

## MESOSCALE FUNCTIONS OF GPS SLANT DELAY

Mariusz FIGURSKI \*, Marcin GAŁUSZKIEWICZ, Paweł KAMIŃSKI  
and Krzysztof KROSCZYŃSKI

*WAT Military University of Technology, Faculty of Engineering Chemistry and Applied Physics,  
Gen. S. Kalinskigo 2st, PL 00 908 Wasaw 49, Poland*

*\*Corresponding author's e-mail: mfigurski@wat.edu.pl*

*(Received May 2007, accepted October 2007)*

### ABSTRACT

The paper presents a computer module for GPS slant delay determination using data from COAMPS (Coupled Ocean/Atmosphere Mesoscale Prediction System) mesoscale non-hydrostatic model of the atmosphere which is run on IA64 Feniks computer cluster in the Department of Civil Engineering and Geodesy of the Military University of Technology. The slant delay is the result of integrating the ray (eikonal) equation for the spatial function of tropospheric refraction along the GPS wave propagation path. The work is a phase of research concerning operational methods of GPS slant delay determination using data from mesoscale non-hydrostatic models of the atmosphere, like COAMPS of the Naval Research Laboratory (NRL) and the Weather Research and Forecasting (WRF).

**KEYWORDS:** slant delay, non-hydrostatic mesoscale model, ray tracing, anisotropy

---

### INTRODUCTION

Precise location by means of GPS technology has to take into consideration measurement corrections related with atmospheric impact on electromagnetic waves propagation from the satellite to the receiver. Atmospheric pressure, temperature and humidity slow down the signal and consequently make the real path of the ray from the satellite to the receiver longer than the geometric path. This phenomenon is called refraction or tropospheric delay. Various models of refraction are used due to nondispersivity of the troposphere for GPS frequency waves. Zenith Total Delay models dominated until recently. Two parts are distinguished here: the dry one (hydrostatic) determined from the pressure at the site and the wet one – more complicated to determine - depending on the spatial distribution of water vapor. Zenith Total Delay is used for determination of Slant Total Delay (STD), i.e. delay for any angle of GPS satellite observation. Mapping functions used for the purpose depend significantly on meteorological models. The coefficients for the Niell's functions (Niell, 1996) were obtained empirically using US Standard Atmosphere 1966 data while for the Isobaric Mapping Functions (IMF) – using NCEP (National Center for Environmental Prediction) numerical meteorological model (Niell, 2001). Recent research (2006) concentrated on tropospheric mapping functions (Boehm et al., 2006) using data from the European Centre for Medium-Range Weather Forecasts model (ECMWF). Coefficients of these functions (Vienna Mapping Functions - VMF) result from applying the ray tracing method. Similarly, R. Eresmaa and H. Järvinen of the Finnish

Meteorological Institute (Eresmaa et al., 2006) use the High Resolution Limited Area Model (HIRLAM) to determine the STD. HIRLAM is a hydrostatic model that does not take accelerations of vertical movements of the atmosphere into account. The atmospheric state parameters are determined at 31 or 40 hybrid levels (pressure-and-altitude). The model uses boundary conditions from the ECMWF. The paper presents research conducted using data from COAMPS ver.3.1 non-hydrostatic mesoscale model of the Naval Research Laboratory. This model enables to parameterize physical processes in the atmosphere (e.g. water phase transformations) more adequately than the hydrostatic models used by Boehm J. et al., R. Eresmaa and H. Jarvinen. Using models like COAMPS or WRF is far-reaching because of their continual development. Plans for COAMPS development include increasing the upper limit level of the model to 100 km (currently 30 km). Such an extension is of crucial importance to GPS signal propagation modeling, especially for numerical methods of the eikonal solving. WRF model is actively developed by numerous powerful meteorological institutions e.g. National Oceanic and Atmospheric Administration, National Centers for Environmental Prediction, Air Force Weather Agency (AFWA) etc. High resolution non-hydrostatic mesoscale models enable to locate anisotropic distribution of water in its various phases. They enable to determine heterogeneity of atmospheric refraction fields (refraction coefficient, propagation speed) of GPS electromagnetic waves which can be used to investigate and simulate propagation of microwave radiation in the atmosphere. The obtained

data may constitute basis for a module of analysis and distribution of tropospheric corrections of GPS signals propagation times.

## 1. STD MODULE

The prototype of the module includes programs for the refraction coefficient and slant delay fields determination. These programs use data from the COAMPS model run on IA64 Feniks computer cluster in the Department of Civil Engineering and Geodesy of the Military University of Technology. The grid of the 31 2D computational surfaces of the model is rectangular and related to the Lambert Conformal projection. Grid of 39 and 13 km are used in the research described here. The surfaces in the bottom part of the model of the atmosphere mimic orography. Nonlinear vertical coordinate is of  $\sigma$  type. It is defined (Hodur et al., 2003) as:

$$\sigma = H(z - z_s)/(H - z_s) \quad (1)$$

$H$  and  $z_s$  correspond to the depth of the atmosphere and the value of terrain at the grid point (Fig. 1). The depth  $H$  is 31.50 km i.e.  $\sim 10$  hPa. In general, the 3D cells of the model are not rectangular prisms because their nodes lie at different altitudes. The COAMPS

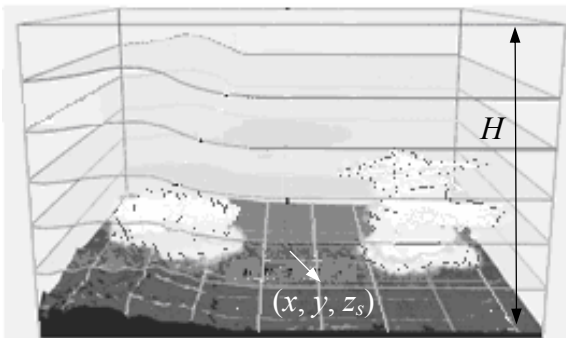


Fig. 1 Computational surfaces of the model.

model provides 3D fields (grid: 112x103x31 and 145x169x31) of forecasted parameters, i.e.: potential temperature -  $\theta$ , Exner pressure -  $\pi$ , specific humidity  $q$  and wind velocity components  $u$ ,  $v$ ,  $w$  used for further computing.

### 1.1. REFRACTION FIELDS SUBROUTINE

Atmospheric refraction coefficient  $N$  is derived from the following equation:

$$N = k_1 \frac{p_d}{T} Z_d^{-1} + k_2 \frac{e}{T} Z_w^{-1} + k_3 \frac{e}{T^2} Z_w^{-1} \quad (2)$$

where:  $p_d = p - e$  and  $e = qp/(0.622 + 0.378q)$  are partial pressures;  $Z_d$  and  $Z_w$  are coefficients of compressibility of dry air and of water vapor;  $p$ ,  $T$  and

$q$  are pressure, temperature and specific humidity of the atmospheric air;  $k_1, k_2, k_3$  constants determined experimentally (Bevis et al., 1994).

The parameters of the atmospheric state  $p$ ,  $T$  and  $e$  are determined by the refraction fields subroutine on computational surfaces of the model using fields of forecasted parameters  $\theta$ ,  $\pi$  and  $q$ . 3D field of refraction  $N$  is determined for 31 computational levels for 39 and 13 km grids and 3-hours intervals. (Bohem et al. 2006) determine refraction  $N$  by means of a geometric method of ray using ECMWF model's fields of  $p$ ,  $T$  and  $e$  on 15 standard pressure levels (approximated from computational surfaces) for two kinds of grid i.e.: global  $2.5^\circ \times 2.0^\circ$  and local  $0.25^\circ$ , and 6-hours intervals.

### 1.2. SLANT DELAY SUBROUTINE

Slant Total Delay is a result of integrating the refraction coefficient  $N$  along the GPS wave propagation path from the receiver to the satellite:

$$STD = \int (n-1) ds = 10^{-6} \int N ds, \quad N = (n-1) \cdot 10^6. \quad (3)$$

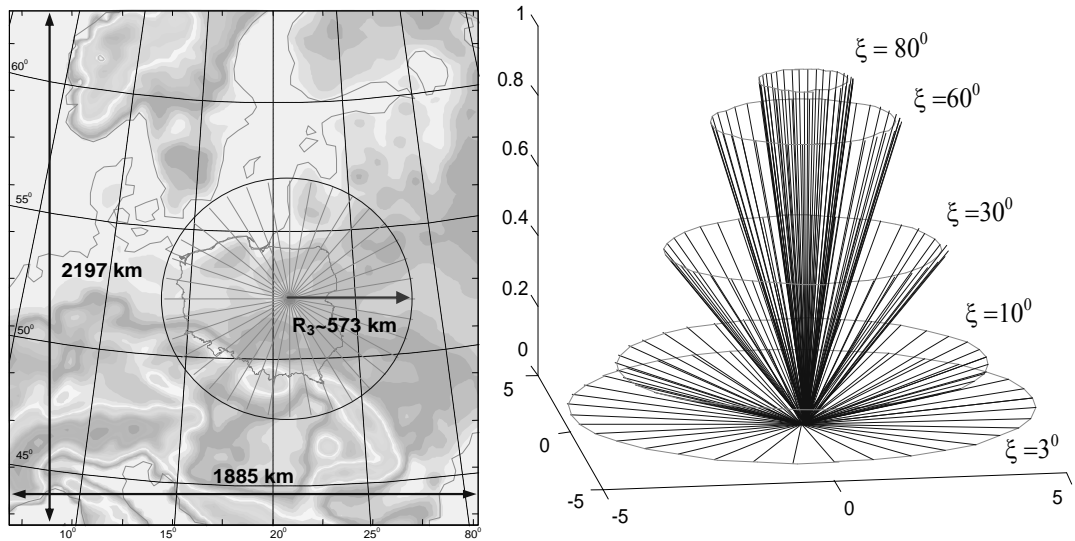
where:  $n$  is atmospheric refraction coefficient,  $ds$  elementary increase of the path  $s$  of the electromagnetic wave along its trajectory.

The signal trajectory is a solution of the following equation system (Zou et al., 1999):

$$\begin{cases} \frac{d\vec{r}}{ds} = \vec{v} \\ \frac{d\vec{v}}{ds} = n(\vec{r}) \nabla n(\vec{r}) \end{cases} \quad (4)$$

where:  $\vec{r} = [x(t), y(t), z(t)]$  is the ray vector with respect to a selected reference point i.e. GPS station coordinates,  $\vec{v}$  - vector tangential to the trajectory at point  $\vec{r}$ .

The equation system (4) for the ray is solved by means of the Runge-Kutta type methods for a known field of the  $n(\vec{r})$  coefficient and initial conditions related with the GPS station location and elevation angles of the satellites observed from the station. Due to discrete and nonlinear nature of the computational grid of the model, the coefficient  $n$  and its gradient  $\nabla n$  are results of two-step approximation in every computational step. In the first step, fields of  $n$  from model surfaces of equal altitudes  $z$  with respect to the terrain are approximated onto parallel surfaces i.e. surfaces lying at the same altitude with respect to the sea level. The number of the surfaces is twice the number of the model levels  $\sigma$ . In the second step, values of  $n$  are determined at the internal points of the approximated grid cells by means of polynomial trilinear interpolation. Polynomial structure of  $n$  enables to compute refraction and 3D refraction gradient at a trajectory point that belongs to such a cell and to determine the next trajectory point



**Fig. 2** Forecast area of the COAMPS mesoscale model and the angles selection scheme.

according to system (4). This process is repeated until the ray leaves the model atmosphere. STD is determined by means of numerical integration using the ray points coordinates and refraction values at the points.

**APPLICATION EXAMPLES**

The module was used to investigate the anisotropic character of the slant delay functions. The functions were determined for selected elevation angles  $\xi$  with  $1^\circ$  and  $5^\circ$  intervals in the ranges from  $3^\circ$  to  $10^\circ$  and  $10^\circ$  to  $90^\circ$  and azimuths  $\phi$  from  $0^\circ$  to  $350^\circ$  with  $10^\circ$  interval for selected points of the mesoscale model area - 1885 km by 2197 km. This area is sufficient to determine slant delay distributions for rays of  $3^\circ$  elevation e.g. for Poland. The lengths of the rays projections are close to the value of: -  $R_3 = H / \tan(3^\circ) \approx 573 \text{ km}$  -H is the depth of the atmosphere (Fig. 2). Figs. 3a and 3b present 2D and 3D polar (cylindrical) distribution of slant delay  $\tau(\xi, \phi)$  obtained for the MUT station. Figs. 3c and 3b present reduced distribution of slant delay  $\tau_R(\xi, \phi)$  determined according to the following formula:

$$\tau_R(\xi = \xi_c, \phi) = \tau(\xi = \xi_c, \phi) - \overline{\tau(\xi = \xi_c, \phi)}, \xi_c = \text{const.} \quad (5)$$

where:  $\tau_R$  is the difference between  $\tau$  and its average value  $\bar{\tau}$  for elevation angle values  $\xi$ . Introducing (4) enables to better represent anisotropic (tensor) distribution of slant delay. The figures show that for small values of elevation angle  $\xi = 3^\circ - 6^\circ$ ,  $\tau$  and  $\tau_R$  depend on observation direction. In this case the GPS wave takes longer to pass the troposphere. The probability that it interacts on its way with water (in any phase) causing the delay is

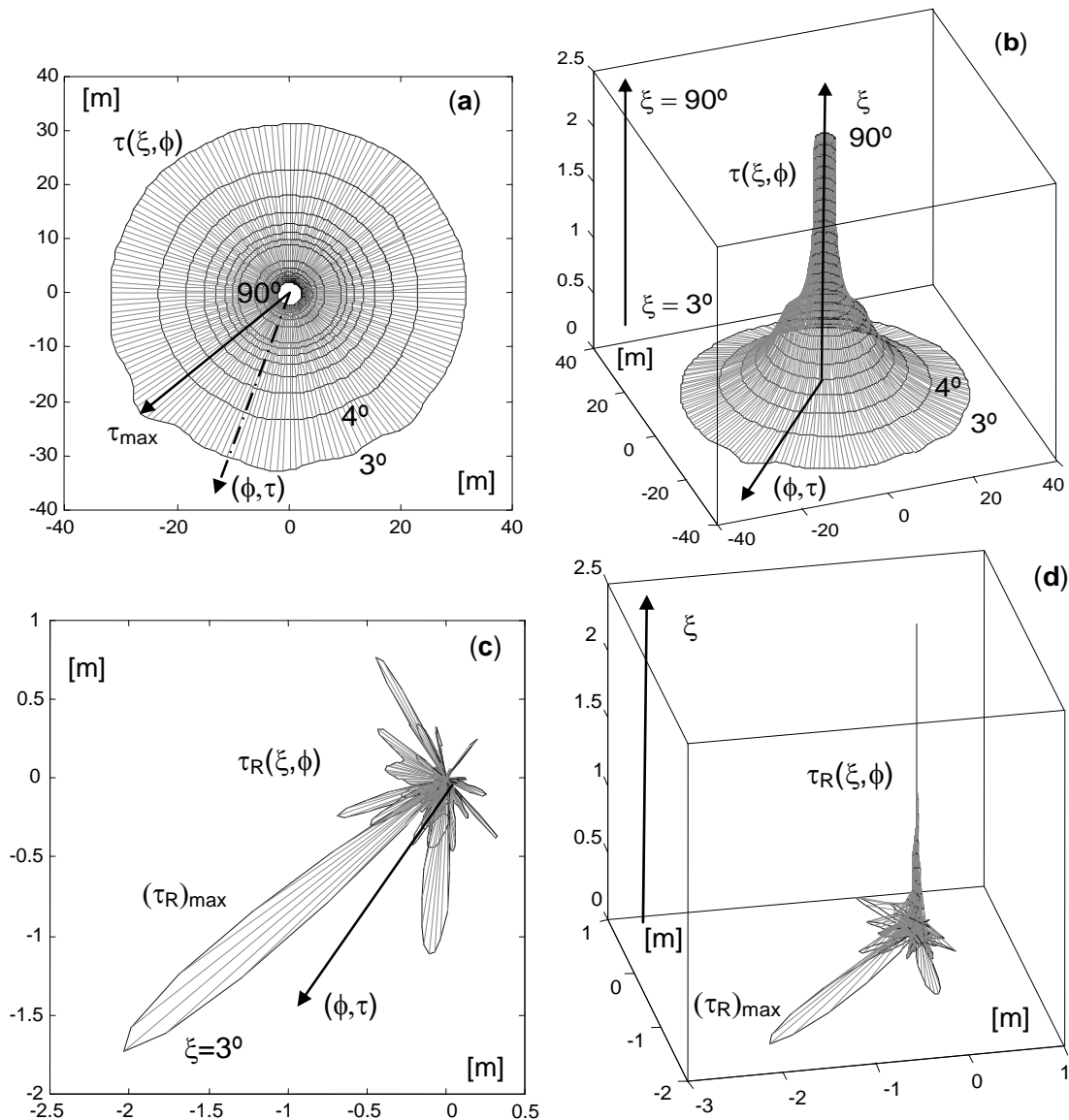
greater. The investigation results show that the difference  $\tau_R$  (the measure of the delay  $\tau$  anisotropy) equals meters. In the presented case, the observed anisotropy of the slant delay function  $(\tau_R)_{\max} = 2.5 \text{ m}$ . It is mainly a representation of spatial differentiation of atmospheric conditions.

**SUMMARY**

A prototype module for slant delay  $\tau$  determination using data from the COAMPS non-hydrostatic mesoscale model is a result of the research. The module enables to investigate angular characteristics of the delay for various atmospheric conditions. The delay may be determined e.g. for any GPS station location in the working area of the model and any elevation angle of a satellite observed from the point. The results show that for small values of the elevation angles the slant delay value depends on the observation direction. Hence, it includes information concerning heterogeneity of atmospheric humidity distribution along the GPS signal path. More precise explanation of the obtained STD distribution heterogeneity requires further research related with e.g. increasing the number of computational levels in the lower part of the model troposphere. It is planned to include in the module subroutines for satellite and radar images analysis for further research of the relation between the slant delay and the atmospheric conditions related especially with cloud systems of atmospheric fronts.

**REFERENCES**

Air/Ocean Coupled Prediction System at NRL MRY, Hodur RM, Naval Research Laboratory Monterey, CA, 19 -21 August 2003.



**Fig. 3** Anisotropic distributions of slant delay  $\tau$ .

- Bevis, M., Businger, S., Chiswell, S., Hering, T.A., Anthon, R.A. and co-authors: 1994, GPS meteorology: mapping zenith wet delays onto precipitable water. *J. Appl. Meteorology*, 33, 379-386.
- Boehm, J., Niell, A.E., Tregoning, P. and Schul, H.: 2006, Global Mapping Functions (GMF): A new empirical mapping function based on numerical weather model data. *Geoph. Res. Letters*, Vol. 33.
- Boehm, J., Werl, B. and Schul, H.: 2006, Troposphere mapping functions for GPS and very long baseline interferometry from European Centre for Medium-Range Weather Forecasts operational analysis data. *J. Geophys. Res.*, 111.
- COAMPS<sup>®</sup> version 3 model documentation, <http://www.nrlmry.navy.mil/coamps-web/web/home>
- Eresmaa, R. and Järvinen H.: 2006, An observation operator for ground-based, GPS slant delays. *Tellus*, 58A, 131-140.
- Niell, A.E.: 1996, Global mapping functions for the atmosphere delay at radio wavelengths. *Journal of Geodesy Research* 101(B2), 3227-3246.
- Niell, A.E.: 2001, Preliminary evaluation of atmospheric mapping functions based on numerical weather models. *Physics and Chemistry of the Earth A*(26), 475-480.
- Zou, X., Vandenberghe, F., Wang, B., Gorbunov, M.E. and co-authors: 1999, September 27, A ray-tracing operator and its adjoint for the use of GPS/MET refraction angle measurements. *J. Geophys. Res.* Vol. 104, No. D18, 301-318.