

## ANISOTROPY OF ELASTIC PROPERTIES OF ROCK MASS INDUCED BY CRACKS

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Fractures commonly existing in rocks flow on their elastic properties and hence on velocity of seismic waves propagating in the rock mass. This relation allows to use seismic methods to determine the fracture density and the orientation of fracture sets.

This paper presents results of the research which concern directional changes of dynamic elastic moduli in sedimentary and igneous rocks from south part of Poland. These moduli depend on density of the rock matrix as well as density and orientation of cracks and flow on seismic wave velocity. The seismic equipment Terraloc MK6 (ABEM) was used for the measurements of seismic wave velocity in the surface layers of rock mass. The research was made along precise oriented radial seismic profiles. P-waves and S-waves velocities were established from recorded seismograms. The values of P and S waves velocity allowed to calculate values of dynamic elastic moduli for all profiles. The results were presented on diagrams of azimuth distribution of elastic moduli, and diagrams of dynamic elastic moduli versus P – wave velocity. The diagrams showed an anisotropy of elastic properties of the investigated rocks. The maximal values of moduli agree with maximal values of velocity and also with orientation of main crack sets or potential directions of weakening of rock mass. Obtained results point that the seismic methods allow to assign the directions of weakening of rocks what can be for example use during preliminary designing, constructing and exploitation of tunnels.

**KEYWORDS:** dynamic elastic moduli, effective moduli, fracturing, seismic wave velocity

**1. INTRODUCTION**

Fracturing is one of the elementary features of rocks which decides about their properties. Physical properties of discontinuous solids such as fractured rock mass are influenced by density and geometry of existing cracks. Especially, cracks have an essential influence on the elastic properties of rock and hence they flow on velocity of seismic waves propagating in the fractured rock mass (Bamford and Nunn, 1979; Idziak, 1992; Idziak and Stan-Kleczeck, 2006b). Velocities of P and S seismic waves can be used to assign dynamic elastic moduli of rock mass. On the other side it is possible to calculate the effective moduli of cracked rocks from crack tensor describing geometry of crack sets (Oda, 1986, 1993).

The aim of our research was to establish directional changes of dynamic elastic moduli in chosen rock mass and compare them with effective moduli obtained from the crack tensor. Investigation was carried out in ten quarries localized in the southwest part of Poland.

**2. DYNAMIC ELASTIC MODULI**

The knowledge of longitudinal and shear waves velocities allow to calculate values of dynamic elastic moduli. Dynamic elastic moduli one can assign from the relations (Plewa, 1992; Burger, 1992):

- Young's modulus:

$$E = \frac{\rho V_S^2 (3V_P^2 - 4V_S^2)}{2(V_P^2 - V_S^2)} \quad (2.1)$$

- shear modulus:

$$\mu = \rho V_S^2 \quad (2.2)$$

- bulk modulus:

$$k = \rho \left( V_P^2 - \frac{4}{3} V_S^2 \right) \quad (2.3)$$

- Poisson's constant:

$$\nu = \frac{V_P^2 - 2V_S^2}{2(V_P^2 - V_S^2)} \quad (2.4)$$

where  $\rho$  is bulk density of rock,  $V_P$  and  $V_S$  are velocities of longitudinal and shear wave respectively.

**3. EFFECTIVE MODULI**

Oda (1986) introduced the model which allowed to use second rank crack tensor to calculate values of



**Fig. 1** The map of investigated area: site 1 – Devonian limestone (Dębnik), site 2 – Triassic limestone (Strzelce Opolskie), 3 – Cretaceous sandstone (Wisła), 4 – Oligocene sandstone (Mucharz), 5 – Oligocene sandstone (Skawce), 6 – Permian diabase (Niedzwiedzia Góra), 7 – Permian porphyry (Zalas), 8 – Paleogene basalt (Gracze), 9 – Paleozoic granite (Strzelin), 10 - Paleozoic granite (Strzegom).

effective Young's modulus for different directions of seismic waves propagation. Directional changes of effective modulus  $E^*$  can be present as quotient effective modulus and its isotropic modulus  $E$ :

$$\frac{E^*}{E} = \frac{1}{1 + \ell(F_1 \cos^2 \alpha + F_2 \sin^2 \alpha)}, \quad (3.1)$$

where  $\ell$  is geometrical parametr of cracks (for elliptical cracks  $\ell = \pi/2$ ),  $F_1$  and  $F_2$  are eigenvalues of second rank crack tensor,  $\alpha$  is the angle between direction of wave propagation and longer axis of crack tensor. In anisotropic medium, the biggest values of effective Young's modulus exist in direction perpendicular to longer axis of crack tensor, the smallest values – along this axis.

In analogical way one can define directional changes of effective shear modulus  $\mu^*$ :

$$\frac{\mu^*}{\mu} = \frac{1}{1 + \frac{\ell}{2(\nu+1)}(F_1 + F_2)}, \quad (3.2)$$

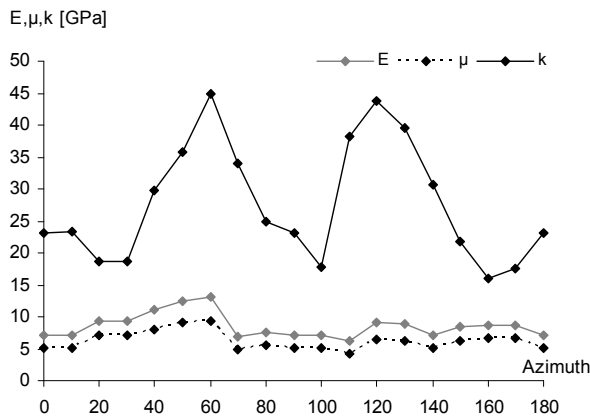
According to model proposed by Oda modulus  $\mu^*$  does not depend on the direction of applied stress when nonlinear effects can be omitted. It means that

effective shear modulus is isotropic and do not depend on crack anisotropy.

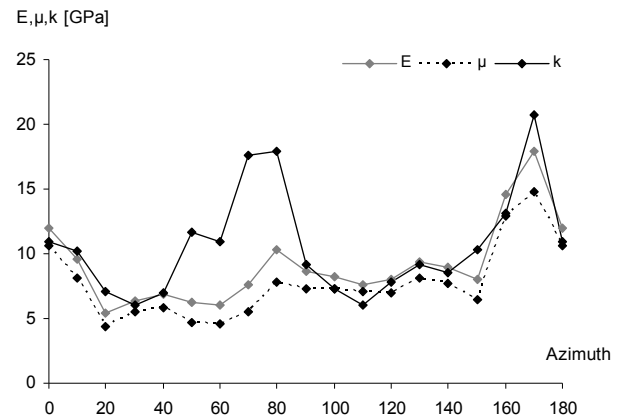
#### 4. THE FIELD STUDY

Velocity of seismic waves and crack systems were assigned for Devonian and Triassic limestone, Cretaceous and Oligocene sandstone, Paleogene basalt, Permian porphyry, Permian diabase and Paleozoic granite (Fig. 1).

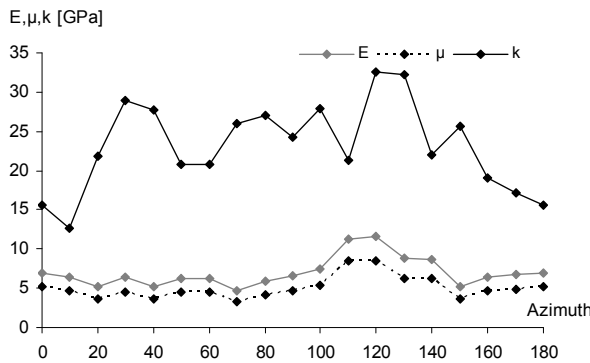
The seismic equipment Terraloc MK6 was used for the measurements of seismic wave velocity in the surface layers of rock mass. The measurements were made along precisely oriented radial seismic profiles. The azimuth interval between profiles was 10 degrees. The seismic data were digitally recorded with up to 12 geophones at a 2-meter spacing so profiles were 24 meter long. Seismic waves were generated by eight kilogram hammer which was hit against a metal plate. The measuring accuracy of seismic velocity depends on precision of apparatus and local wave velocity changes along profile. The first break times of P-waves were read from recorded seismograms. Besides P-waves also S-waves were identified on the seismograms. Wave velocities were calculated from a slope of linear refraction travell-time diagrams obtained by least-squares fitting to experimental data. Obtained P-waves and S-waves velocity data were



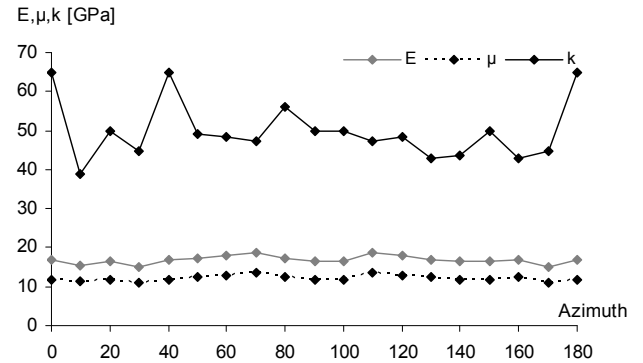
**Fig. 2a** Azimuth distribution of dynamic elastic moduli for limestone from Debnik.



**Fig. 2b** Azimuth distribution of dynamic elastic moduli for sandstone from Skawce.



**Fig. 2c** Azimuth distribution of dynamic elastic moduli for diabase from Niedźwiedzia Góra.



**Fig. 2d** Azimuth distribution of dynamic elastic moduli for granite from Strzegom.

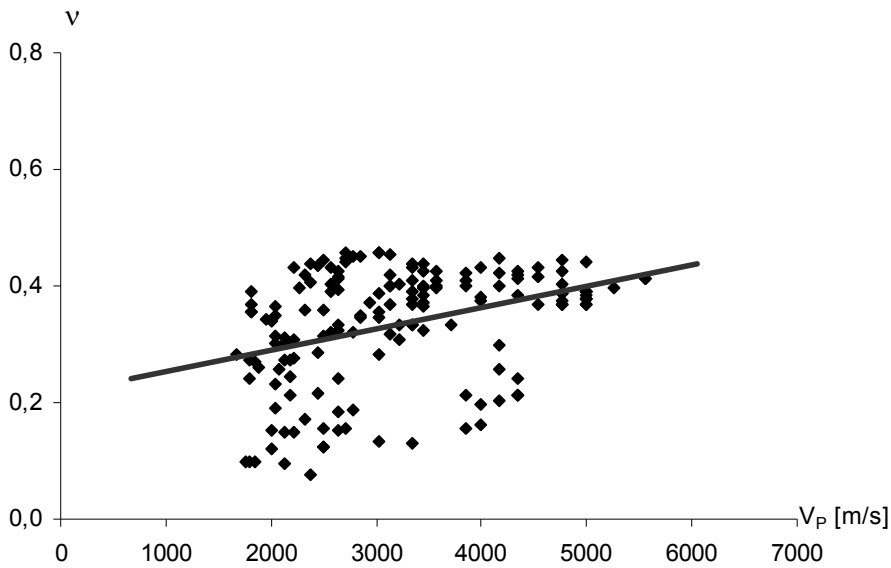
used for calculating the dynamic elastic moduli. Due to used sampling time the first breaks for P-wave were read with accuracy of 0.5 ms whereas the accuracy of S-wave breaks was about 1ms. It allowed to establish wave velocities with standard deviation 200m/s for P-wave and 100m/s for S wave. The mezostructure data were collected from the same quarry in which the seismic measurements were done. The strike azimuth and dip angle were measured with a geological compass. About 100 cracks were measured in every quarry. The surface orientation diagrams were drawn on the base of crack orientation data. The results of crack systems orientation and their geometrical parameters measurements were utilized to calculate the crack tensor. During our research we were only interested in fractures perpendicular to the layering. It allowed to calculate two-dimensional crack tensors in the planes parallel to the layer surfaces. Crack tensor allowed to calculate effective Young's modulus. The obtained values of seismic

wave velocity and directional distribution of cracks were presented in earlier publications (Stan and Idziak, 2005; Idziak and Stan-Kleczek, 2006a; Stan and Idziak, 2006).

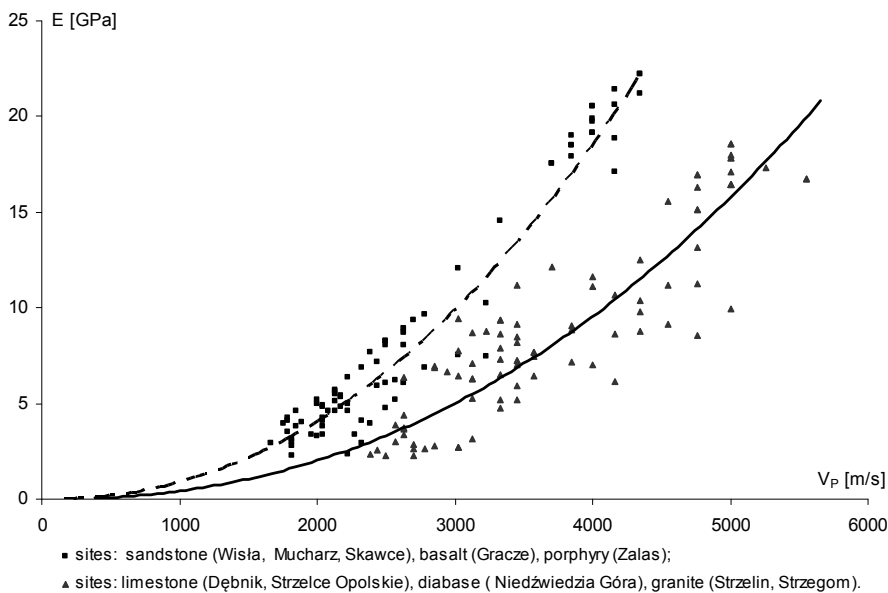
## 5. RESULTS

The most characteristic azimuth distribution of dynamic elastic moduli obtained for investigated sites are presented in Figures 2a-2d. This figures present variations of elastic moduli for different measurement directions and they show anisotropy of elastic properties of rocks under study.

The results obtained for the Debnik quarry show that azimuth distribution of bulk modulus  $k$  have two maxima which agree with strike of main crack sets (Fig. 2a). The azimuth distribution of Young's modulus  $E$  and shear modulus  $\mu$  are characterized by occurrence of only one maximum corresponding to fractures oriented about  $60^\circ$ .



**Fig. 3** The relation between Poisson’s constant and P-wave velocity. Grey line is regression straight line ( $R^2=0.14$ ).



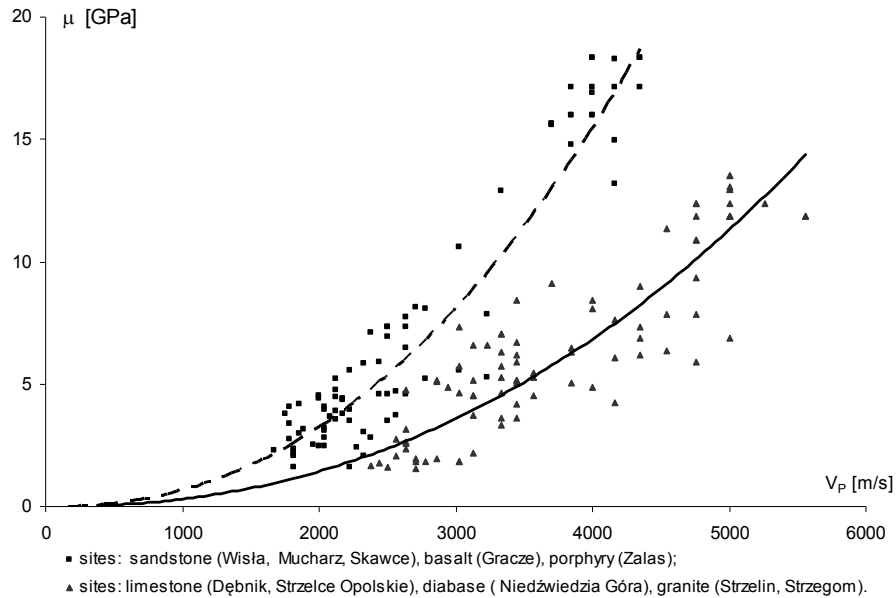
**Fig. 4** The relation between Young’s modulus and P-wave velocity. Regression curve for squares - dashed line ( $R^2=0.90$ ), regression curve for triangles – solid line ( $R^2=0.77$ ).

Different situation one can observe for sandstone from Skawce (Fig. 2b). Azimuth distributions all of the dynamic elastic moduli are characterized by presence of two maxima corresponding to strike of the two existing crack systems. For more, maximal values obtained for azimuth  $170^\circ$  are similar for all presented moduli.

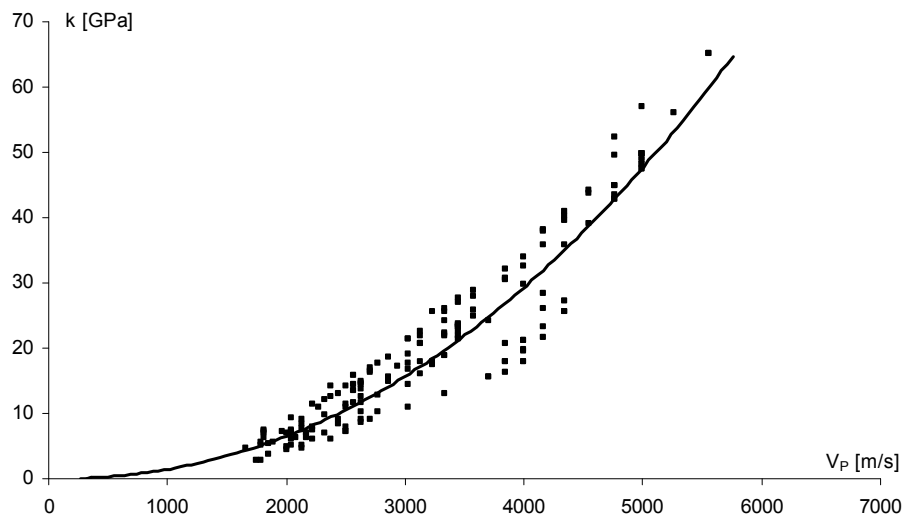
On the azimuth distribution of Young’s modulus  $E$  and shear modulus  $\mu$  for diabase from Niedźwiedzia

Góra we can observed one maximum (Fig. 2c). The distribution of bulk modulus  $k$  is very diversified but it can be possible to distinguish two maxima which agree with strike of two existing crack sets.

For granite from Strzegom, the azimuth distributions of Young’s modulus  $E$  and shear modulus  $\mu$  are rather constant (Fig. 2d.). On distribution of bulk modulus  $k$  one can observed two maximum corresponding to fracture orientations.



**Fig. 5** The relation between shear modulus and P-wave velocity. Regression curve for squares - dashed line ( $R^2=0.84$ ), regression curve for triangles - solid line ( $R^2=0.74$ ).



**Fig. 6** The relation between bulk modulus and P-wave velocity. Regression - solid line ( $R^2=0.90$ ).

## 6. DISCUSSION

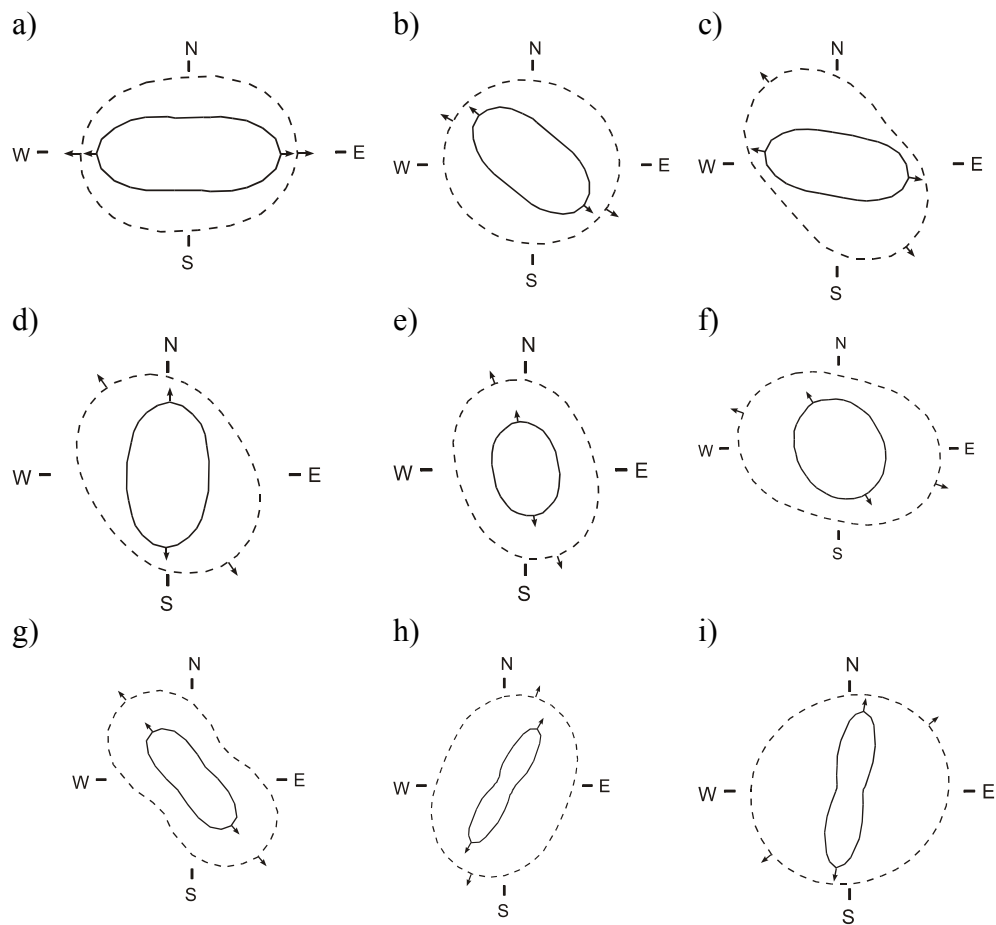
Obtained data allowed to create diagrams of the relations between dynamic elastic moduli and P-wave velocity.

As one can see in Figure 3 the Poisson's constant is not correlated to P-wave velocity and crack density. Values of Poisson's constant vary in interval between 0.1 and 0.46. The similar results were obtained for carbonate rocks from Uppersilesian Coal Basin, Poland (Idziak, 1988).

Very surprising results were obtained for Young's modulus  $E$  (Fig. 4.) and shear modulus  $\mu$  (Fig. 5). The relation between Young's modulus and P-wave velocity should have the character of square

function. In the case one can observe that experimental values of Young's modulus correspond to theoretical relation only for some rock mass (square). For the other rocks (triangle), the regression curve like power function with power less than two is more proper. The similar situation we observed for relation between shear modulus  $\mu$  and P-wave velocity.

This diversification is not observed for the relation between bulk modulus  $k$  and P-wave velocity. Modulus values concentrate around one regression curve (Fig. 6.). Present investigation do not allow to explain observed effect.



**Fig. 7** The relation between distribution of squared P-wave velocity (dashed line) and effective Young's modulus in investigated rock mass:  
 a) limestone (Dębnik), b) limestone (Strzelce Opolskie), c) sandstone (Wisła),  
 d) sandstone (Mucharz), e) sandstone (Skawce), f) diabase (Niedźwiedzia Góra),  
 g) basalt (Gracze), h) granite (Strzelin), i) granite (Strzegom).

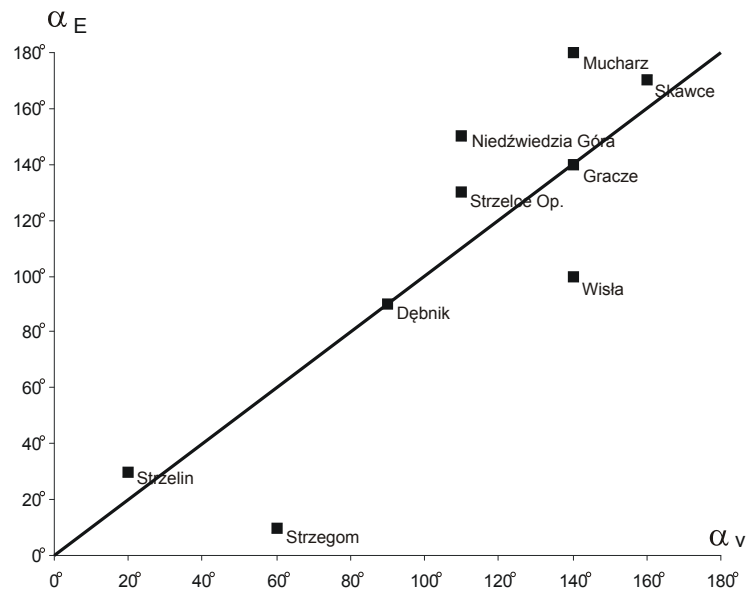
On the base of established crack tensors the azimuth distributions of effective Young's modulus were calculated for investigated rock mass and compared with azimuth distribution of squared velocity (Figure 7).

Obtained directional distributions of effective Young's modulus showed distinct similarity to distribution of square P-wave velocity described by two rank velocity tensor for limestones from Dębnik and Strzelce Opolskie, sandstone from Skawce and basalt from Gracze. Figure 8 showed relation between direction of maximum values of squared P-wave velocity and maximum effective Young's modulus assigned from crack tensor in investigation rock mass. In Figure 8 solid line presented the theoretical relation resulted from the Oda's model. The most of results obtained for investigated rock mass agree with the model.

## CONCLUSION

Presented study proved fracturing as one of the most important factor influencing on anisotropy of physical properties of rocks. The fracture-induced anisotropy of effective elastic moduli generates anisotropy of seismic wave velocity which can be observed in field measurements. Seismic measurements carried out along oriented seismic profiles allow to establish azimuth distribution of longitudinal and shear waves velocities and thus to calculate spatial distribution of effective dynamic moduli.

The anisotropy of elastic properties is the most visible for effective bulk modulus. If existing crack systems are nearly perpendicular, maximum effective elastic moduli are observed for resultant directions of strike azimuth of both crack systems but if cracks belonging to different systems cross by small angle



**Fig. 8** The relation between maximum values of squared P-wave velocity direction and effective Young's modulus in investigation rock mass. Line represents theoretical dependence.

maximum effective elastic moduli are observed for diagonal direction of strike azimuth of cracks. It is possible to assign anisotropy of elastic parameters directly on the basis of crack tensor. For some rocks under study this anisotropy well agree with anisotropy of elastic moduli obtained from seismic measurement. It means that in these rocks fracturing is the main factor inducing anisotropy of their elastic properties. For several investigated rocks mass, the differences between directional distributions of elastic moduli calculated using both methods are significant. It can be caused by another factors influencing on elastic properties of rocks.

#### REFERENCES

- Bamford, D. and Nunn, K.R.: 1979, In situ seismic measurement of crack anisotropy in the carboniferous limestone of North-West England. *Geophys. Prospec.*, 27 (1), 322–338.
- Burger, H.R.: 1992, *Exploration geophysics of the shallow subsurface*, Prentice Hall PTR, 7–20.
- Idziak, A.: 1988, Seismic wave velocities in fractured sedimentary carbonate rocks, *Acta Geophysica Polonica*, vol. XXXVI, no.2, 101–114.
- Idziak, A.: 1992, Seismic wave velocity anisotropy and its relation to crack orientation of rock masses. Katowice: Silesian Univ. Publishing, (in Polish with English abstract).
- Idziak, A. and Stan-Kleczyk, I.: 2006a, Geomechanical properties of fractured carbonate rock mass determined by geophysical methods, in: A.V. Cotthem et al (eds), *Multiphysics Coupling and Long Term Behaviour in Rock Mechanics*, Taylor & Francis, London.
- Idziak, A.F. and Stan-Kleczyk, I.: 2006b, Physical properties of fractured rock mass determined by geophysical methods. in: C.F. Leung, Y.X. Zhou (eds.) *Rock Mechanics in underground construction*. World Scientific, New Jersey - Singapore, 301–309.
- Oda, M., Yamabe, T. and Kamemura, K.: 1986, A crack tensor and its relation to wave velocity anisotropy in jointed rock masses, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, 23 (6), 387–397.
- Oda, M., Yamabe, T., Ishizuka, Y., Kumasaka, H., Tada, H. and Kimura, K.: 1993, Elastic stress and strain in jointed rock masses by means of crack tensor analysis, *Rock. Mech. Rock Engng.*, 26(2), 89–112.
- Plewa, M. and Plewa, S.: 1992, *Petrofizyka*, Wydawnictwa Geologiczne, 273–309.
- Stan, I. and Idziak, A.: 2005, Anisotropy of seismic waves velocity due to the fracturing in chosen rock mass, in P. Konecny (ed), *Impact of Human Activity on the Geological Environment*, A.A. Balkema Publishers, London.
- Stan, I. and Idziak, A.: 2006, Anisotropy of seismic waves velocity in chosen rock mass, *Publications of the Institute of Geophysics Polish Academy of Sciences*, M-29 (395), 211–222.