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DETERMINATION OF NEAR-SURFACE APPARENT RESISTIVITY BY MEANS OF HIGH FREQUENCY MUTUAL IMPEDANCE MEASUREMENTS

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ABSTRACT
A method of determination the apparent resistyvity of the ground's near-surface layers from measurements of the mutual impedance of a loop antenna system operating at a high frequency is presented. An analysis of the sensitivity of mutual impedance to variation in the parameters of conducting half space at different frequencies had been carried out. Additionally influence of displacement current on measured mutual impedance values was discussed. A frequency of measuring system was selected in order to obtain a high sensitivity of the measuring system to detection inhomogeneities characterised by small dimensions and low apparent resistyvity contrast. The measuring system and the method of interpreting measurement results were described. The results of field measurements were presented.

KEYWORDS: geophysical method, mutual impedance, loop antennas, apparent resistivity

INTRODUCTION

The measurement of the mutual impedance of loop antennas situated near a conducting half space is a well-known technique used in geophysical investigations. There are relationships for the mutual impedance of loop antennas situated near a conducting medium with a varied structure (Keller and Frischknecht, 1966; Ward and Hochmann, 1988; Verma and Sharma, 1995). Also the different aspects of mutual impedance measurement and the equipment used for this purpose have been described (Spies and Frischknecht, 1991). All the above works deal with antenna systems operating at a low frequency, defined herein as the quasi-static case where displacement currents are negligible. Few papers are devoted to the analysis of the mutual impedance of an antenna system operating at a high frequency where displacement currents cannot be ignored (Wait, 1954). Spies and Wait (Spies and Wait, 1972a, 1972b) described a simple technique of determining a homogenous conducting medium's conductivity and permittivity through the measurement of the mutual impedance of horizontal coplanar loops laid on its surface. Fuller and Wait (Fuller and Wait, 1972) provide a relationship for the mutual impedance of a horizontal coplanar antenna system operating at a high frequency, placed on the surface of a conducting half space whose parameters vary exponentially with depth.

Several papers (Anderson, 1991 and 1995; Steward et al., 1994) deal with the application employing high-frequency electromagnetic techniques to the sounding of the earth's near-surface layers. It is significant that in Steward at al. (Steward et al., 1994), an assessment of the three most popular techniques was made, i.e., the ground penetrating radar (GPR), the time-domain electromagnetic method (TEM) and the frequency-domain electromagnetic method (FEM), as applied to near-surface geophysical measurements. The authors of the latter paper emphasized that only FEM can be used to examine layers of ground lying very close to the surface and those located at a depth of several meters.

Low operating frequency of the antenna system is highly advantageous – it allows one to examine deep layers of the ground since the depth of penetration of the electromagnetic wave is inversely proportional to the square root of its frequency. But in some cases it is necessary to investigate thoroughly the layers lying close to the surface of the ground. Such investigations are impossible at low operating frequencies since near-surface, low-contrast inhomogeneities of small dimensions do not significantly affect, in comparison with the depth of penetration of the wave, the impedance value. Consequently, the result is averaged to a high degree because all layers of the medium which the electromagnetic field penetrates influence the measured impedance value. If the operating frequency of system is increased, the sensitivity of the mutual impedance function to changes in its parameters also increases. Thus, at high operating frequencies of the antenna system, one can examine the near-surface layers of a conducting layered earth in more detail and detect inhomogeneities of linear dimensions in their structure. One should, however, bear in mind that as the operating frequency increases, the depth of penetration of the electromagnetic wave decreases. Thus the measurement of the mutual impedance of loop antennas operating at a high frequency introduces limitations which make the examination of the deep layers of a conducting medium impossible. A multi-frequency system overcomes these limitations to some extent.

MUTUAL IMPEDANCE OF LOOP ANTENNA SYSTEM PLACED ON SURFACE OF CONDUCTING HALF SPACE

The antenna systems used for measuring mutual impedance are shown in Figure 1.

Let us assume that the antenna systems shown in Figure 1 are situated in free space over an N-layered

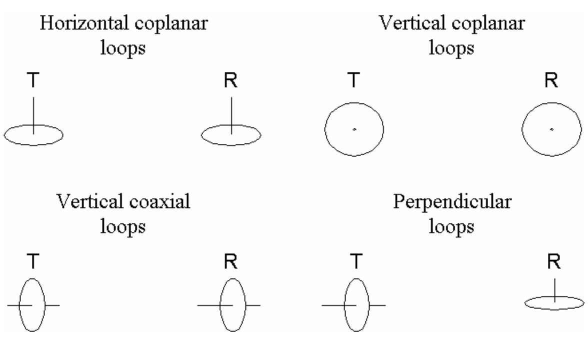

Fig. 1 Transmitter (T) and receiver (R) antennas configurations.

conducting half space interface. The free space parameters are as follows: permittivity $\varepsilon_0 = 8.854 \cdot 10^{-12} [F/m]$, magnetic permeability $\mu_0 = 4\pi \cdot 10^{-7} [H/m]$ and conductivity $\sigma_0 = 0$. Each layer with thickness d_N is characterized by the following parameters: ε_N, ε_N and σ_N. The distance between the antennas is r. The antenna being the source of the electromagnetic field ("transmitting antenna") consists of N_1 loops (turns) and has area A_1. The field measuring antenna ("receiving antenna") is made up of N_2 loops (turns) and has area A_2. An alternating current with variable intensity $I = \sin(\omega t)$ and frequency $f = \omega / 2\pi$ flows through the transmitting antenna. In order for the antennas to be considered as elementary magnetic dipoles their diameters must be much smaller than the distance between them. At the same time the distance between the antennas must be much smaller than the length of the wave radiated by the transmitting antenna.

Under such conditions, neglecting the displacement currents in the air and in the conducting earth, the normalized mutual impedance of the loop antenna system placed on the surface of a multilayered conducting medium is expressed by the following relationships (Verma and Sharma, 1995):

- for horizontal coplanar antennas:

$$\frac{Z}{Z_0} = 1 - r^3 \int_0^\infty \lambda^2 R(\lambda) J_0(\lambda r) d\lambda \tag{1}$$

- for perpendicular antennas:

$$\frac{Z}{Z_0} = -r^3 \int_0^\infty \lambda^2 R(\lambda) J_1(\lambda r) d\lambda \tag{2}$$

- for vertical coaxial antennas:

$$\frac{Z}{Z_0} = 1 - \frac{r^2}{2}\left[\int_0^\infty \lambda R(\lambda) J_1(\lambda r) d\lambda - r\int_0^\infty \lambda^2 R(\lambda) J_0(\lambda r) d\lambda\right] \tag{3}$$

- for vertical coplanar antennas:

$$\frac{Z}{Z_0} = 1 + r^2 \int_0^\infty \lambda R(\lambda) J_1(\lambda r) d\lambda \tag{4}$$

where: J_0, J_1 are Bessel functions of the zero and first order, respectively; *R(λ)* is the kernel of an integrand, dependent on the parameters of the investigated medium's particular layers and the frequency. *R(λ)* is calculated from this recurrent relationship:

$$R_{i-1,N}(\lambda) = \frac{V_{i-1,i} + R_{i,N}(\lambda) e^{-2d_i V_i}}{1 + V_{i-1,i} R_{i,N}(\lambda) e^{-2d_i V_i}}. \tag{5}$$

where: $i = N, N-1, \ldots\ldots, 1$; $R_{N,N}(\lambda) = 0$; $v_i = \sqrt{i\omega\mu_0\sigma_i}$ is the wave number for a given layer of the medium; $V_i = \sqrt{\lambda^2 + v_i^2}$; $V_{i,k} = \frac{V_i - V_k}{V_i + V_k}$. Parameter λ is a Hankel integral transformation variable. Z0 is the impedance of the antennas located in a clear area:

$$Z_0 = \frac{A_1 N_1 A_2 N_2 \mu_0 i\omega}{4\pi r^3}. \tag{6}$$

For vertical coaxial antennas the mutual impedance is:

$$Z_0 = \frac{A_1 N_1 A_2 N_2 \mu_0 i\omega}{2\pi r^3}. \tag{7}$$

where $i = \sqrt{-1}$. The mutual impedance of a system of perpendicular antennas located in free space is equal to zero. For this configuration Z_0 is assumed to be the same as for coplanar loops. If the antenna system is located at height *h* above a conducting half space, electric parameters of the first layer in relationship (5) are assumed the same as for the free space and its thickness is $d_1 = h$.

SELECTION OF ANTENNA SYSTEM OPERATING FREQUENCY

The discussed method was adjusted to detection of inhomogeneities in levees. Thus, all the numerical calculations were performed for these ranges of earth parameters that basically refer to such objects. The operating frequency of the measuring system was selected by taking into consideration two aspects: firstly - increasing sensitivity of mutual impedance function to low-contrast inhomogeneities with small dimensions present in the examined conducting medium; secondly - neglecting displacement currents in both the free space and the conducting medium. The lower limit of the frequency range was determined through numerical calculations of the sensitivity of the mutual impedance function described by equations (1)-(4), for conducting media differing in their parameters. Upper limit of the frequency range was determined by analysing the effect of conduction and displacement currents on the wave number versus frequency for parameter ranges typical for levees.

SENSITIVITY OF MUTUAL IMPEDANCE FUNCTION

The sensitivity of the impedance functions of all the antenna systems was analysed for the conducting medium model, shown in Figure 2. The model has a three-layer structure with the middle layer being one-tenth of the thickness of the first layer. The first layer and the third layer were assumed to have the same resistivity. Then sensitivity of mutual impedance function to ten per cent changes in the resistivity of middle layer versus frequency were determined relative to a resistivity equal to the resistivities of the other layers. Thus one can say that the middle layer represents an inhomogeneity with a low conductivity contrast and small thickness – as compared with the whole medium.

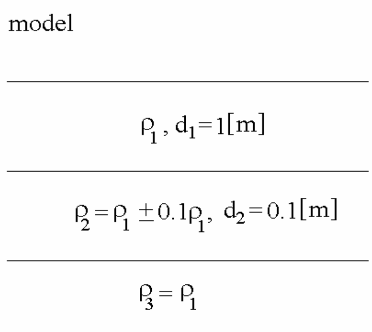

Fig. 2 Conducting medium model used for determining mutual impedance function sensitivity.

As mentioned, the analysed method was adapted to investigation of levees, thus the clue aspect is the earth resistivity range that characterises a conducting medium such as levees. In order to determine this range many measurements of selected sections of levees were performed with the D.C. resistivity method (Pralat, 1994 and 1997). The results showed that the values of the measured resistivity usually amount to a few dozens ohmo-meters and they rarely exceed 100 Ωm. Due to this the resistivity range that was assumed to the calculations was $(1 \div 100)[\Omega m]$. The mutual impedance sensitivity for all the antenna systems with their antennas separation $r = 5$ m, situated at height $h = 1$ m over the considered conducting medium was calculated from this relationship (Ralston, 1965):

$$S = \frac{\rho_2}{Z / Z_0} \frac{d\left(Z / Z_0\right)}{d\rho_2} , \tag{8}$$

where the antennas' mutual impedances are described by relationships (1)-(4) and $\rho = \sigma^{-1}$ expresses the relationship between resistivity and conductivity. The results of sensitivity variation versus a global parameter frequency/resistivity ($f\rho^{-1}$) are presented in Figure 3.

For all the antenna configurations, even slight changes in the resistivity of middle layer are reflected in changes of mutual impedance starting from a frequency/resistivity about 10^4. The value corresponds to the frequency 1 MHz, taking into account the given range of resistivity $(1 \div 100)$ [Ωm].

Thus we can assumed that the sensitivity of the mutual impedance function for shallow and thin layers with a low contrast increases for frequencies equal or greater than 1 MHz. On the basis of these results, the lower limit of the frequency range to be adopted for examining the near-surface layers of the ground was set at 1 MHz.

NEGLECTING OF DISPLACEMENT CURRENTS

The upper range of the operating frequency of an antenna system is restricted due to the necessity of taking into account the occurrence of displacement currents in both the free space and the conducting medium (Steward et al., 1994). Therefore prior to the determination of the upper range of the operating frequency of antenna system the influence of the displacement current and the conduction current on the propagation of an electromagnetic wave in a conducting half space having the same parameters as the ground was analysed for different frequencies.

Similarly as for the case of determination the sensitivity of mutual impedance function, the resistivity belongs to the range $(1 \div 100)$ [Ωm] (conductivity $\left(0.01 \div 1\right)$ [S/m]). The maximum value of

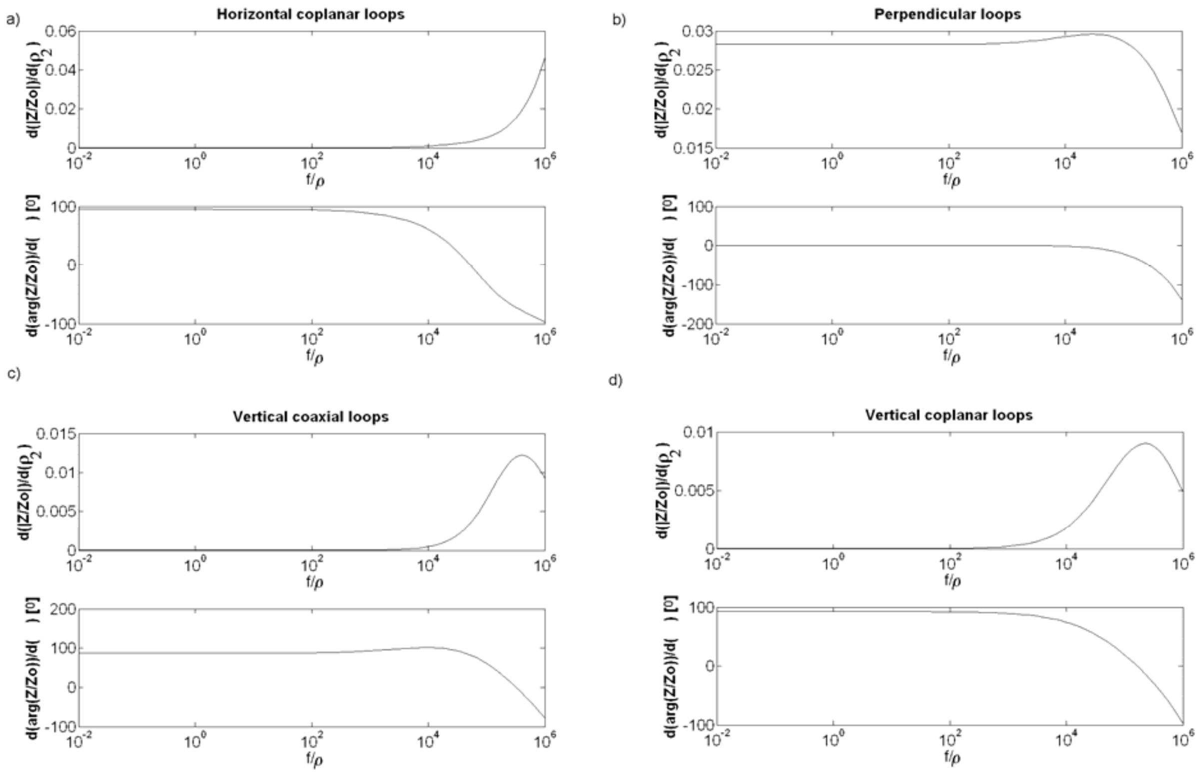

Fig. 3 Logarithmic sensitivity of mutual impedance as function of frequency to resistivity ratio, calculated for: a) horizontal coplanar loops, b) perpendicular loops, c) vertical coaxial loops and d) vertical coplanar loops.

permitivity was assumed $\varepsilon_r = 10$. This is a large value of permitivity for typical terrain material (McNeill and Labson, 1991).

The relationship below describes the squared wave number:

$$\gamma^2 = \omega^2 \mu_0 \varepsilon_r \varepsilon_0 - i\omega\mu_0\sigma = \alpha - i\beta . \qquad (9)$$

The ratio of a displacement to conducting current is given by:

$$\frac{\alpha}{\beta} = \frac{\omega\varepsilon_0\varepsilon_r}{\sigma} = \omega\varepsilon_0\varepsilon_r\rho . \qquad (10)$$

If the ratio exceeds 5% the displacement currents cannot be neglected (Huang and Freser, 2002). For the given maximum values $\varepsilon_r = 10$, $\rho = 100$ Ωm:

$$f_{\max} = \frac{0.05}{2\pi\varepsilon_0\varepsilon_r} = 898kHz . \qquad (11)$$

On the basis of this simple numerical analysis 898 kHz was determined as the upper limit of the frequency range to be adopted for measurements of the ground's near surface layers. At this frequency no significant error is made if the occurrence of displacement currents is neglected in the analysis of measurement results.

ULTIMATE CHOICE OF OPERATING FREQUENCY OF MEASURING SYSTEM

The frequency range for the equipment to be used for examining the ground's near-surface layers was determined on the basis of two numerical analyses: a) an analysis of the mutual impedance function's sensitivity and b) an analysis of the influence of displacement currents on the electromagnetic wave's propagation constant. The results showed that the sensitivity of a mutual impedance function increases above frequency 1MHz. But frequency 898 kHz is the highest for which the displacement currents can be neglected. Ultimately, 1 MHz was selected as the measuring system's frequency. At this frequency a high sensitivity of the mutual impedance function is obtained and at the same time no large error (5.6%) is introduced into the analysis when displacement currents are neglected.

MEASURING DEVICES

System for mutual impedance measuring is similar to described by Wright and Chew (Wright and Chew, 2002), but it works only at one frequency 1 MHz. This system has been developed at Wroclaw University of Technology in Years 1994-1997 (Pralat, 1994; 1997). It measures magnitude and phase of mutual impedance for: horizontal coplanar, vertical coplanar, vertical coaxial and perpendicular loop antennas systems. Spacing between antennas can be changed from 2 to 6 meters. Antennas are placed 0.5 meter above the ground. The magnetic moment of source antenna can be changed from 0.17 to 3.6 [Am^2]. The system includes system displacement measuring unit with a measuring resolution adjustable from 0.05 m to 1.6 m.

The interpretation of mutual impedance measurements consists in finding the best matching between the measured data and the theoretical curves (Kruk et al., 2000; Wilt and Stark, 1982). Results of field measurements The measuring system is used mainly for investigating the structure of levees because of the high flood hazard in Poland. Since most of the levees are up to 8 m high the method is highly suitable for this purpose. Presented below are the results of investigations of two selected levees. The examined levee sections are characterised by visible strong structural inhomogeneity. Obtained result showed possibility to use described method to detect near-surface inhomogeneties in investigated ground.

First measurements were carried out above water culvert (Fig. 4), placed inside an embankment of Odra River in Wroclaw, Poland. In Figure 5a are presented recorded traces magnitude and phase of mutual impedance of coplanar horizontal antenna system with spacing of 2 m. In Figure 5b are presented results of interpretation measured data.

Second example of examination of the levee by means of described method concerns section of Bystrzyca River in Wrocław, Poland. This set of measurement results was used to determine the relationship between sensitivity of mutual impedance function and the depth at which strong inhomogeneity is located in the structure of the investigated conducting medium. For this reason a levee section with an inactive water culvert was selected. A sketch of the investigated levee section and the mutual impedance measurement results are presented in respectively in Figures 6 and 7a. The examination was performed using perpendicular loops spaced 6 m apart, at every 0.05 m. Similarly as above, the interpretation can be slightly corrupted with small errors from neglecting the displacement currents in the measured mutual impedance ($\rho > 100$ [Ωm]).

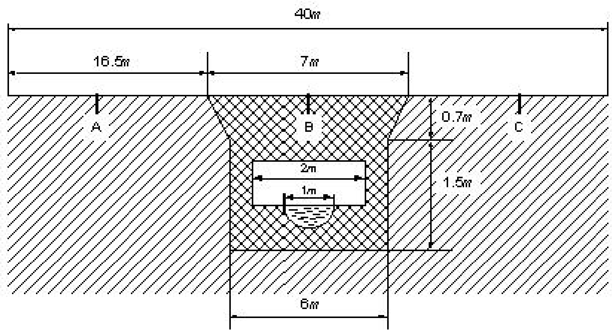

Fig. 4 Sketch of investigated section of the Odra River's levee in Wrocław, Poland.

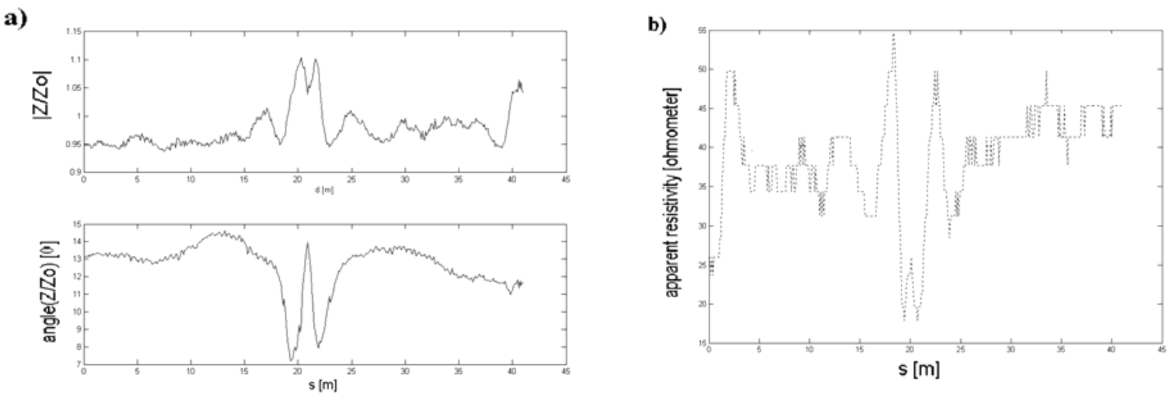

Fig. 5 Measurement results for section of the Odra River's levee in Wrocław, Poland: measurement trace of mutual impedance of coplanar horizontal antenna system with spacing of 2 m, b) apparent resistivity traces obtained through interpretation of electromagnetic measurement results.

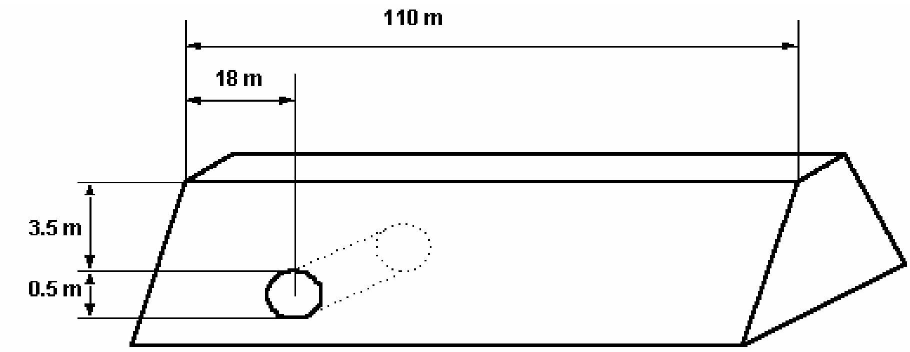

Fig. 6 Sketch of investigated section of the Bystrzyca River's levee in Wrocław, Poland.

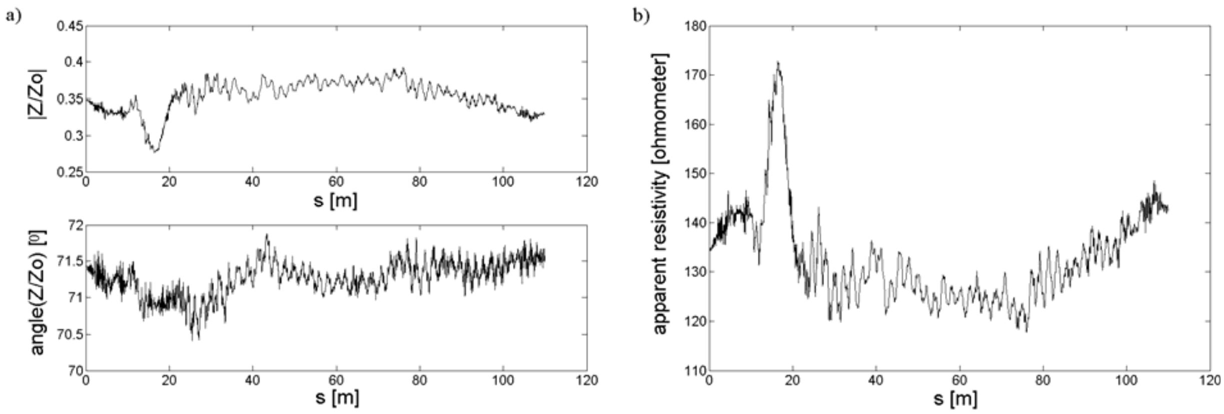

Fig. 7 Measurement results for section of the Bystrzyca River's levee in Wrocław, Poland: measurement trace of mutual impedance of perpendicular antenna system with spacing of 5 m, b) apparent resistivity traces obtained through interpretation of electromagnetic measurement results.

The location of both water culvert is clearly visible in the apparent resistivity curves (Fig. 5b, Fig. 7b) despite the fact that the culverts were ceramic and filled with air. The possible reason is the depth at which the culverts is situated. For perpendicular and horizontal coplanar loops, the penetration depth, is equal to the spacing between the loops. The water culvert in Figure 6 is situated at half the penetration depth of the antennas. This kind of inhomogeneity could be caused, e.g. by animals living in neighbourhood of water reservoirs. This makes weaknesses of a levee, which may result in breaking of a levee during a high level of water. Measurements of mutual impedance make possible to find this kind of inhomogeneity but, as showed in this example, the mutual impedance function is most sensitive to inhomogeneities situated close to half-penetration depth. Hence one can conclude that separation between antennas should be at least equal to a height of levee.

SUMMARY

A method of determining the apparent resistivity of a conducting medium's near-surface layers through the measurement of the mutual impedance of a system of loop antennas operating at a high frequency has been presented. Relationships for the mutual impedance of loop antennas placed on the surface of an inhomogeneous conducting half space were given. The operating frequency of antenna system, equal to 1 MHz, was selected on the basis of a numerical analysis of the influence of displacement currents on wave propagation in the conducting half space. The measuring system and the method of interpreting measurement results were described. In comparison to other existing EM geophysical systems, system works at high frequency. Because of it EM wavelength in the ground is shorter than in the case of low frequency systems and therefore method allows to detect near surface, small dimension and low contrast inhomogeneities.

Two examples of the measurement of a conducting medium's apparent resistivity by the presented method were given. Obtained results showed that described method could be used to detect inhomogeneities in near – surface ground.

Authors applied proposed method to the investigation of the inhomogeneity of levees, but there are many other potential applications of the method, e.g. in road construction and in other industries, generally there where near-surface layers of the earth need to be investigated.

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