STABILITY OF PERIOD AND QUALITY FACTOR OF FREE CORE NUTATION

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ABSTRACT

In our study we find, from the analysis of VLBI observations, small quasi-periodic fluctuations of the period and quality factor of retrograde Free Core Nutation (FCN), ranging mainly between 429.8 to 430.8 days and 17000 to 21000, respectively. To this end, we use resonant effects in several dominant forced nutation terms to calculate the period and quality factor of FCN in running 6-year intervals. We also recently demonstrated that the atmospheric and oceanic excitations are capable of exciting FCN. Both amplitude and phase of the geophysically excited motion are consistent with the values observed by VLBI, in the interval of tens of years. The geophysical excitations are now numerically integrated, using Brzeziński's broadband Liouville equations, and removed from the observed celestial pole offsets. The remaining part is then used to derive the period and quality factor of FCN in running intervals, and to study the temporal stability of these important Earth parameters. It is demonstrated that the observed quasi-periodic variations of both parameters are probably not caused by these geophysical excitations.

KEYWORDS: Earth orientation, nutation, geophysical excitation, VLBI

1. INTRODUCTION

There is a strong resonance in near-diurnal part of the spectrum (in terrestrial frame) in celestial motion of the Earth's spin axis (precession-nutation), due to the existence of a flattened fluid outer core of the Earth. This resonance, together with a partial viscosity of the mantle, leads to such effects as

- Modification of amplitudes and phases of the forced (by external torques exerted by the Moon, Sun and planets) nutation terms, with respect to the solution calculated for the rigid Earth model. The largest influence is observed for the retrograde annual term that is closest to the resonance.
- Free Core Nutation (FCN), whose period *P* and quality factor *Q* depend dominantly on the flattening of the core and viscosity of the mantle. This free motion has, in absence of excitations, a damped sinusoidal character.
- Amplification of any excitation with frequency close to the resonance, namely the effects of the atmosphere and oceans, leading to small additional modifications of some of the forced nutation terms.

If the core were in hydrostatic equilibrium, the FCN period (in non-rotating celestial frame) would be equal to about 460 days (Wahr, 1981). This was the value used to derive the older model of nutation IAU1980. The observations by Very Long-Baseline Interferometry (VLBI), so far the only observational technique that is capable of observing celestial motion of the Earth spin axis with sufficient accuracy, revealed that the period is somewhat smaller, only about 430 days (Roosbeek et al., 1999; Hinderer et al., 2000; Mathews et al., 2002; Vondrák and Ron, 2005; Vondrák et al., 2005), which corresponds to the core's flattening about 4 per cent larger than the hydrostatic equilibrium value (Herring et al., 1986; Gwinn et al., 1986). The effects of the atmosphere and oceans in nutation were recently studied by many authors, e.g., by Bizouard et al. (1998), Yseboodt et al. (2002), Lambert (2006) or Vondrák and Ron (2006, 2007, 2008). From these studies it follows that the effects are most significant in annual and semi-annual terms of nutation, and are of the order of a hundred microarcseconds. They are very small compared to the dominant influence of external torques exerted by solar system bodies, but now detectable by VLBI.

But is the period *P* (and quality factor *Q*) really stable in time? This question was recently addressed, e.g., by Lambert and Dehant (2007). They conclude that the resonant period is stable within less than half a day, but differences in approach of different analysis VLBI centers have an impact of the same order of magnitude. They also find that the contribution of the atmosphere is negligible. Here we study the problem of stability in more detail to find to which extent these conclusions are correct.

2. RESONANCES IN EARTH ROTATION AND ESTIMATION OF *P, Q*

The resonances are most complexly given by Mathews et al. (2002). They derived so called Mathews-Herring-Buffet (MHB) transfer function in the form

$$T(\sigma) = \frac{e_R - \sigma}{e_R + 1} N_0 \left[1 + (1+\sigma) \left(Q_0 + \sum_{j=1}^{4} \frac{Q_j}{\sigma - s_j} \right) \right], \quad (1)$$

expressing the ratio of the non-rigid amplitude of a forced nutation term with terrestrial frequency σ (in cycles per sidereal day – cpsd) to its rigid Earth value. Here e_R denotes the dynamical ellipticity of the rigid Earth used to compute the rigid solution, N_0, Q_j are complex "strength" parameters, and s_j are four complex resonance frequencies corresponding to Chandler Wobble (CW, with terrestrial period of about 435 days), retrograde FCN (with celestial period of about 430 days), Prograde Free Core Nutation (PFCN, with celestial period of about 1020 days) and Inner Core Wobble (ICW, with terrestrial period of about 2400 days), respectively. In our case, only $s_2 \approx -1.0023$ cpsd (FCN frequency) is interesting since it is close to the frequencies of all nutation terms and, at the same time, the corresponding coefficient Q_2 ($\sim 4.89\times10^{-2}$) is two orders of magnitude larger than Q_3 ($\sim 2.96\times10^{-4}$), corresponding to PFCN. Eq. (1) was used, together with a rigid-Earth solution by Souchay et al. (1999) to derive the presently adopted model of nutation IAU2000A.

The nutation angles are obtained from VLBI observations in the form of celestial pole offsets (CPO), which are the differences between the observed position of celestial pole from its predicted position with the IAU2000A model of nutation and IAU2006 model of precession. They are given as two small angles dX, dY, representing a mixture of several influences:

- Free Core Nutation;
- Deficiencies of the IAU model of precession-nutation;
- Neglected geophysical (atmospheric, oceanic…) excitations.

If we forget for a moment the third source, we can analyze CPO to obtain amplitudes of several circular nutation terms with frequencies corresponding to the five terms mentioned below:

$$\begin{aligned} A^+ &= \frac{1}{2}\left[S_x - C_y + \mathrm{i}(C_x + S_y)\right], \\ A^- &= \frac{1}{2}\left[-S_x - C_y + \mathrm{i}(C_x - S_y)\right]. \end{aligned} \quad (2)$$

Adding these amplitudes to the corresponding values of IAU2000A model, we get the complete observed amplitudes.

To verify how much the results can be influenced by different software, and analysis configuration (station motions, source instabilities, atmospheric loading …) used by different analysis centers of the International VLBI Service for Geodesy and Astrometry – IVS (Schlüter and Behrend, 2007), we used the following VLBI-based series of celestial pole offsets:

- OCCAM software:
  - Geoscience Australia (AUS), 1984.0 – 2008.8;
  - Institute of Applied Astronomy, St. Petersburg (IAA), 1984.0 – 2008.8;
  - St. Petersburg University (SPU), 1989.0 – 2008.8;
- CALC/SOLVE software:
  - U.S. Naval Observatory (USNO), 1984.0 – 2008.8;
  - Goddard Space Flight Center (GSFC), 1984.0 – 2008.8;
  - Observatoire de Paris (OPA), 1984.0 – 2008.7;
  - Bundesamt für Kartographie und Geodäsie (BKG), 1984.0 – 2008.8;
- IVS combined solution `ivs08q2X` (IVS), 1984.0 – 2008.3.

All these data are originally given in unequally spaced intervals, 1-7 days long, sometimes with large outliers. Therefore, we first cleaned the data by removing CPO values exceeding 1 mas, and then interpolated them to 3-day intervals, by using a weak smoothing and cubic spline function, to get equally spaced data.

MHB transfer function (1) can be used to derive the parameters on the right-hand-side, provided the value $T(\sigma)$ is known for several different frequencies σ (for more details see, e.g., Vondrák et al., 2005). Here we use the amplitudes and phases of five dominant nutation terms (with periods 365.26, 182.62, 121.75, 27.55 and 13.66 days) as observed by VLBI, both for positive and negative frequencies. We estimate them in six-year running intervals, using the weights computed from VLBI formal standard errors. To account for possible longer term variations, we insert also a bias and a linear trend to the least-squares fit, and, to avoid possible aliasing, we also estimated a term with FCN period. Dividing the estimated nutation amplitudes by their rigid-Earth values by Souchay et al. (1999) we get the values T. Necessary to note that MHB solution contains a relatively simple model of atmospheric excitation in the form of so called Sun-synchronous correction. It is a prograde annual term with the amplitude of about 100 μas, obtained as an empirical correction removing obvious residuals at this frequency. Quite naturally, we removed this correction from VLBI-based celestial pole offsets before determining the values T. Thus, we obtain ten

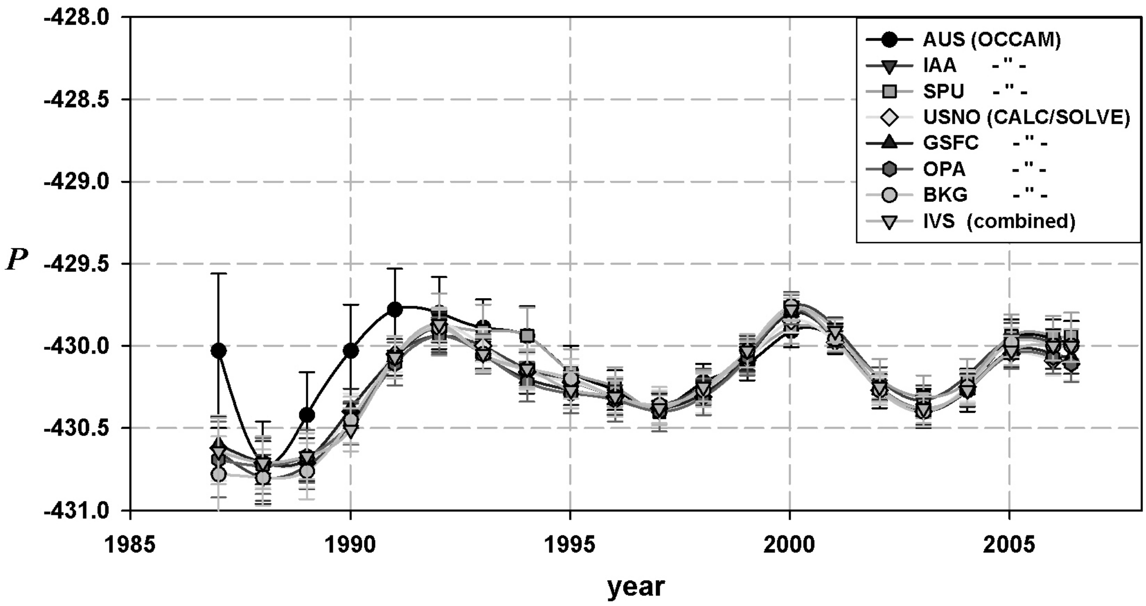

Fig. 1 Period *P* of Free Core Nutation from VLBI solutions, with MHB Sun-synchronous correction removed.

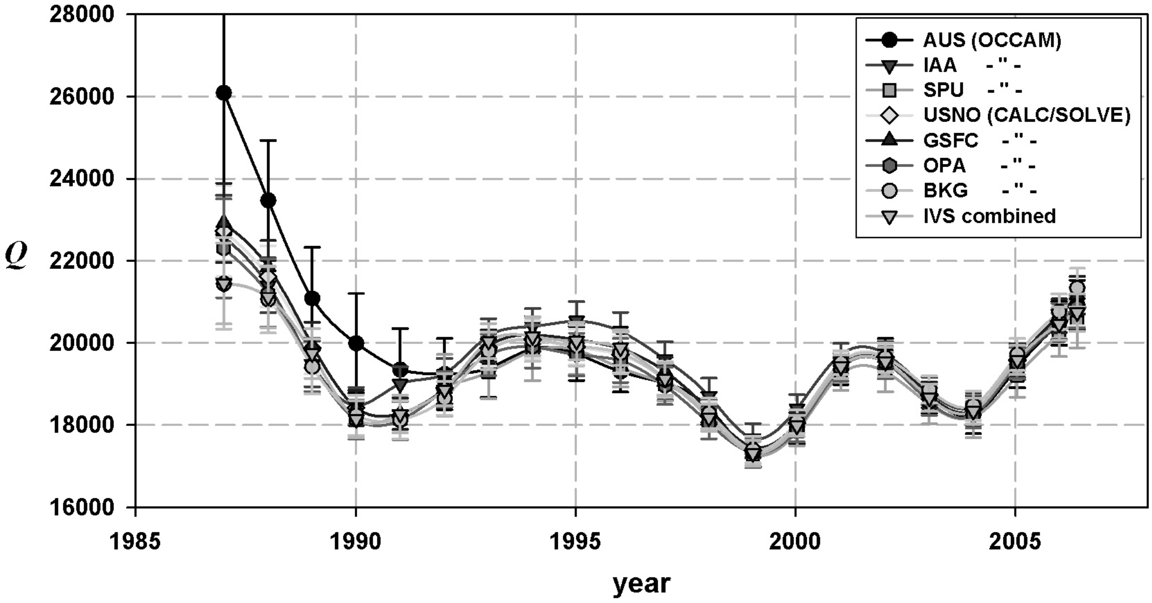

Fig. 2 Quality factor *Q* of Free Core Nutation from VLBI solutions, with MHB Sun-synchronous correction removed.

different complex values of *T* (i.e., twenty different "observables"). These, in turn, were used to estimate the complex frequency s_2 in a weighted least-squares solution, in six-year moving intervals; all remaining parameters of Eq. (1) were fixed to MHB values. Once we have the complex values of resonance frequency s_2, we can easily compute the period and quality factor from the simple equations

$$P = 0.99727/[\mathrm{Re}(s_2)+1], \quad Q = -\mathrm{Re}(s_2)/2\,\mathrm{Im}(s_2), \tag{3}$$

where *P* is given in solar days.

The results, with their error bars, are graphically depicted in Figures 1 and 2. The uncertainties, represented as error bars, are not based on formal errors of individual VLBI-based CPO; they are computed from the dispersion among different values of *T* for different nutation frequencies. All series display similar quasi-periodic changes, giving almost identical results (their agreement is fully consistent within error bars). These variations are most probably not caused by instabilities of the observed radio sources, since different VLBI solutions use different strategies of aligning their celestial reference frame

Table 1 Comparison of amplitudes/phases of some of the diurnal tidal terms of AAM.

		pressure				wind			
		NCEP		ERA		NCEP		ERA	
wave	Per.[d]	Amp.	Phase	Amp.	Phase	Amp.	Phase	Amp.	phase
S_1	365.26	1.092	125	1.275	133	10.246	263	8.715	318
P_1	182.62	0.475	117	0.597	122	16.577	215	17.595	213
π_1	121.75	0.244	346	0.177	350	0.429	55	0.326	0
O_1	13.66	0.179	125	0.192	123	1.082	151	1.589	132
ψ_1	–365.26	0.120	166	0.132	162	1.732	194	0.804	171

with ICRS. There might be several reasons causing these changes, but here we concentrate on just one question: Can the variations of *P, Q* be caused by the influence of the atmosphere and oceans that can significantly perturb estimates of the forced nutation amplitudes?

3. GEOPHYSICAL EXCITATIONS AND THEIR INTEGRATION

To verify the above mentioned possibility, we need time series of that part of nutation that is due to geophysical excitations. To this end, we use the atmospheric and oceanic angular momentum functions that are now available from different agencies. Since the most important part of excitation is in near-diurnal part of the spectrum (in terrestrial frame), we need the data sampled with sub-diurnal interval. Namely we use the following data, available in 6-hour intervals:

- Atmospheric angular momentum functions (pressure + wind terms):
  - NCEP/NCAR re-analysis, 1984. –2008.8 (Salstein, 2005);
  - ERA, 1984.0 – 2008.0 (Thomas et al., 2007; Dobslaw and Thomas, 2007; re-analysis model before 2001.0, operational model afterwards);
- Oceanic angular momentum functions (matter + motion terms):
  - ECCO model, 1993.0 – 2008.2 (Gross et al., 2005);
  - OMCT model, 1984.0 – 2008.0 (Thomas et al., 2007; Dobslaw and Thomas, 2007; driven by re-analysis atmospheric model before 2001.0, by operational model afterwards).

All these time series are given in terrestrial frame, so we have to transform them into celestial (non-rotating frame). Thus, the near-diurnal variations become very long-periodic ones. Because we are interested only in long-periodic motions, we apply the smoothing (Vondrák, 1977) to remove all periods shorter than 10 days. It is also worth mentioning that ECCO model is forced by the NCEP/NCAR data, so they should be used together; a similar argument holds for OMCT/ERA combination.

The comparison of spectral analyses of AAM models NCEP/NCAR and ERA in the interval 1985.6 – 2008.0 (in celestial frame) for the main tidal diurnal waves S_1, P_1, π_1, O_1, ψ_1 shows the consistency of both models in pressure term. Slight inconsistency is however seen in wind term, where the amplitude of ψ_1 for ERA is about a half of that of NCEP and the phases differ by up to 50° (see Table 1, where amplitudes are given in mas, phases in degrees).

For solving theoretically the rotation of non-rigid Earth model, Brzeziński (1994) proposed so called broad-band Liouville equations, considering only the two main resonances – Chandler wobble and FCN. In celestial frame and complex notation, they read

$$\ddot{P} - \mathrm{i}(\sigma'_C + \sigma'_f)\dot{P} - \sigma'_C\sigma'_f P = \\ = -\sigma_C \begin{Bmatrix} \sigma'_f(\chi'_p + \chi'_w) + \sigma'_C(a_p\chi'_p + a_w\chi'_w) + \\ + \mathrm{i}[(1+a_p)\dot{\chi}'_p + (1+a_w)\dot{\chi}'_w] \end{Bmatrix}, \quad (4)$$

in which $P = \mathrm{d}X + \mathrm{i}\mathrm{d}Y$ is the excited motion of Earth's spin axis in celestial frame; $\sigma'_C = 6.32000 + 0.00237i$, $\sigma'_f = -0.0146011 + 0.0001533\mathrm{i}$ rad/day are the complex Chandler and FCN frequencies in celestial frame, respectively, whose imaginary parts are closely related to the corresponding quality factors (these values are based on observed polar motion and celestial pole offsets);
$\sigma_C = \sigma'_C - \Omega$ is the Chandler frequency in terrestrial frame, where $\Omega = 6.30038$ rad/day is the angular speed of Earth's rotation;
χ'_p, χ'_w are excitations (matter and motion term, respectively) in celestial frame,
and $a_p = 9.2 \times 10^{-2}$, $a_w = 5.5 \times 10^{-4}$ are dimensionless numerical constants, expressing the response to a matter and motion excitation, respectively.

The numerical integration of Eq. (4) is made by fourth-order Runge-Kutta method with 6-hour step. Namely we use the procedure `rk4` from Numerical Recipes (Press et al., 1992) that we adapted to our purpose by rewriting it into complex form. To obtain two first-order equations, instead of a second-order one given by Eq. (4), we use the substitutions

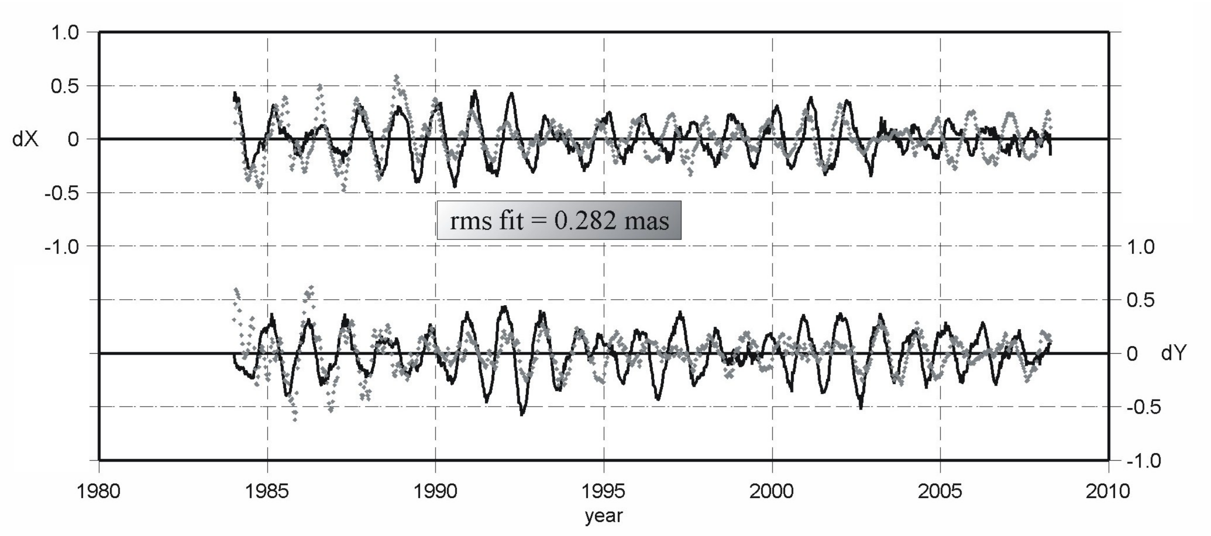

Fig. 3 Observed and integrated celestial pole offsets with NCEP AAM (pressure + wind).

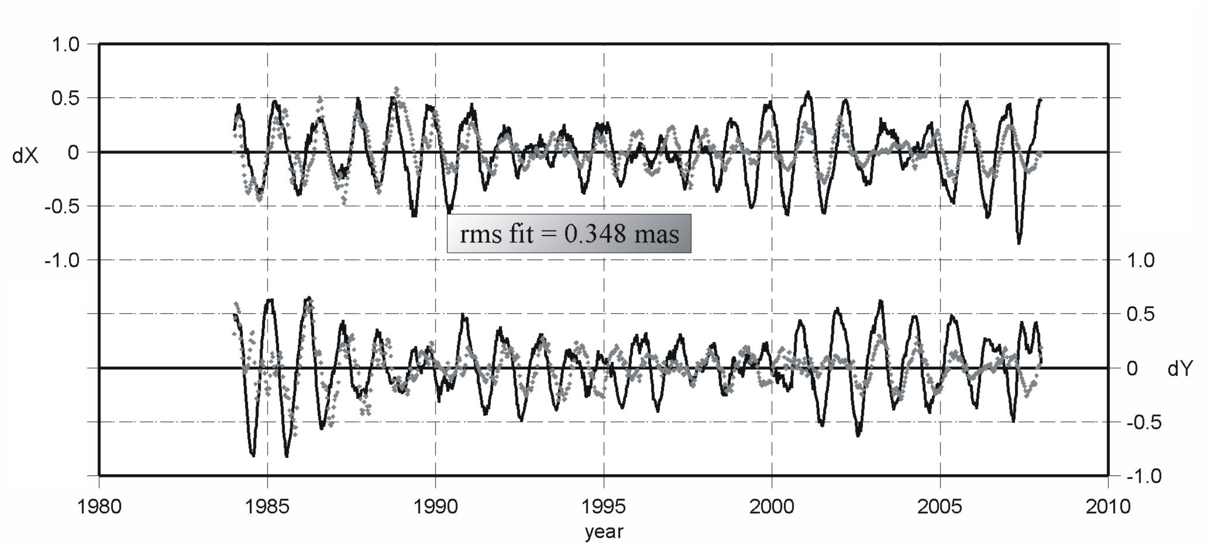

Fig. 4 Observed and integrated celestial pole offsets with NCEP AAM (pressure with IB correction + wind).

$y_1 = P$, $y_2 = \dot{P} - \mathrm{i}\sigma'_C P$, leading to differential equations for two complex functions y_1, y_2

$$\dot{y}_1 = \mathrm{i}\sigma'_C y_1 + y_2$$
$$\dot{y}_2 = \mathrm{i}\sigma'_f y_2 - \sigma_C \begin{Bmatrix} \sigma'_f(\chi'_p + \chi'_w) + \sigma'_C(a_p\chi'_p + a_w\chi'_w) + \\ + \mathrm{i}[(1+a_p)\dot{\chi}'_p + (1+a_w)\dot{\chi}'_w] \end{Bmatrix} \quad (5)$$

The solution generally yields two free damped circular motions: rapid prograde Chandler wobble and slow retrograde FCN with celestial frequencies σ'_C and σ'_f, respectively. To integrate the system (5), we need to choose the initial values – two complex constants, defining the amplitudes and phases of both free motions. We are not interested in rapid (nearly diurnal) Chandlerian motion, so we choose only one, pole position at initial epoch P_0; its first derivative is constrained so that the Chandlerian amplitude disappears. This is assured by choosing the values $y_1(0) = P_0$, $y_2(0) = \mathrm{i}(\sigma'_f - \sigma'_C)P_0$. The final choice of P_0 is made by repeating the integration with different values P_0 until the fit of the integrated motion to VLBI observations attains a minimum.

Four different series of geophysical excitation were used to integrate the equations. The results are shown in Figures 3 through 6, in which the observed CPO values are compared with the integrated geophysical excitation. VLBI observations are depicted as gray dots, integration as black full line. The *rms* fit between the two series is displayed in a box inside each graph. Figure 3 shows the

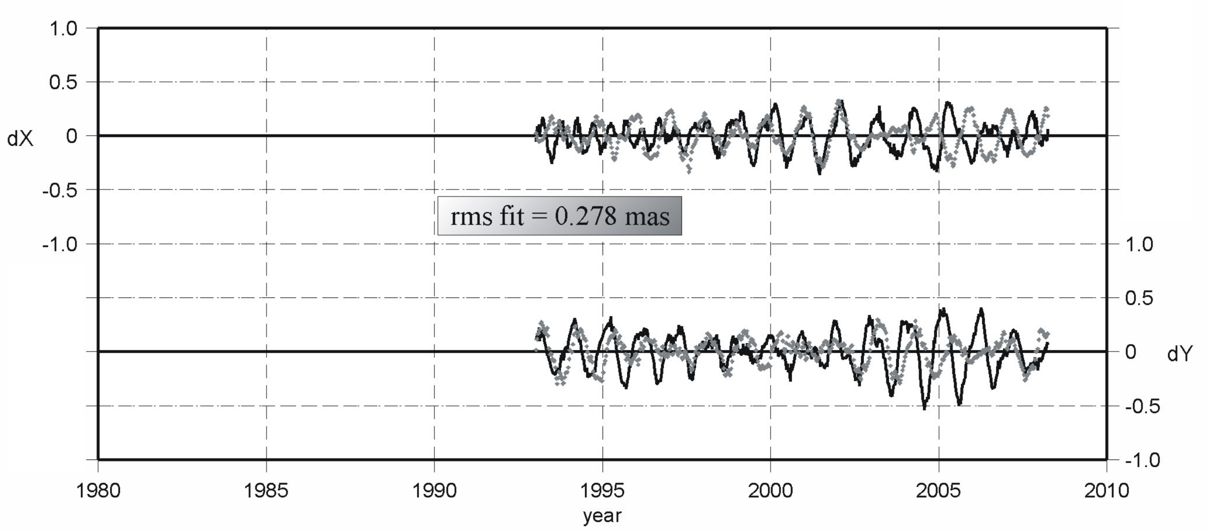

Fig. 5 Observed and integrated celestial pole offsets with NCEP AAM (pressure with IB correction + wind) + ECCO OAM.

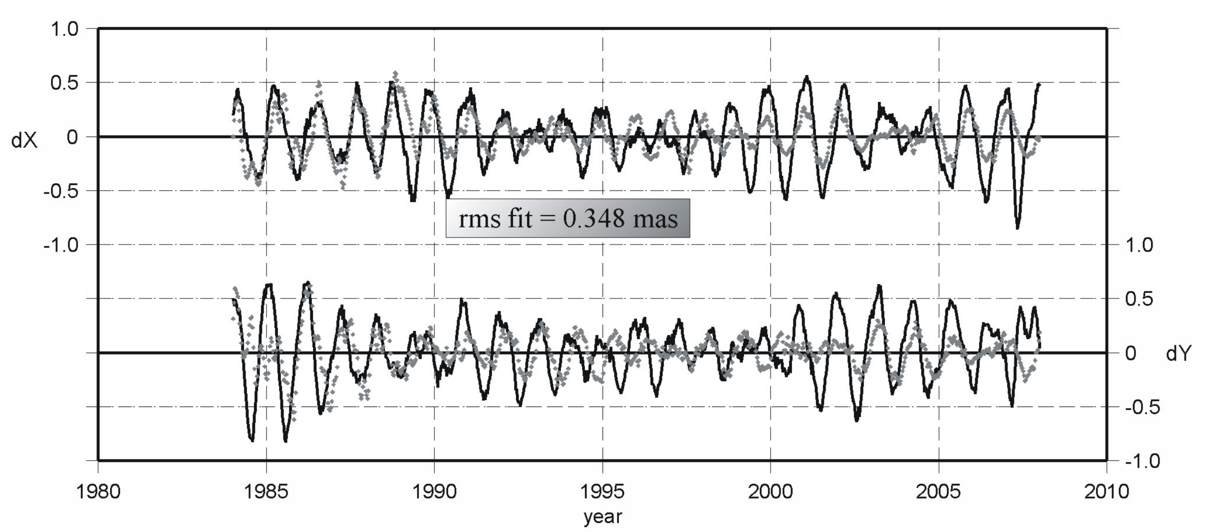

Fig. 6 Observed and integrated celestial pole offsets with ERA AAM + OMCT OAM.

integration with only NCEP atmospheric effects (pressure and wind) taken into account, in which the effect of the oceans is completely neglected. Pressure term is taken over the whole globe, and the behavior of the oceans is taken as if it was completely frozen. Figure 4 depicts the same, but with pressure term corrected for inverted barometer (IB) correction. In fact, it is a very simple model of the oceans in which they react to pressure changes inversely to barometric pressure changes, thus fully compensating its influence over the oceans, for all frequencies. In Figure 5 the NCEP atmosphere is combined with oceanic ECCO model, and finally Figure 6 shows the same, but integrated with ERA model of the atmosphere combined with OMCT model of the oceans (notice that the change from re-analysis to operational model at 2001.0 does not introduce any visible step).

Thus, we have four different time series describing the effects of geophysical excitation in nutation. We removed these integrated values from VLBI observations (this time we use only the combined IVS solution since the other ones contain potentially the same information, as demonstrated in Figures 1 and 2), instead of MHB Sun-synchronous correction, and repeated the same estimation of P, Q in six-year running intervals as in Section 2. The results are shown in Figures 7 and 8. In both figures, the lines denoted as MHB Sun-synchr. are identical with the lines IVS (combined) of Figures 1 and 2.

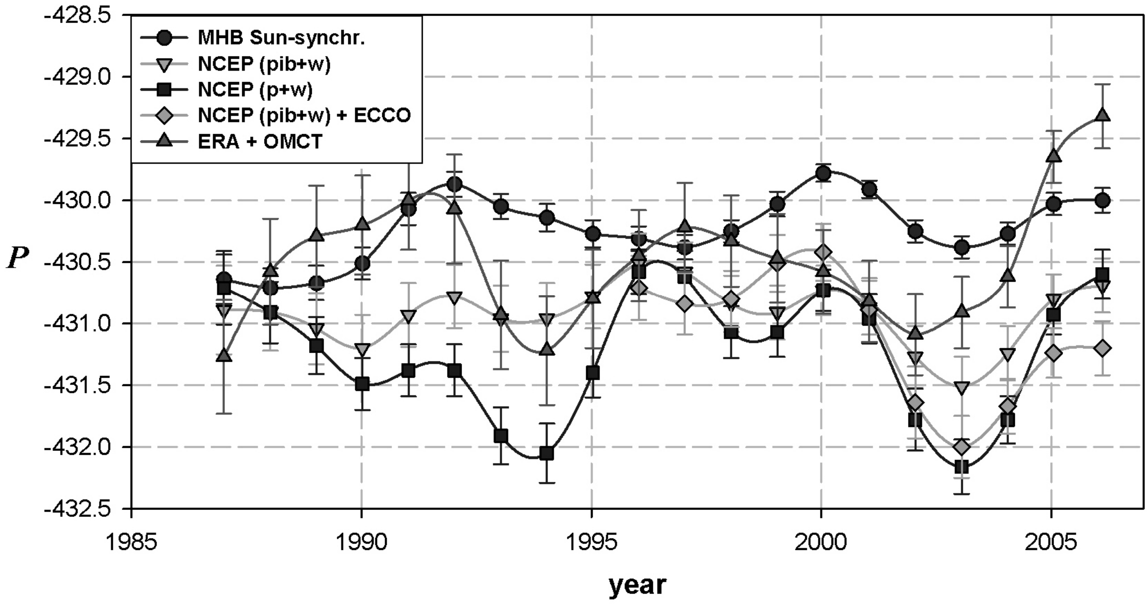

Fig. 7 Period *P* of Free Core Nutation from VLBI solutions, with geophysical excitations removed.

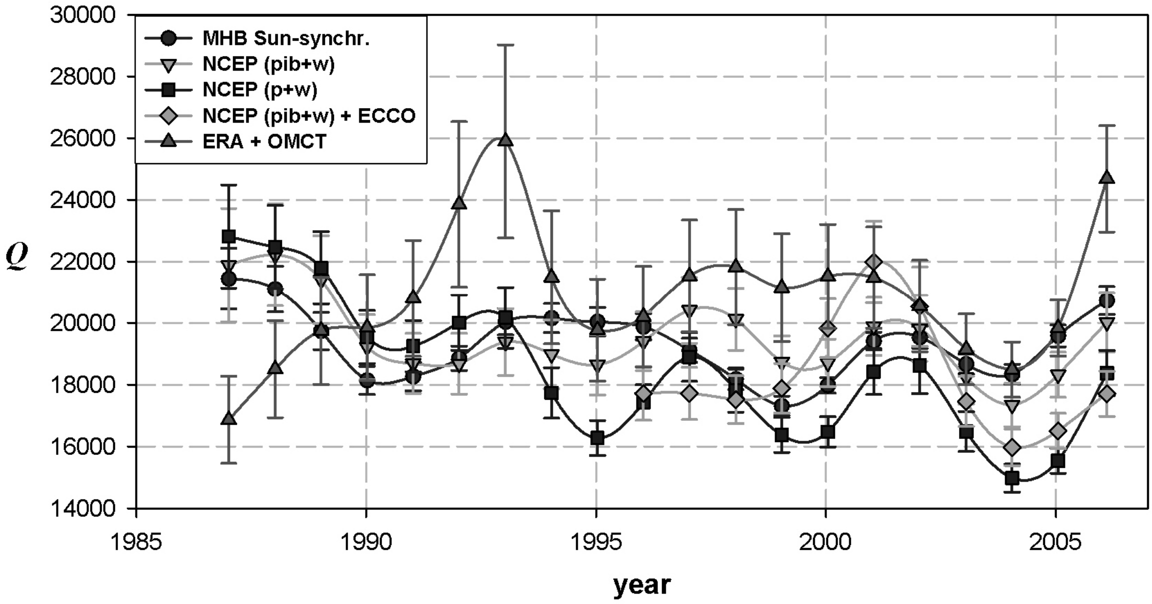

Fig. 8 Quality factor *Q* of Free Core Nutation from VLBI solutions, with geophysical excitations removed.

From Figures 7 and 8 we see that using the real atmospheric and oceanic excitations instead of MHB model does not improve the temporal stability of the results, and different models of atmosphere/ocean yield results that differ significantly. This also demonstrates how much the atmospheric and oceanic excitations at near-diurnal band from different agencies diverge. It is in agreement with the findings by Yseboodt at al. (2002) that different agencies providing atmospheric excitations differ significantly at near-diurnal band.

4. CONCLUSIONS

We demonstrated that all VLBI solutions yield similarly varying values of estimated period *P* and quality factor *Q* of Free Core Nutation. The agreement among different IVS analysis centers is very good, the differences do not exceed their formal uncertainties. These quasi-periodic variations keep within the limit of 429.8 – 430.8 solar days in case in the period, and 17000 – 21000 in case of the quality factor. Both temporal stability of the results and agreement among different analysis centers seem to improve in time.

Forced nutations due to excitation by the atmosphere and ocean are significant, especially at annual and semi-annual periods, but different models give significantly different results. The best agreement with VLBI-based celestial pole offsets is obtained for atmospheric excitation with IB correction (see Fig. 4). If these excited nutations are used instead of MHB correction, and new computation of P, Q is made, a systematically longer period of FCN is obtained for all models used (by about 0.5 d), but no systematic change of quality factor is observed. The temporal variations of P, Q are thus heavily dependent on the removed AAM/OAM effect from different sources. Again, the atmospheric excitation with IB correction leads to the smallest temporal variations of P, Q (see Figs. 7 and 8); on the other hand, if IB correction is not applied, the variations become very large. This is somehow surprising, since it is generally admitted that the redistribution of oceanic water due to atmospheric pressure changes takes a few days. However, the variations of P, Q never disappear, no matter which atmospheric/oceanic model is used. We conclude that the current AAM and OAM data, at the present state of their modeling, are not able to explain the observed variations of P and Q. The role of so far unknown processes (as. e.g., fluctuations of the core flattening caused by mantle convection at core-mantle boundary) is open.

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