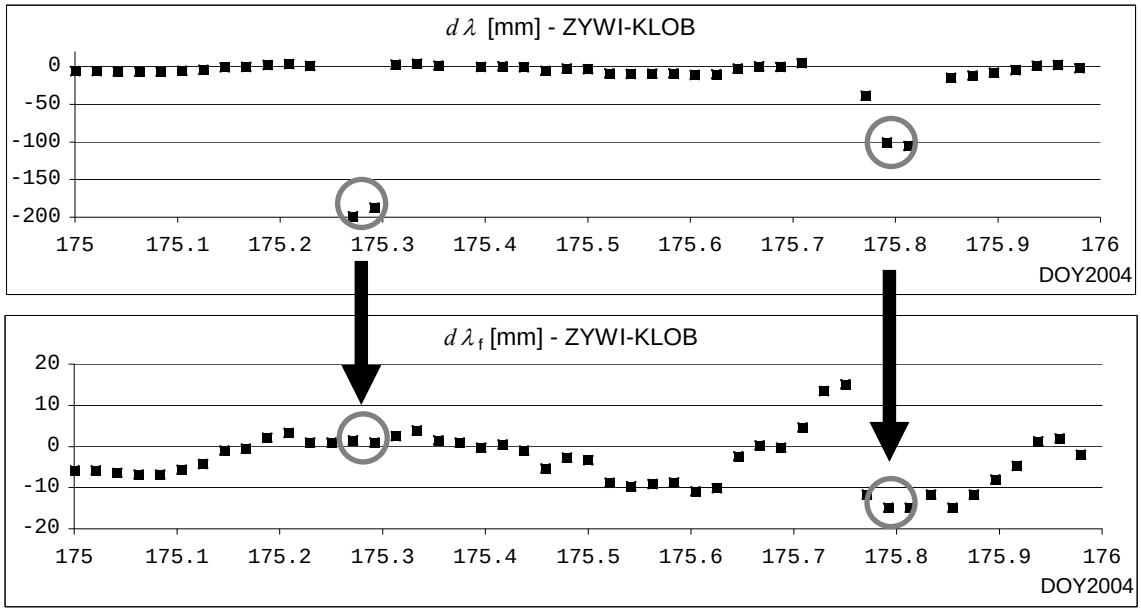


**Fig. 9** The example of impulse noises elimination using median filter method and breaks filling for VSTSGPS of ZYWI-KLOB vector,175-176 DOY 2004.

smoothness of time series is checked using the VSTSGPS second derivatives function. The creating of others admissibility conditions depends on necessity and can be made in unrestricted way.

## 5. CONCLUSIONS

To VSTSGPS breaks filling, the geometrical dependences in vectors network were used. The high efficiency of geometrical and approximation methods of reconstruction of entire VSTSGPS promises high efficiency of filling for several finding breaks. Therefore, the simple methods like “mirror reflecting” filling method and median filter smoothing method confirm that sometimes the simple methods are sufficient enough. All of the presented time series preparing techniques gave good results because they used the features of measured objects. That rule could be useful for every kinds of time series analysis.

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