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ACCURACY OF THE RELATIVE GRAVITY MEASUREMENT

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ABSTRACT

Precise relative gravimeters achieve the internal precision about a few μGal [1], even in field conditions. Nevertheless this precision is in fact concerned with the instant of measurement and can not be confused with the accuracy of the gravity at the gravity station, which is influenced by other effects. The best approach of these two values is question of high-quality elimination of instrumental errors and time-variable disturbing effects affecting the relative gravity measurements.

KEYWORDS: accuracy, precision, relative gravimetry

1. INTRODUCTION

A precision of relative gravity measurements on the level of a few μGal offers application also for geodynamic purposes. The question remains whether the inherent instrument precision (root mean square error of one measurement) corresponds to the real accuracy of the gravity at the station. For instance, the precision for gravimeter Scintrex Autograv CG-5 No. 10125 and gravimeter LaCoste & Romberg G No. 1068 (LCR) reaches values:

Gravimeter	Number	Precision
Scintrex Autograv CG-5	10125	2.1 ± 1.1μGal
LaCoste & Romberg	1068	8.9 ± 4.2μGal

The precision in table means an average root mean square error for one measurement in one typical day unit. The final value in the table was computed as an average from four years gravity data (Fig. 1). Preservation of so high precision (Scintrex) needs a consistent application of mathematical corrections and an elimination of other effects, which can affect the final accuracy.

2. CORRECTIONS OF GRAVITY MEASUREMENT

According to (Olejník and Diviš, 2002) the gravity acceleration g at the station is given by the equation

$$g = \hat{g}_r + \sum o_i + Z(t) \quad , \tag{1}$$

where $\hat{g}_r$ stands for the relative value of the gravity acceleration, $\sum o_i$ sum of corrections and $Z(t)$ the drift of the gravimeter represented as the time function. The relation (1) is specified to the following form

$$g = \hat{g}kk_0 + \sum o_i^I(t) + \sum o_i^{II} + Z(t) \ , \tag{2}$$

where $\hat{g}$ is the reading in [CU], k the given scale factor, k_0 the additional scale factor, $\sum o_i^I(t)$ sum of corrections caused by external influences (Earth and ocean tides, air pressure changes and other) and $\sum o_i^{II}$ sum of corrections caused by internal influences (barometrical effect, mechanical hysteresis and other). Keeping with the formula (2) the possible error sources of relative measurements is possible to divided to errors caused by

- external influences,
- internal (instrumental) influences.

2.1. *EXTERNAL INFLUENCES IN GRAVITY MEASUREMENT*

External influences cause real gravity changes and it is necessary to remove them from the measurement. These effects are independent on the used gravimeter.

EARTH AND OCEAN TIDES

Application of tidal corrections is possible with the accuracy better then 1 μGal. Provided that we have to well know parameters δ and κ of tidal waves. For the accuracy better then 1 μGal is also needful to measure time with the accuracy 0.5 minute and a latitude with the accuracy ten arc second. According to (Torge, 1989) the basic term for gravity changes caused by tidal forces is

$$\Delta g_t(t) = \sum_{i=1}^{n} \delta_i A_i \cos(\omega_i t + \Phi_i + \kappa_i) \ , \tag{3}$$

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[1] 1μGal = 10^{-8} ms^{-2}

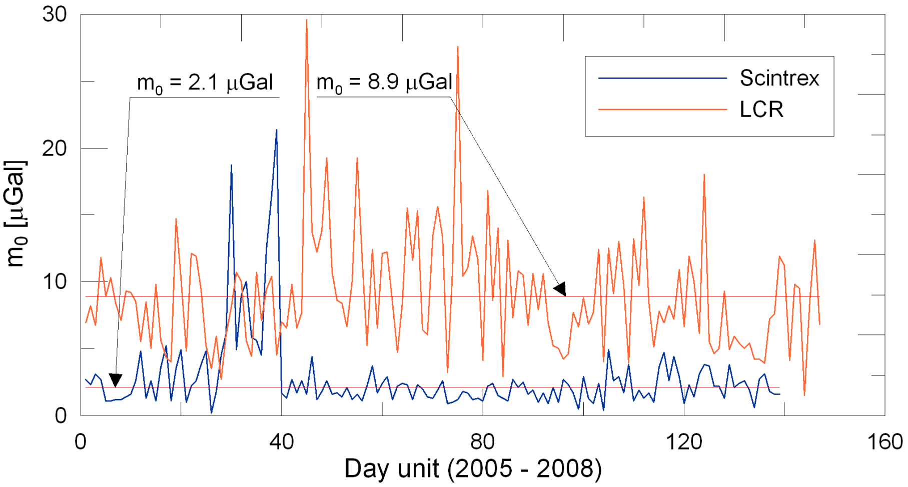

Fig. 1 RMS evolution during time period 2005 – 2008.

where i is the index of the tidal wave, A symbolizes the theoretical amplitude, ω is the circular frequency, Φ the theoretical phase in time $t = 0$, δ the amplitude factor and κ the phase shift.

Computation of tidal corrections is not a restricting factor for the accuracy of relative measurements.

HYDROGEOLOGICAL INFLUENCES

A hydrological influence in gravity is quite big and its magnitude significantly exceeds the measurement accuracy (measuring over longer period). The amount of the groundwater and the soil moisture correlates with many factors (morphology, geological subsoil and other) and it is almost impossible to mathematically simulate potential gravity changes. The global models (dynamic models from the satellite mission GRACE or global hydrological models, (Andersen et al., 2005)) can help partly. Unfortunately the gravity change is most sensitive to changes in the station vicinity (60 % of the whole hydrological effect is up to 1 km) and therefore we need additional information.

Measurements of additional physical quantities (precipitations, soil moisture, groundwater and other) nearby the gravity station bring at least partial information about the mass changes. Then we are able to find a possible correlation of one or more measured quantities with the gravity change. The correction is possible expressed in a simplified way in term

$$\Delta g_{Hydro} = f(P, E, Q) , \qquad (4)$$

where P is the precipitation, E the water evaporation and Q the water outflow in the given area. The possible gravity changes have a seasonal character, when the amplitude can reach up to 15 μGal, (Harnisch and Harnisch, 2002).

ATMOSPHERIC PRESSURE CHANGES

Atmospheric pressure changes produce firstly direct gravity effect (Lederer, 2004). The pressure variations change in periods from a few hours to one year and reach values about a few hPa. For the computation of the correction is commonly used the empirical form

$$\Delta g_p = 0.3(p - p_n) \qquad [\mu\text{Gal}] , \qquad (5)$$

where p [hPa] is the measured pressure and p_n [hPa] normal atmospheric pressure for the international standard atmosphere (ISA). In accordance with the form (5) the pressure change 3 hPa causes the gravity change ≈1 μGal. During the day unit is sometime possible to expect bigger changes of atmospheric pressure then 3 hPa. Neglecting of atmospheric pressure changes can produce an error in the gravity overhangs 1 μGal.

OTHER EXTERNAL INFLUENCES

Among other less important or difficultly registered effects, which can influence the gravity acceleration, belong:

- pole motion,
- mass changes inside the Earth's body
- seismical effects,
- vertical movement of the Earth's crust,
- seasonal periodical and quasi-periodical influences (if they are not covered elsewhere).

2.2. INTERNAL INFLUENCES IN GRAVITY MEASUREMENT

2.2.1. INSTRUMENTAL ERRORS

Instrumental errors mean changes of the recorded gravity produced by the technical imperfection of gravimetrical instruments.

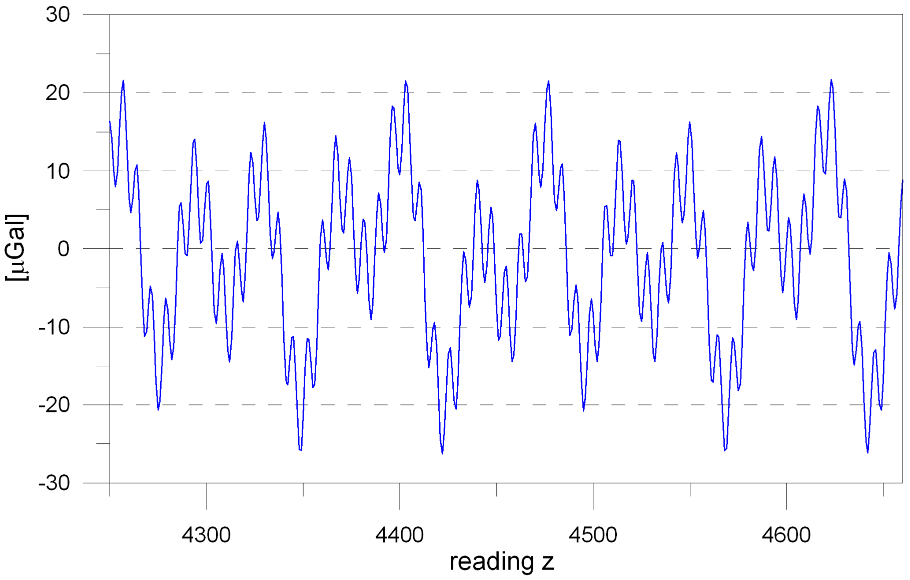

Fig. 2 LCR G No. 1068 periodical errors.

CALIBRATION FUNCTION

The calibration function serves for converting the reading from CU (Counter Unit) to the physical unit (usually to mGal = 10^{-5} ms^{-2}). The function is usually derived from the lab measurement performed by the manufacturer. It is used a special additional equipment to obtain a relative calibration table/curve. The calibration table is unique for each gravimeter and represents the realization of non-linear components of the transformation from CU to the physical unit (accuracy 10^{-3}. . . 10^{-4}). The calibration function has the following form

$$g = F(z) ,$$

where z is the reading in [CU]. For an approximation is usually used 3^{rd}... 5^{th} order function.

In addition to this transformation an additional linear factor (scale factor) is determined at gravimetrical baselines every year (Lederer, 2002). The scale factor (SF) changes with time and should be influenced by other effects, for instant humidity. The SF accuracy derived from the main gravimetric baseline is about 1.10^{-4}. It represents possible error 10 µGal for the gravity difference Δg = 100 mGal. An error on the level 10^{-5} should be caused by using just the linear scale factor model. It can produce the error a few µGal for the gravity difference Δg = 100 mGal.

New LCR gravimeters are equipped with the capacitance beam position indicator (CPI). It reduces operator's fatigue and the galvanometer can be set to have greater sensitivity then the optical system. The CPI must be regularly calibrated by the reading screw.

With the feedback (magnetic induction or electrostatic force is used to balance the beam in the zero position) is possible to achieve the most accurate reading. The feedback linearity and the prime calibration are supposed.

PERIODICAL ERRORS OF THE READING SCREW (GRAVIMETERS WITH GEAR BOX)

Formally the periodical errors belong to the calibration errors, because they negatively impact the reading conversion to the physical unit.

We assume that the periodical errors are simply functions of the reading screw position. Due to imperfections of the gear box and the screws (we do not consider the backlash now), the handling of the sensor is in another position then the reading screw shows. Each specific effect of the periodical error of the expected (known) period P and then the frequency $f = 1/P$ is possible to express by the phase shift sine wave

$$X = x + A\sin(f(x+\varphi)), \qquad (6)$$

where A is the amplitude, φ the phase of the error, x the original reading and X its real value. The periodical error omission can produce four times bigger error then is the derived amplitude in the extreme case (Träger et al., 1978). For example, the gravimeter LCR G No. 1068 has the amplitude equal 5 µGal (Lederer, 2005) for the period P = 7.33 CU. The amplitudes of some LCR meters exceed value 10 µGal (Lederer, 2005; Torge, 1989; Figure 2).

DRIFT

In stationary and field operation, gravimeters exhibit a temporal variation in the display of the zero position, which is called the gravimeter drift. The drift is caused by fading of spring tensions and by uncompensated or unshielded external effects. Type and magnitude of the gravimeter drift are the function of:

- type and characteristic of the specific instrument (quartz systems have larger drifts than metal spring gravimeters),

- age and usage of the meter (generally the drift is decreasing with age),
- external temperature fluctuations,
- air humidity,
- air pressure changes,
- changes of the voltage power supply.

The spring reacts on these disturbances with change of its physical characteristics which leads to the reading change. For modelling of the gravimeter drift is commonly used the following formula (Torge, 1989)

$$z(t) = z(t_0) + a(t - t_0) + b(t - t_0)^2 + c(t - t_0)^3 + \dots$$

where t_0 is the reference time and $a, b, c, \dots$ are the unknown drift coefficients. The drift coefficients are determined by repeated measurements distributed appropriately during the measurement period. Various measurement schedules were developed to obtain the drift coefficients.

TILT EFFECT

The accuracy is dependent on the quality of levelling. By well calibrated levels (calibration better than ±10") the tilt error 10" causes the residual error smaller than 2 μGal (Torge, 1989).

The regular level checking and calibration eliminates a potential error. Gravimeter Scintrex CG-5 has an automatical reduction for errors caused by imperfect levelling of the meter. For the LCR meter is possible to correct the reading mathematically (using electronic level record).

BAROMETRICAL EFFECT

The barometrical effect is caused by an imperfect isolation of the sensor from outer air pressure changes. The effect of the imperfect shielding against the air pressure causes:

- own barometrical effect (reading changes depending on pressure changes),
- continuing gravity change after end of the pressure change,
- hysteresis by the returnable pressure changes.

All above mentioned effects can be summarized in one simple term *hysteresis*. The simple barometrical effect is often approximated by the linear form, using so-called barometrical coefficient *B*, which should be determined in the barometrical chamber. The barometrical correction is then

$$\Delta g_P = B(p - p_0)\ [\mu\text{Gal}] \quad , \tag{7}$$

where p_0 is the air pressure at the first station during the day unit and p the pressure at the actual station. The correction according to formula (7) is possible to applied just in case that the linear dependency is strong, possibly strong appearance to the hysteresis. Otherwise it is not suitable to use the simple linear form for the calculation of the barometrical effect.

MECHANICAL HYSTERESIS

When the beam is clamped, the main spring is stretched compared to its length when the beam is unclamped. It would take time for the meter to stabilize for a reading after unclamping. The mechanical hysteresis is the common effect which occurs after break (clamping/unclamping) between measurements. To eliminate this effect is proper to set the expected reading before unclamping and usage of the symmetrical reading (after the same time interval).

TEMPERATURE EFFECTS

Temperature variations cause changes of spring characteristics. The final reading change is function of the temperature, time gradient of the temperature and temperature transmissions inside the sensor. It should be reduce using

- isolated sensor (usually placed in Dewar flask),
- suitable materials (for example bi-metal),
- thermostat for the stable temperature inside the meter.

The temperature changes are mostly indicated by the drift changes. Thermostat is an essential precondition for the precise measurement with the relative spring gravimeters. There are some methods to find some relation between gravity changes and temperature changes but the expression is too complicated and without satisfactory results till now. Therefore the best solution remains a thermostat (optionally double).

MAGNETIC FIELD

A magnetic field represents the physical field, which influences gravimetric instruments (Pálinkáš et al., 2005). Earth's magnetic field reaches values 48 μT (vertical component 44 μT and horizontal component 20 μT) with changes about 3 μT in the Czech Republic. An artificially excited magnetic field (sources inside building, transmitters an so on) should be also meaningful sources of errors. There is possible to expect big changes of the magnetic field nearby artificial sources.

The elimination of the magnetic field influences is usually secured by an orientation of the gravimeter to the same azimuth at the gravity station. Worst results should be obtained inside building with sources of an artificial magnetic field. For instant, sensitivity to the magnetic field of the gravimeter LCR G No. 137 reaches values about 0.5 μGal/μT, in the vertical as well as in the horizontal direction.

OTHER EFFECTS

The final accuracy of the gravimeter can also affect some further effects such as:

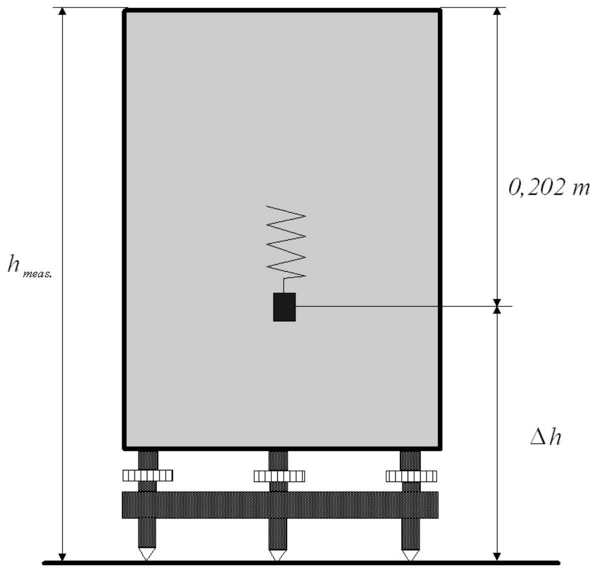

Fig. 3 Gravity transfer scheme (Scintrex CG-5).

- unstable voltage,
- reading errors,
- backlash of the measurement screw,
- shocks (sudden reading change).

2.3. VERTICAL GRADIENT

A possible error caused by neglecting of the correct vertical gradient is not produced by the gravimeter imperfections but depends on the gravimeter design.

The sensor of the gravimeter is in the specific height Δh above the gravity station (reference height) during the measurement. This distance is typical for each type of the gravimeter. It should be from a few centimeters (gravimeter LaCoste&Romberg without a plate) to a few tens of centimeters (gravimeter Scintrex CG-5 with an original tripod). The needful correction $\Delta g_{W_{zz}}$ is possible calculate by well known form

$$\Delta g_{W_{zz}} = W_{zz} \Delta h, \tag{8}$$

where W_{zz} is the vertical gradient and Δh the vertical distance between the gravity station and the gravimeter sensor (Fig. 3).

A *normal vertical gradient* for a latitude $\varphi = 50\ °$ and for the ellipsoid GRS80 is equal to the value $W_{zz} = 3085\ \mathrm{E}$ [2] (GRS80, 1980). The normal gradient is used for the transfer of the measured gravity to the reference, using the formula (8), when the vertical gradient was not measured directly. In the Czech Republic the vertical gradient ranges from 2500 to 4500 E (Olejník, 1988).

We consider the gravity measurement with the gravimeter Scintrex CG-5 (Fig. 3) at the station with an extreme vertical gradient ($W_{zz} = 4500\ \mathrm{E}$) for the model example. The common measured instrument's height is $h_{meas.} \cong 47$ cm. The difference from the normal gradient $\Delta W_{zz} = 1415\ \mathrm{E}$ together with $\Delta h = h_{meas.} - 0.202 \cong 0.27$ m causes the error 38.2 μGal only by applying the normal vertical gradient instead of the directly measured one.

3. RECOMMENDATIONS

The enumeration of disturbing influences (Chapter 2) itself shows the complexity of the theme. It takes big demands on the measurement and the post processing of the relative gravity measurement. We are not able to eliminate or evaluate all effects satisfactory yet. Therefore, for the high measurement accuracy, we must try to eliminate disturbing effects every which way.

The recommendations are focused on the relative gravity measurement in the gravity or geodynamical networks, for that reason are not all of them needful for gravity prospecting.

3.1. ELIMINATION OF EXTERNAL EFFECTS

A good knowledge of the tidal parameters δ, κ and application of the latest tide potential development allows us to remove *tidal effects* without an accuracy losses.

Removing of the *hydrogeological influences* remains the hot issue. The topic is really complex and needs a lot of additional measurements, such as a groundwater level, precipitations, temperature, soil moisture and so on. In addition to that, the quantities have to be measured permanently. It is not possible to fulfill these demands for relative gravity measurement. The global gravity and hydrological changes models help partly, but the gravity is most sensitive to mass changes in the near vicinity. The hydrogeological changes have a strong seasonal effect and they may significantly influence the relative gravity measurement performed in stages. For the best elimination is useful to measure every stage in the same time of the year. In case we know the time behavior of the gravity change (nearby GGP [3] stations) we can extrapolate seasonal gravity variations for close relative stations. However we are still talking about the approximate solutions.

Air pressure changes are not so critical according to the magnitude of the effect. Applying of the formula (5) secures the sufficient accuracy (less than 1 μGal).

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[2] $1\ \mathrm{E} = 10^{-9}\ \mathrm{s}^{-2}$

[3] Global Geodynamic Project, stations with absolute and superconducting gravimeters

Fig. 4 Gravimeters Scintrex Autograv CG-5 (left) and ZLS Burris (right).

3.2. INSTRUMENTS EQUIPMENT

It is important to notify that the relative gravimeters work on the accuracy level 10^{-8}. Such a high accuracy of the sensor is paid by the sensitivity to other physical quantities. A manufacturer is trying to shield the sensor against disturbing effects, but very often the protection does not work accurately, how was proved by many lab tests.

For the precise relative gravity measurements is possible to use gravimeters Scintrex CG-3M (Hugill, 1988) and Autograv CG-5, gravimeter Burris ZLS (Jentzsch, 2007) and gravimeters La-Coste&Romberg G and D (Šimon, 1995). These gravimeters should have some errors described in paragraph 2.2, which is needful to eliminate. From this reason is recommended to do:

- periodic checking of the meters according to manufacturer’s instruction,
- annual probing of the scale factor,
- regular lab tests,
- periodical errors checking [4] (gravimeters with gear box).

Derived correlations are necessary to mathematically remove from the realized measurements.

3.3. MEASUREMENT

An appropriate measuring schedule can partly eliminate the instrumental errors. We can summarize some recommendation in a few items:

- First reading carried out in the office (spring relaxation after long clamping).
- Suitable measuring schedule secures the best *drift* approximation. Stable conditions during the measurement (temperature, humidity) and protection against meteorological conditions (wind, sunshine) help to minimize drift changes.
- Identical instrument orientation minimizes *magnetic field influences*.
- Setting the approximate reading before unclamping and symmetrical reading after unclamping reduces the *mechanical hysteresis* of the sensor.
- Usage of directly measured *vertical gravity gradient* is necessary condition in order to guarantee the best accuracy.

On average, the gently handling with the gravimeter during the measurement and the transport a priori decreases errors.

3.4. GRAVITY CONTROL OPTION

Type of the stabilization and the station locality selection can also influence the final accuracy. The locality choice has to fulfill demands, which have to guarantee the best quality of gravity measurements.

It should be chosen an undisturbed place in a sufficient distance from artificial disturbing effects (e.g. big cities, railway and highways). The places situated nearby possible mass changes [5] such as dams, mines, rivers are also unsuitable. Higher altitudes are not fit for the relative measurement, if it is not necessary. A scale factor uncertainty and big meteorological changes in mountains can significantly decrease the expected accuracy.

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[4] Turning clockwise with the nulling dial before reading is important for elimination of the backlash

[5] If it is not the research goal

Table 1 Overview of disturbing effects and its elimination estimation.

Effect	Magnitude [μGal]	Elimination [μGal]	Depend on
Earth and Ocean Tides	280	< 1	δ a κ accuracy
Hydrogeological influences	tens	≈ 5	Hydrogeology
Air pressure changes	units	< 1	Air pressure
Calibration function	tens	1…10/100mGal	Gravity difference
Periodical errors	tens	1…5	Gravimeter
Gravimeter drift	hundreds	1…5	Gravimeter
Tilt effect	units	< 2	Observer's care
Barometrical effect	tens	units	Air pressure
Mechanical hysteresis	1..3	< 1	Gravimeter
Temperature effects	tens	units	Temperature
Magnetic field	units	< 1	Orientation
Other effects	units	1…2	Gravimeter
Vertical gradient error	tens	1…2	Gravimeter, terrain

The stabilization itself is very important for gravimetric surveying. A solid rock or a concrete pillar strongly embedded in the ground ((Zeměměřický úřad, 2003), type II stabilization) represents good base for the precise gravity measurement. The pillar is not suitable for the gravity measurement because of possible troubles with the vertical gradient measurement.

3.5. SUMMARY

All disturbing effects described in Chapter 2 are transparently summarized in Table 1. It is very complicated to estimate the extent of elimination and it depends on many other parameters (used gravimeter, topography, morphology, weather conditions and so on). Therefore the values in the Table 1 (column Elimination) are just the qualified estimates.

4. CONCLUSION

It is hard to determine the achievable accuracy of relative gravity measurements. There are many external and internal factors which can affect the final accuracy. Considering recommendations mentioned in previous chapters we can try to estimate the accuracy for gravimeters LaCoste&Romberg No. 1068 and Scintrex Autograv CG5 No. 10125. The values from the Table 1 and the precision mentioned in Chapter 1 are used for the estimation. The estimated accuracy for the Scintrex meter is about 6 μGal and for LCR meter about 11 μGal in ideal field conditions (all effects are eliminated as good as we can). These values belong to specific instruments and can not be simply generalized because each meter has its specific qualities. Also is impossible to include some aspects to the final accuracy (for instance an error from poor knowledge of the scale factor increases with the gravity difference).

In conclusion we can say that the precise relative gravity measurement is possible to use for geodynamic purposes, but in close cooperation with the absolute gravity measurement. The unstable field conditions, scale factor and drift uncertainty and also the necessity of the reference value are the most important effects which have negative influence to the final accuracy of relative gravity measurements. Hydrological changes are most important for longer or periodical measurements where they can cause big differences for stages measurements. It is also the main restricting factor in accuracy for precise relative and also absolute instruments.

Relative gravity measurements should be used for extending of absolute measurements, possibly for smaller local controls, where the aim is focused to relatively quick and bigger changes of gravity (for instance fast changes of the river surface, undermined areas and so on). Application of a group of gravimeters (at minimum 2 instruments) is necessary.

Relative gravimetry is also possible to use for regions where is possible to detect larger changes than approx. 2 - 3 μGal per year (for example Fenno - Scandinavian uplift).

An important role of relative gravimeters is for gravity gradient measurement. The precise gravity value transfer needs very well knowledge of the vertical (possibly horizontal) gravity gradient. There is not another such precise method than the relative gravity measurement.

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