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RELOCATION OF MINING-INDUCED SEISMIC EVENTS IN THE UPPER SILESIAN COAL BASIN, POLAND, BY A DOUBLE-DIFFERENCE METHOD

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ABSTRACT

The application of the double-difference (DD) algorithm to the relocation of induced seismic events from the Upper Silesian Coal Basin is discussed. The method has been enhanced by combining it with the Monte Carlo sampling technique in order to evaluate relocation errors. Results of both synthetic tests and relocation of real events are shown. They are compared with estimates of the classical single-event (SE) approach obtained through the Monte Carlo sampling of the *a posteriori* probability. On the basis of this comparison we have concluded that the double-difference approach yields better estimates of depth than the classical location technique.

KEYWORDS: seismic event location, double difference algorithm, Bayesian inversion

1. INTRODUCTION

Location of seismic events is the first step in analyses of natural and induced seismicity. Various location techniques are used to find the sought solution. The methods can be classified into two basic groups: single-event techniques and relative location methods. The first group consists of locate algorithms which treat each event separately and try to locate it using seismic records from a set of seismic sensors. The second class consists of location methods which locate seismic sources either with respect to other, already located events (for example, the master event technique (Gibowicz and Kijko, 1994; Mendecki, 1997), or simultaneously locate a group of events. The newly developed double-difference (DD) algorithm proposed by Waldhauser and Ellsworth (2000), is an example of a modern algorithm of this sort. We expect some improvements in location accuracy when the DD approach is applied to mining-induced events. The reason for this is that mining-induced seismicity tends to form spatial clusters (Gibowicz and Kijko, 1994; Lasocki, 1992; Mortimer and Lasocki, 1996) and the DD algorithm is designed for such circumstances. Moreover, the DD algorithm makes the location results less dependent on the velocity structure when compared to the single-event techniques.

In the case of both natural and mining seismicity, depth is the parameter most difficult to estimate reliably. This is partially caused by an almost planar distribution of mining seismic networks, usually set up at production level. Moreover, depth is very strongly correlated with velocity (Dębski et al., 1997) and thus very sensitive to any complexity and simplification of the rock mass velocity structure. Verification if the DD algorithm can improve the accuracy of depth location was one of the motivations of this work. To answer this question we have carried out synthetic tests inverting numerically generated "observational" data as well as using the real data recorded by the regional seismic network (USRSN) of the Central Mining Institute, Katowice (CMI) operating in the Upper Silesian Coal Basin. The paper starts with a short presentation of the DD location algorithm and the Bayesian approach to inverse problems. Then the synthetic tests are discussed, and finally the location of real events induced by coal mining is considered.

2. LOCATION ALGORITHMS

The hypocenter of mining seismic tremors can be located on the basis of various information read from seismograms (Lomax et al., 2000). In the simplest case, the arrival times of the first recorded seismic phase are used. In this paper we follow this approach, as it is most often used in mining practice because of its robustness and simplicity (Gibowicz, 1990). Denoting by t^i_k the arrival time of the first phase from the *i-th* event recorded by the *k-th* station, its relation to the origin time T^i when the event occurred and to the seismic velocity v reads

$$t_k^i = T^i + \int \frac{ds}{v} \qquad (1)$$

under the ray approximation (Aki and Richards, 1985; Gibowicz and Kijko, 1994). From a set of recorded t^i_k one can subsequently estimate 4 hypocenter parameters: three spatial coordinates (X^i, Y^i, Z^i) and the origin time T^i for each event separately. In the most classical approach this is achieved by minimizing the travel time residua:

$$r_k = \left\| t_k^{obs} - t_k^{th}(m,v) \right\| \qquad (2)$$

where $\mathbf{m}=(X,Y,Z,T)$ are the sought parameters: epicentral coordinates (X,Y), depth (Z) and the origin time (T), and ||.|| stands for a norm in the data space Dębski (1997). As it follows from Eq. 1 the results of inversion depend on the velocity model used to calculate the theoretical travel times. Different velocity models may lead to different location results and complicate and increase the estimate of the location accuracy (Dębski, 1999; Husen et al., 2003). Thus, if the velocity structure of rock mass is poorly known we can expect quite large location errors (Dębski, 1997; Lomax et al., 2000). These errors can be diminished if the relative location technique is used (Waldhauser and Ellsworth, 2000). One such method is the so-called double-difference algorithm (Waldhauser and Ellsworth, 2000). In the DD technique, instead of locating the hypocenters of each event separately by minimization of the temporal residua, as described above, we locate all events simultaneously by minimizing the norm of the following double-difference residua:

$$dr_k^{ij} = \left(t_k^i - t_k^j\right)^{obs} - \left(t_k^i - t_k^j\right)^{th} \qquad (3)$$

where dr^{ij}_k are the residua between observed and theoretical travel time differences for all pairs of considered events i, j. This means that with the DD algorithm, instead of estimating four parameters (X, Y, Z, T) for each event separately, we are simultaneously inverting 4xN parameters where N is the number of relocated events. One of the advantages of this approach is visible if the location and simultaneous velocity inversion is performed (Dębski, 1997). In such a case the classical approach may lead to a different velocity estimate for each event, while the DD algorithm will use information from all observations to estimate the best "apparent" velocity model. This seems to lead to more consistent results. Obviously, as the same velocity model is used for locating all events, the more clustered the events are in space, the more efficient the diminishing of the influence of velocity errors on location errors.

3. INVERSION ALGORITHMS

As mentioned earlier, classical location is achieved by minimization of the temporal residua. This is a very classical inversion method (Dębski, 1997). The alternative, Bayesian approach, although computationally more complicated is able to provide reliable estimation of inversion uncertainties (Tarantola, 2005; Dębski, 1999). This is why we decided to use this approach. The Bayesian approach considers the *a posteriori* probability density function (PDF) $\sigma(\mathbf{m})$ as the solution of the inverse problem at hand. It describes the probability of the given model **m** being the true one and reads (see, e.g. Lomax et al., 2000; Tarantola, 2005; Dębski, 2004)

$$\sigma(m) = const. f(m) L(m) \qquad (4)$$

where $f(\mathbf{m})$ is the PDF function which describes *a priori* constraints and $L(\mathbf{m})$ is the likelihood function which "measures" the misfit between observed and predicted data. While the *a priori* PDF has an arbitrary form which depends on the available information, the likelihood function reads, in the simplest case (see, e.g. Jackson and Matsu'ura, 1985; Tarantola, 2005):

$$L(m) = \exp(-\left\| t^{obs} - t^{th} \right\|) \qquad (5)$$

where t^{obs} is the vector of observed arrival times, t^{th} is the vector of theoretically predicted travel times (for example, according to Eq. 1) and ||.|| stands for a norm in the data space (Tarantola, 2005). The role of the *a priori* PDF is to introduce into the final, *a posteriori* solution our knowledge about the sought quantities coming from other sources than the measurement. It includes our experience gained from previous or similar analyses, evaluation of data information content, expectation, etc. It thus provides the mechanisms to restrict and numerically stabilize the final solution, so its role is partially similar to the regularization procedure in the classical, optimization-based inversion.

Knowledge of the $\sigma(\mathbf{m})$ distribution allows not only to find the most likelihood model for which:

$$m^{ml} := \sigma(m^{ml}) = \max \qquad (6)$$

which corresponds to the classical minimization-based solution, but also other characteristics of the solution, like the average model:

$$m^{ave} = \int_M m\sigma(m)dm \qquad (7)$$

and its variance:

$$C_{post}^{mn} = \int_M \left(m_m - m_m^{ave}\right)\left(m_n - m_n^{ave}\right)\sigma(m) \qquad (8)$$

which provides a convenient measure of the inversion accuracy (Dębski, 2004).

4. CASE STUDY

The DD method was used to relocate events induced by coal mining in the Upper Silesian area. The arrival times recorded by the Upper Silesian Regional Seismic Network (see Fig. 1) were used to perform event relocation.

To evaluate the effectiveness of the DD algorithm in improving location accuracy, we have

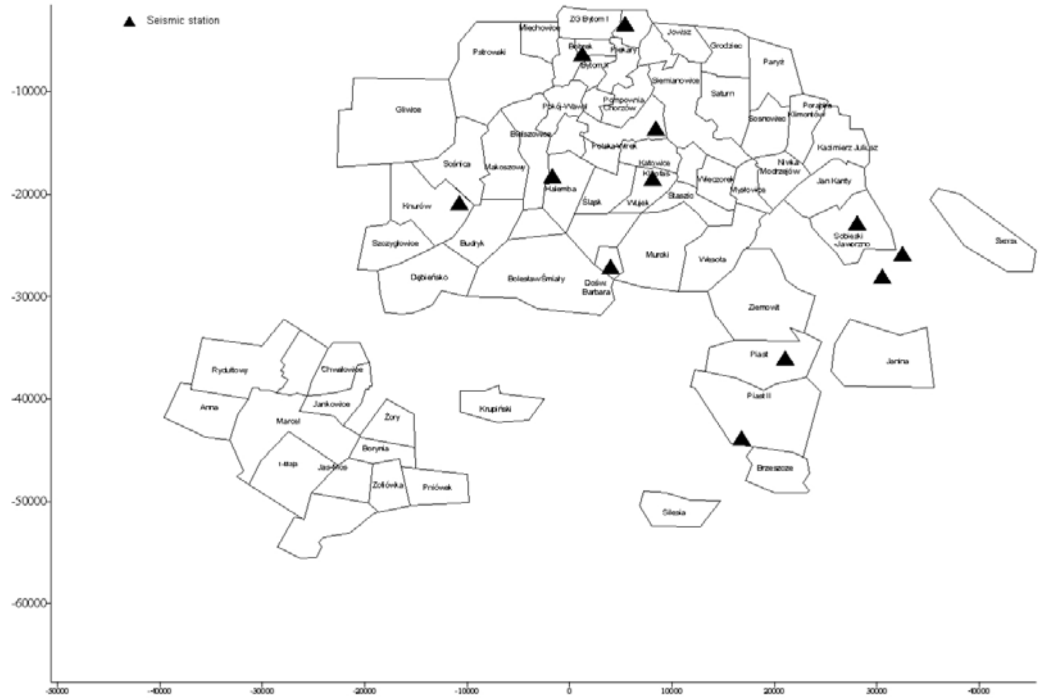

Fig. 1 Location of the seismometers of the Upper Silesian Regional Seismic Network (triangles) over the sketch of different mines.

performed three tests. First, synthetic tests based on inversion of numerically generated data were performed. Second, the real events were located separately, and finally the DD algorithm was applied to real data. All inversions were performed within the Bayesian framework by using the Metropolis algorithm to sample the *a posteriori* PDF. In this way we were able to calculate all the statistical estimators of the sought solutions (Eq. 6-8). To construct the likelihood function the l_2 norm was used, so for the classical, single-event location it reads

$$L(m) = \exp\left(-\sum_k \left(\frac{t_k^{obs} - t_k^{th}}{s_k} \right)^2 \right) \tag{9}$$

while for the DD algorithm we have used

$$L(m) = \exp\left(-\sum_{ijk} \left(\frac{\left(t_k^i - t_k^j\right)^{obs} - \left(t_k^i - t_k^j\right)^{th}}{s_k} \right)^2 \right) \tag{10}$$

The parameter s_k defining $L(\mathbf{m})$ was assumed to be constant for all events and equal $s_k = 1$. The Metropolis algorithm was always tuned to keep the acceptance ratio in the range of 30-70% (Dębski, 2004).

4.1. SYNTHETIC TESTS

Synthetic tests used the USRSN station configuration, but we have assumed that 8 considered events form a cross-like cluster shown in Figure 2, with the master event located in the center of the cluster.

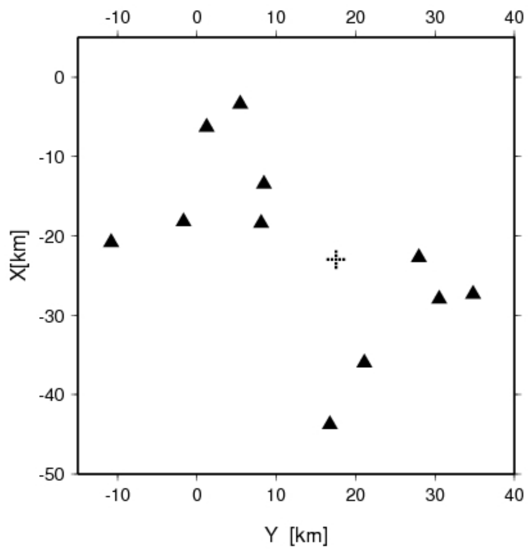

Fig. 2 Cross-like cluster of 8 seismic events (+) used in the synthetic test of the DD location algorithm. Triangles denote the location of the USRSN seismic stations used.

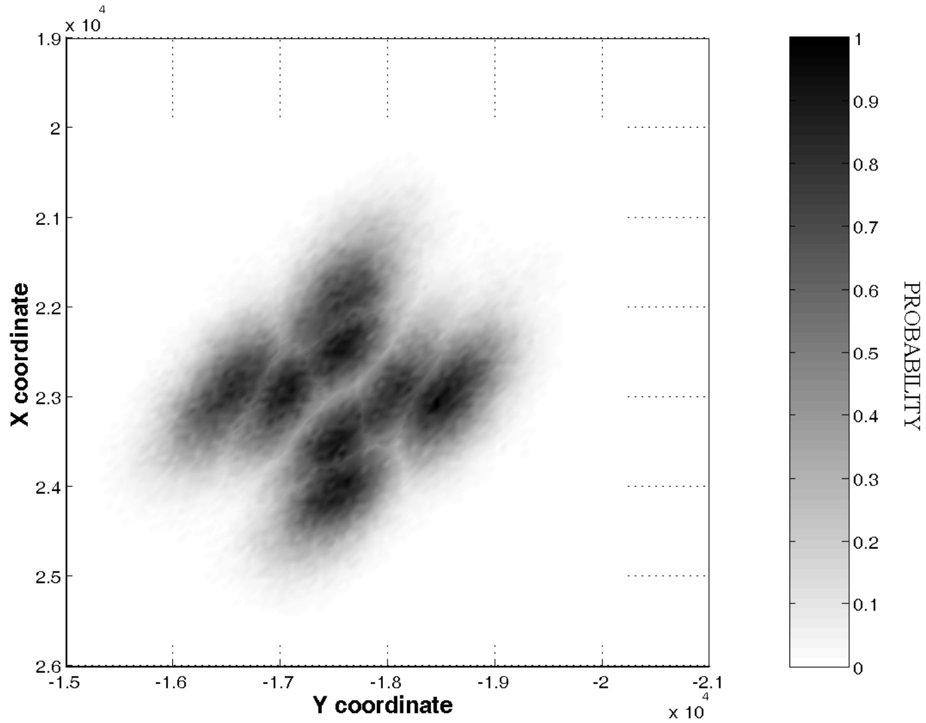

Fig. 3 2-D *a posteriori* PDF for epicentral coordinates of the relocated 8 events used for the DD synthetic test. Note that the imaging errors being the "measure" of high probability regions are less than the distances between events, allowing to resolve the structure of the cluster.

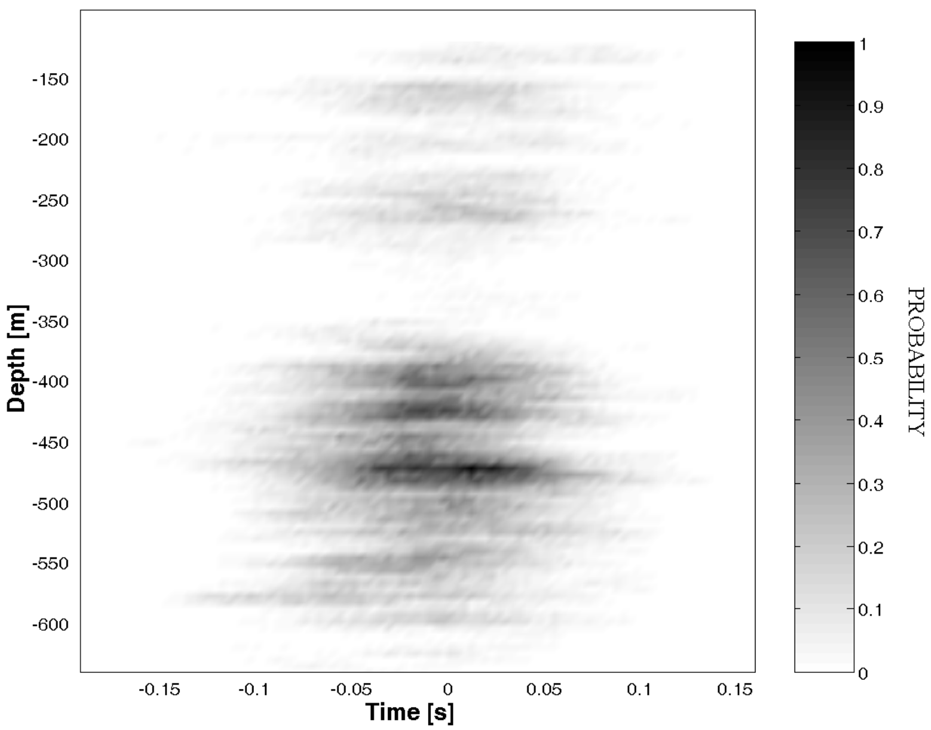

Fig. 4 2-D *a posteriori* PDF for origin time and depth (*T*,*Z*) for one of the events from the cluster obtained using the DD algorithm. There is almost no correlation between *Z* and *T*, which means that the depth is resolved independently from *T*. This is quite different from results of single-event location techniques (see Fig. 6), (Dębski, 2004) .

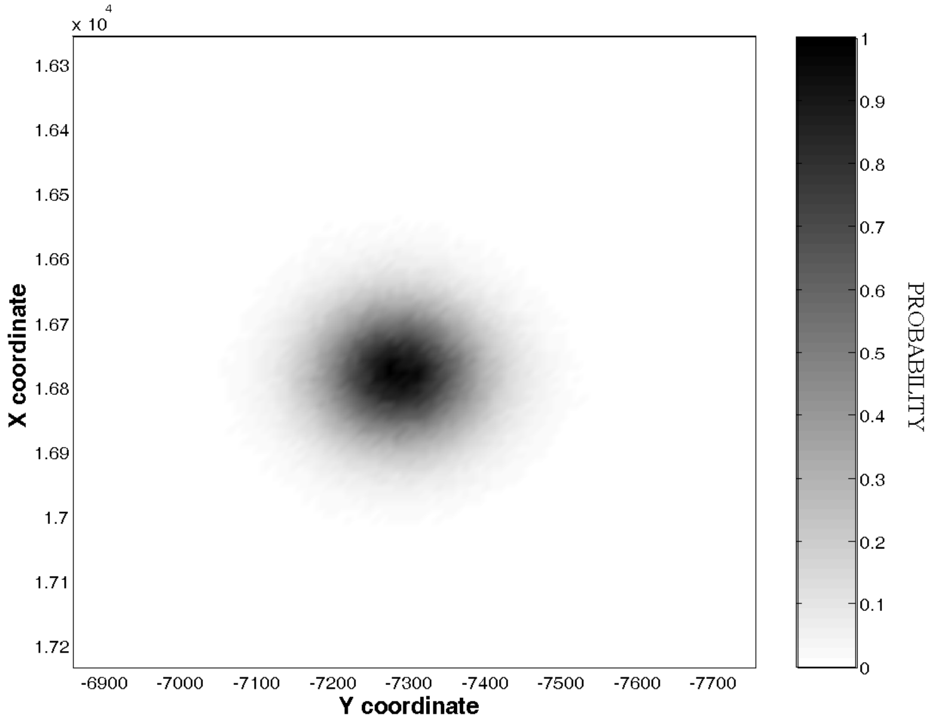

Fig. 5 2-D *a posteriori* PDF for epicentral (*X,Y*) coordinates for an event from the cluster. The location errors are quite small.

At this stage we did not invert the data for velocity but instead used the empirical model proposed by Kornowski (1985) which considers velocity as a function of epicentral distance, describing the apparent velocity as a function of the event epicenter-station distance:

$$v = 6.5 - 3.175\exp(-0.075d) \qquad (11)$$

where d is the epicentral distance to the station. All hypocenters in the cluster were assumed to have the same depth Z=500m. We assumed no *a priori* information, thus $f(\mathbf{m})=const$ (see Eq. 4).

The 2-D marginal PDF for the epicentral parameters being the results of the inversion are shown in Figure 3.

The relocated events form a clearly visible cross-like cluster with location uncertainties smaller than the distance between events in the cluster. Taking into account the size of the cluster (2 km) with respect to the average distance to the stations (30 km), the results of the inversion are satisfactory. The result shown in Figure 4 is very interesting, where the 2D marginal distribution for depth and origin times (Z, T) is plotted. It shows that in this case no correlation occurs between Z and T. Taking into account that T correlates with velocity in the case of extended location with simultaneous velocity inversion (Dębski et al., 1997), this picture shows that the depth estimator obtained by the DD algorithm is much less strongly dependent on the velocity than in the case of the standard location techniques.

4.2. SINGLE-EVENT LOCATION

In the next step we used the classical single-event location algorithm to discriminate the location of events from the same cluster. In this case we incorporated the *a priori* information about the depth of the event assuming $f(\mathbf{m})$ to be a Gaussian function with Z^{apr} = 500m and the dispersion s_z = 300m. We had to make this *a priori* assumption because with no *a priori* information the solution was numerically unstable and often led to spurious values of the *a posteriori* depths. The results are shown in Figures 5 and 6. Figure 5. shows the 2D *a posteriori* PDF for the (X,Y) epicentral coordinates for one of the events from the cluster. The location error seems to be slightly smaller than in the case of the DD solution (Fig. 3). The big difference between the DD and the single-event location algorithm is visible in Figure 4, where the 2D *a posteriori* PDF for (Z, T) coordinates is plotted. Contrary to the DD solution, a clear, weakly nonlinear correlation between these two parameters is visible. These results confirm again (Dębski et al., 1997) that the hypocenter depth is strongly correlated to the velocity structure of the rock mass and is poorly resolved if the structure is insufficiently known.

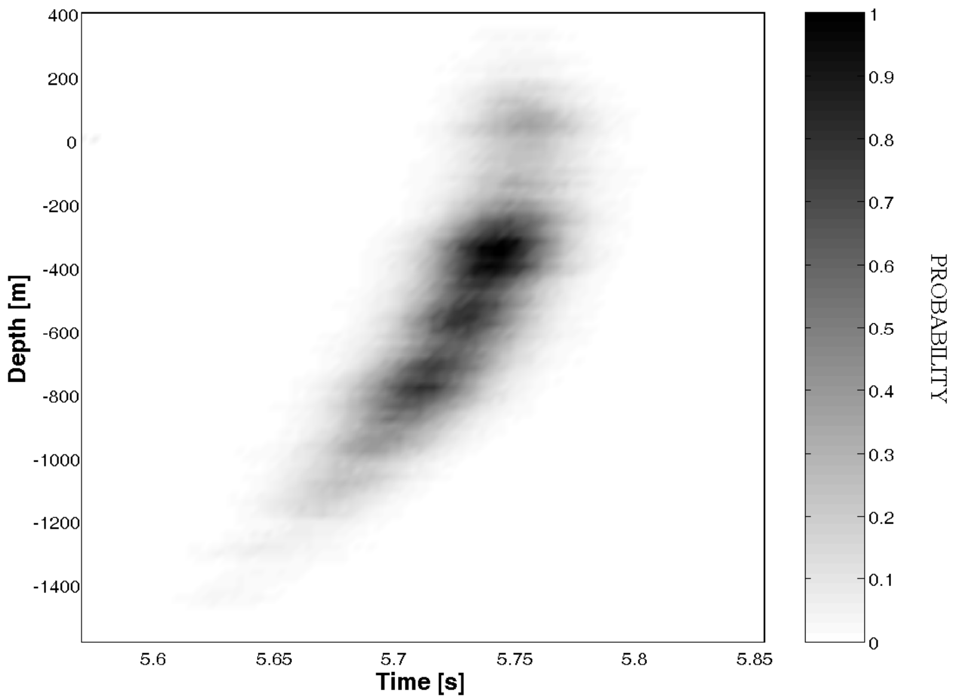

Fig. 6 2-D *a posteriori* PDF for the origin time *T* and depth *Z* coordinates obtained by a single event location algorithm. Note the strong and nonlinear correlation between these two parameters which is not visible in the DD solution shown in Figure 4.

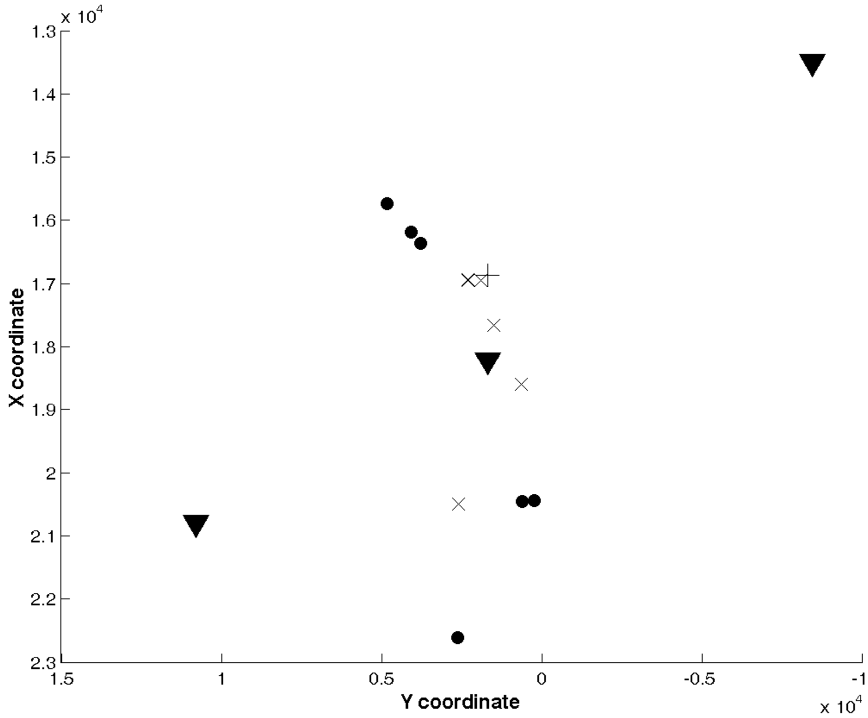

Fig. 7 *A priori* location of seismic tremors by CMI (x) and the final relocation by the DD algorithm (circle). Note the shift of the *a posteriori* solution around 2 km with respect to the CMI solution with a tendency to greater scattering. Triangles denote the location of the three nearest seismic stations and + is the master event.

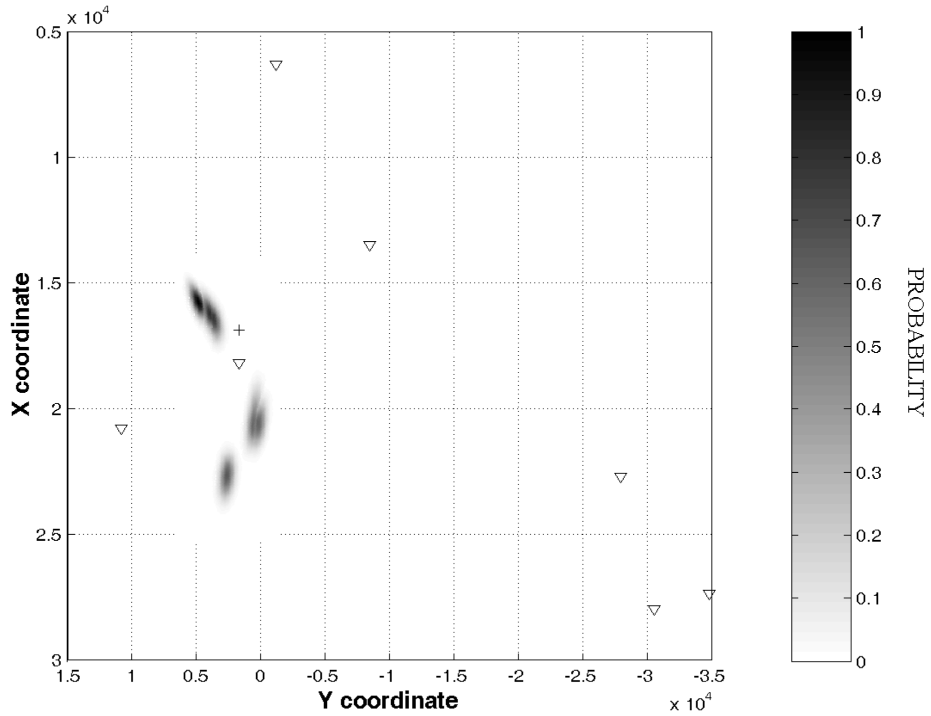

Fig. 8 2-D *a posteriori* PDF for epicentral (*X,Y*) coordinates for seven relocated events.

4.3. RELOCATION OF REAL EVENTS

Finally, the DD algorithm was used to relocate real seismic events which occurred at the Main Syncline - the central part of the Upper Silesian Coal District. The inversion used the *a priori* solution for the *X* and *Y* coordinates obtained by the CMI. The *a priori* locations and the results of the relocation are shown in Figure 7. Note the systematic shift of the relocated events with respect to the solutions of the CMI based on the standard single-event location algorithm. The corresponding 2-D *a posteriori* PDF functions for the epicentral (*X,Y*) coordinates of all seven relocated events are shown in Figure 8. Similarly to the previously studied numerical example (Fig. 4), also in the case study we observe decoupling of the *T* and *Z* parameters, as shown in Figure 9. In addition, an analysis of the PDF distribution shown in Figure 9 shows the existence of secondary maxima in the *T-Z* PDF distribution. Thus, the depth errors are quite complicated and cannot be simply summarized by a single "number" like the *a posteriori* variance. However, if we restrict ourselves *a priori* only to the solutions around the exploitation level, which is often quite a reasonable assumption (Gibowicz and Lasocki, 2001), which corresponds to the position of the main maximum in Figure 9, we obtain depth estimation errors of around $\delta z \approx 50$m, which is a very good result.

5. CONCLUSIONS

The analysis presented in this paper was aimed at evaluating the effectiveness of the DD algorithm for the relocation of mining-induced seismic events recorded by a regional seismic network. The greatest attention was paid to estimating the accuracy of hypocenter depths. As a result of the synthetic tests, we have found that compared to the classic single-event location method, the DD algorithm provides much better and more stable depth estimation. While the DD algorithm was able to provide the correct depth of events without any *a priori* information in the considered cases, the SE technique even required an additional *a priori* restriction imposed on *Z*. In addition we have observed that the *Z* parameters decouple in the DD solution from the origin time. Consequently, the solution is much less sensitive to velocity uncertainties than in the SE method. These results prove that the DD method is more reliable and more accurate for the routine analysis of mining-induced seismicity.

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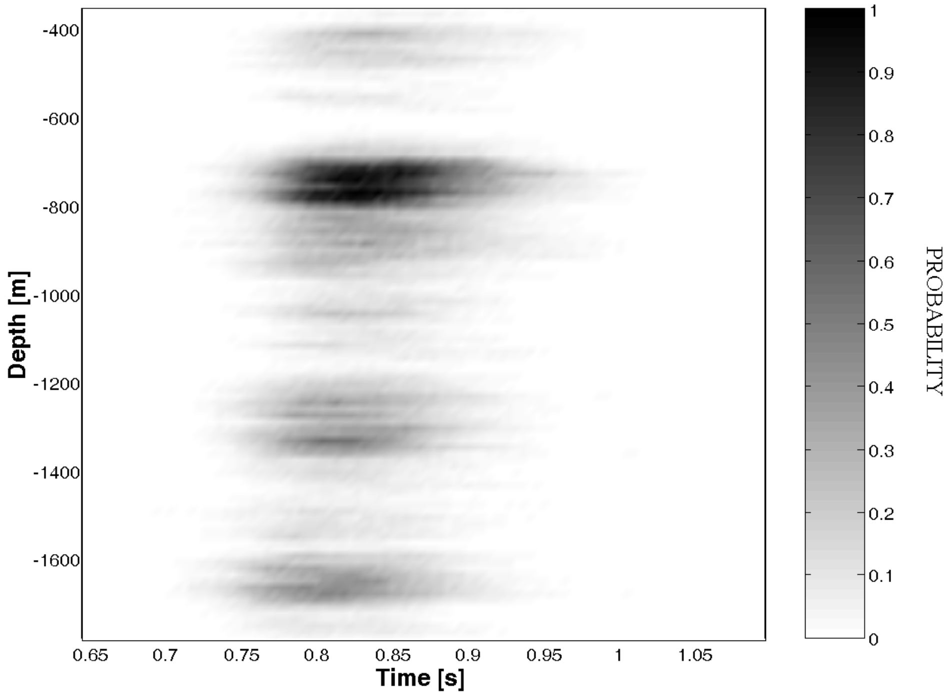

Fig. 9 2-D *a posteriori* PDF for the origin time *T* and depth *Z* coordinates for one of the events from the cluster. Similarly to the DD solution for the synthetic test in Figure 4, no correlation between *T* and *Z* is visible. The multimodality for the depth solution is clearly visible. Its removal requires additional *a priori* constraints to be imposed on the solution.

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