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COMBINATION OF DIFFERENT SPACE GEODESY TECHNIQUES FOR EOP AND TERRESTRIAL REFERENCE FRAME DETERMINATION

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ABSTRACT
The combination method of results of different space geodetic techniques gives two kinds of products. On the one hand, the Earth orientation parameters (EOP) that define the orientation of the Earth in space and, on the other, the coordinates of collocation stations by them the ITRF is realized. Obtained results are based on the method developed by authors, so called "non-rigorous" combination of the data. Approximately eight-year data was successively processed in order to obtain solutions of both products, which were then compared with the results given in ITRF 2005 solution.

KEYWORDS: Earth orientation parameters, combination of space geodesy techniques, station coordinates, ITRF 2005

1. INTRODUCTION

The body of the Earth is changing its orientation in the space. The orientation can be defined by many ways, for example, by three Euler's angles. This way is not very practical because of quick changing of the values of these angles. The best manner is using five rotation angles, called Earth orientation parameters, EOP, which tie the Earth-fixed coordinate system ITRS to the celestial reference frame. The EOP are two coordinates of the intermediate pole with respect to the ITRS, x_P, y_P, and angle, which characterizes irregularity of the Earth's proper rotation, ERA, and, finally, two components of the celestial pole offset, dX, dY, which denote the observed corrections to the adopted precession-nutation model.

Historically, the orientation of the Earth was determined from conventional astronomical measurements. In the recent decades, however, a number of sophisticated space geodetic techniques replaced the older optical methods to produce well-resolved and highly accurate values of EOP.

These techniques are: the Global Position System (GPS), Very Long-Baseline Interferometry (VLBI), Satellite and Lunar Laser Ranging (SLR, LLR) and Doppler Orbitography and Radio-positioning Integrated by Satellite (DORIS). Each technique has analytical center (in the frame of IERS – International Earth Rotation and Coordinate Systems Center) which publish their products, primarily, EOP and station coordinates.

There are basically two possibilities of how to derive one representative set of EOP and station coordinates from all the contributing techniques.

- The rigorous approach needs either to process the original data at a) the level of observation equations or b) to solve a new system of normal equations created from the normal equations or covariance matrices of individual techniques (*e.g.* Gambis et al., 2006). It yields exact solution so that efforts are made by several groups to develop the necessary algorithms, but:
  The first approach is very complicated. Observation equations are complex and additional "inner" unknowns are necessary. Up to now, it has only been tested on a very limited network.
  The " normal equation" approach is apparently simpler but even in this case the problem of applying properly all specific constraints to the new system to improve its generally lower stability remains still open.
- It is also possible to derive an approximate solution by combining the *results* of individual techniques omitting co-variances, i.e. omitting interrelations between the input parameters,

which are treated as independent. We use this so called "non-rigorous approach" because it yields a stable solution, if some simple constraints are applied.

2. NON-RIGOROUS COMBINATION METHOD

The basic idea of the method is to combine station position vectors in the celestial reference frame (Pešek and Kostelecký, 2006). In the celestial frame, the station position vectors are function of all unknowns to be solved. Hence, it is suitable for a common adjustment. All techniques aspire to relate their results to the same reference frame. Nevertheless, small deflections still exist. Thus for each technique, a set of parameters **p** of a seven-parametric transformation is derived, instead of individual station coordinates, which makes combination more stable.

The transformation from ITRS to GCRS, i.e. $\mathbf{x}_T \rightarrow \mathbf{x}_C$, in the concept of non-rotating origins, then reads:

$$\mathbf{x}_C = \mathbf{Q}(t)\mathbf{R_3}(\mathrm{ERA})\mathbf{R_3}(-s')\mathbf{R_1}(y_P)\mathbf{R_2}(x_P)\mathbf{R}(\mathbf{p})\,\mathbf{x}_T \ . \tag{1}$$

Here denote $\mathbf{Q}(t)$ the precession-nutation matrix, which is a function of the celestial pole offset, X, Y. ERA is the Earth rotation angle, which is a linear function of UT1, s' shifts from ITRF x axis to the terrestrial non-rotating origin, TIO, along the intermediate equator, and x_P, y_P are coordinates of the CIP pole. $\mathbf{R_i}$ is matrix of rotation along the i-th axis. Finally, $\mathbf{R(p)}$ is matrix of the seven-parametric transformation, which is considered be a linear function of time,

$$\mathbf{p} = \mathbf{p}(t_0) + \dot{\mathbf{p}}(t - t_0)$$
$$\mathbf{p}(t_0) = p_1, \ldots\ldots, p_7 \text{ , and } \quad \dot{\mathbf{p}} = \dot{p}_1, \ldots\ldots, \dot{p}_7 \, .$$

Input data for the combination consists of ***M*** sets of EOP (x_P, y_P, UT1-UTC, X, $Y)_{\boldsymbol{m}}$ and corresponding sets of station coordinates $(x)_{\boldsymbol{m}}$, $m = 1, \ldots, M$, as derived by analysis centers for individual techniques.

Partial derivatives of the formula (1) with respect to any unknown, U, yields observation equations of the form:

$$\sum_j \frac{\partial x_C}{\partial U_j} dU_j \quad = \quad x_C \big|_{obs} - x_C \big|_0 \quad + \quad v \ , \tag{2}$$

where the "observed" vectors x_C are calculated from the respective input data and $x_C\big|_0$ are functions of adopted *a priori* values of the unknowns and v is residual. The unknowns are:

- daily values of x_P, y_P
- daily values of (UT1 – UTC)
- values of seven-parameter transformation $\mathbf{p}(t_0)$ and $\dot{\mathbf{p}}$ for each technique, determined once for a whole period of processing. Station positions change very slowly so that the rates $\dot{\mathbf{p}}$ are included only in the case of longer data span, when they can be derived reliably.

The EOP are calculated for each individual epoch independently of the others. As a consequence, errors in the input data, including station coordinates, are transferred to the EOP and increase their scatter substantially. The effect can be reduced by including constrains, in the form of additional observation equations (pseudo-observations) which are based on Vondrák (1977) smoothing, and have a form of the third derivative of third-order Lagrange polynomial $L_i(x)$,

$$L_i''' = \sum_{k=0}^{3} \left(6 \prod_{j=0, j\neq k}^{3} \frac{1}{\left(x_{i+k} - x_{i+j} \right)} \right) E_{i+k} \ .$$

They tie values of the respective EOP, E, at four adjacent epochs. The constraints were weighted to retain in the solution as much as 99% of the signal with period greater than 5 days.

To remove singularity of the system (2), a no net-rotation constraint, minimizing mutual shifts and preserving the system as a whole, has to be introduced,

$$\sum \mathbf{p}^T \mathbf{p} = \min \ , \tag{3}$$

which stabilizes calculation of the station coordinates.

The system of observation equations and all additional constraints is solved using modified Cholesky decomposition proposed by Čepek (2005). It can solve the sparse matrix of normal equations very effectively, but the constraints (3) have to be transformed to pseudo-observations instead of the rigorous constraints as used in Štefka et al. (2007). Effect of the pseudo-observations is quite insensitive of the weights, provided the weights exceed some critical level. Weight 10^6 was used in this case.

3. DATA

We used the following data covering the period 2000–2008: GPS and VLBI data were taken from the IERS Combination Pilot Project database (Data-I). For SLR, the constrained *ilrsb* solution was used, as published by ILRS analysis center (Data-II). The GPS and SLR data is weekly SINEX (Solution in Independent Exchange format) solutions, from which the EOP and station coordinates were extracted. VLBI data consists of *per seance* singular normal equation matrices. They were regularized by constraining the station coordinates to the VTRF 2005 frame (Nothnagel, 2005) with the a priori precision of 5 mm. As none of the techniques currently provides celestial

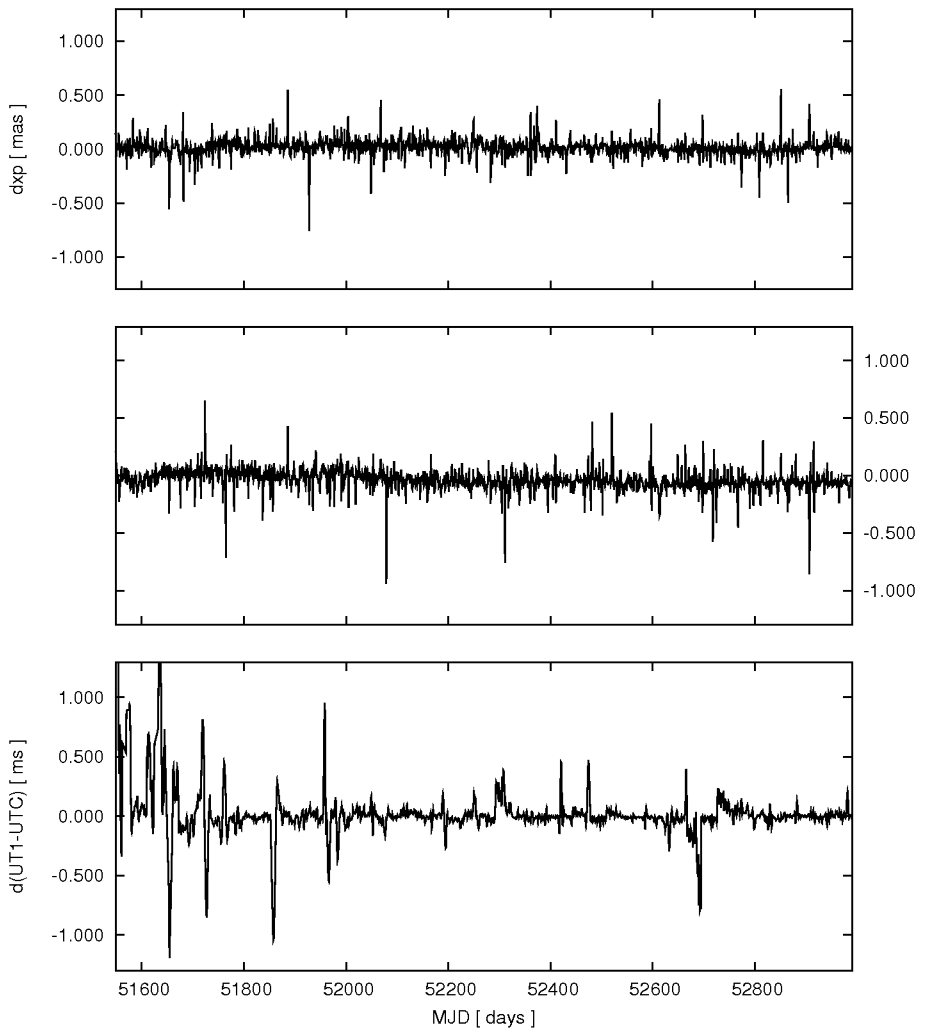

Fig. 1 Monthly solutions over the eight-year period were merged together in order to compare them with the ITRF 2005 solution of EOP. Mean squared differences are 0.148 mas, 0.154 mas and 0.162 ms for the polar motion x_P (top), y_P (center) and the time correction UT1−UTC (bottom), respectively.

pole offset, only the x_P, y_P, and UT1-UTC are solved for.

ITRF 2005 (Altamimi at al., 2007) is used as a representative solution for comparison of our results as it gives a possibility to compare both EOP and station coordinates.

4. NUMERICAL SOLUTION

We used the eight-years data in two different ways. Firstly, the data was successively processed in order to obtain monthly solutions of both products (EOP and **p**). Secondly, we adjusted all the data in one step to obtain one common solution over the whole time period, of EOP, **p** and $\dot{\mathbf{p}}$.

The techniques enter the adjustment with following weights: 1.44 for GPS, 0.8 for SLR, and 1.0 for VLBI. The values do not differ very much from those applied to the IERS Dynamo program (Richard et al., 2008). Thus we used our values to keep continuity with our previous analyses.

Monthly solutions of EOP were merged together and compared with the common solution. As a result of choosing the same level of smoothness, the differences were negligible so that we only use the monthly solution for comparisons with the ITRF series of EOP. The comparison is depicted in Figure 1 where only part of our solution could be compared because the ITRF ends in 2005. Except a few peaks exceeding the level of 1 mas and 1.5 ms, the differences are smaller then 0.5 mas and 1.0 ms for polar motion and time correction, respectively.

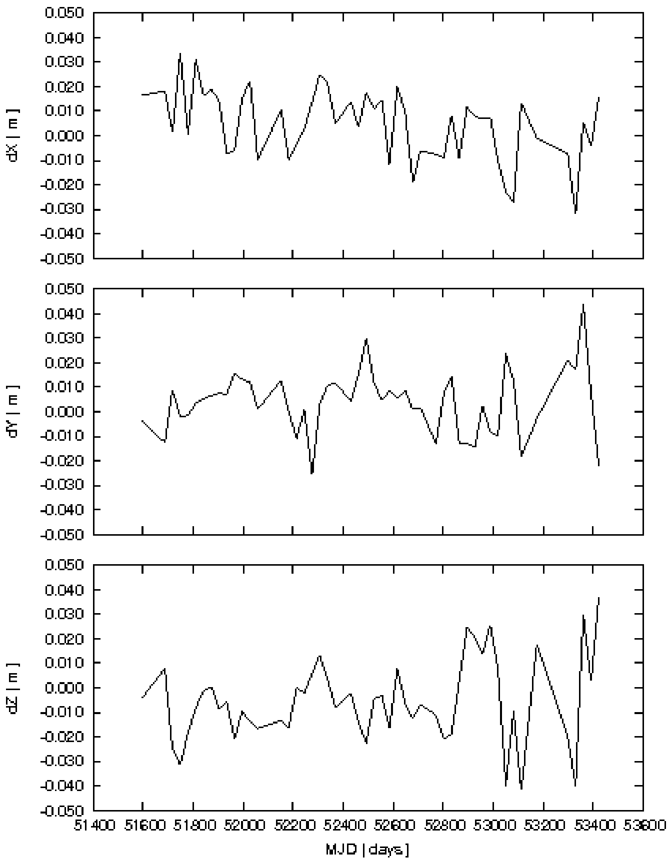

Fig. 2 Example of station coordinates evolution. For each epoch of the monthly solution, the input coordinates of SLR station 21605 were transformed using transformation parameters **p** as obtained from the monthly and the common solutions. Displayed are differences between the two solutions, in the sense 'monthly minus common'. Differences rise primarily from changing station number and distribution in the individual monthly solutions. Rms of differences is 0.010 m.

As example of station coordinates evolution, station 21605 is shown in Figure 2. Two coordinate time series correspond to the monthly and common solutions. The rms of differences between the two series was about 0.01 m. It was due mainly to several peaks appearing in monthly solutions, which were caused by bad station distributions. After removing those peaks, the rms decreased to a level of 0.005 m. Further, the monthly solutions are used since they can show more details of the evolution.

From the monthly solutions, final coordinates for each collocation station were computed as a weighted mean of transformed coordinates of the techniques contributing to the particular solution. From them, biases and linear trends were computed relative to the epoch J2000. Data for stations 21701 and 66008 were not long enough to enable computation of trends and biases with satisfactory precision. Other two stations, 40408 and 42202, show a stepwise change of coordinates, for which we could not get appropriate local ties. Thus these stations were excluded prior to the final comparison.

Comparison with ITRF 2005 is shown in Figures 3 and 4. Figure 3 shows differences of individual station coordinates between our solution and ITRF 2005 at boundary and middle epochs, i.e. MJD 51550, 52980 and 54400. The inconsistency between both solutions is of the level of 0.020 m.

Figure 4 displays comparison of station coordinates evolution at the station 12205, equipped by one GPS and one SLR instrument, with the ITRF positions and velocities. The differences vary slightly in time. Calculated for the beginning, the middle and the end of the data they are: 0.015 m, +0.005 m, –0.006 m in x, –0.005 m, –0.004 m, –0.005 m in y, and +0.004 m, +0.004 m and +0.004 m in z.

The comparisons are affected not only by differences in the combination algotithms. ITRF

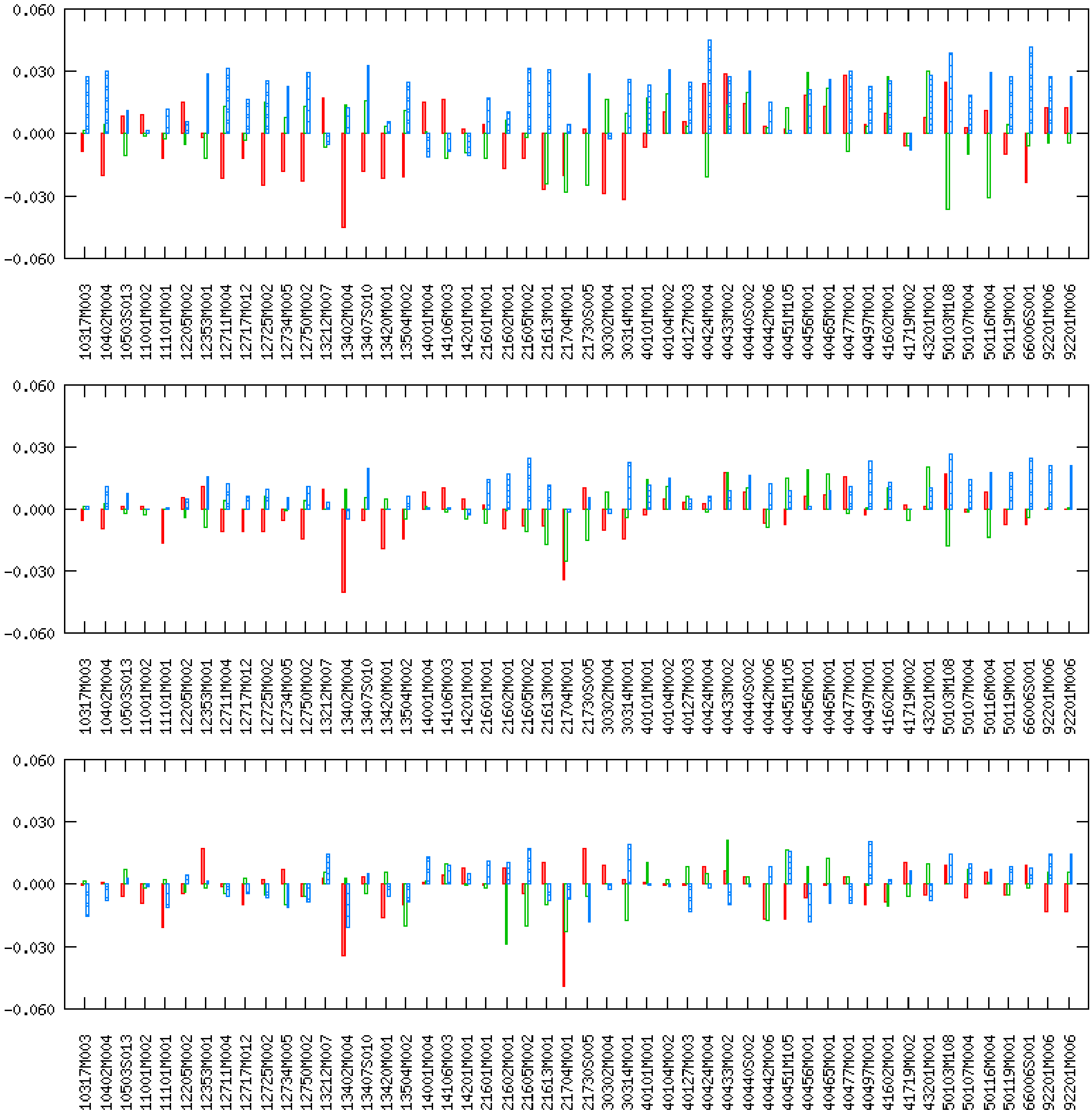

Fig. 3 Station coordinates differences between the present and ITRF 2005 at the beginning (MJD 51550), middle (MJD 52980) and the end (MJD 54400) of the eight-year period. (x red, y green, z blue. Differences in metres).

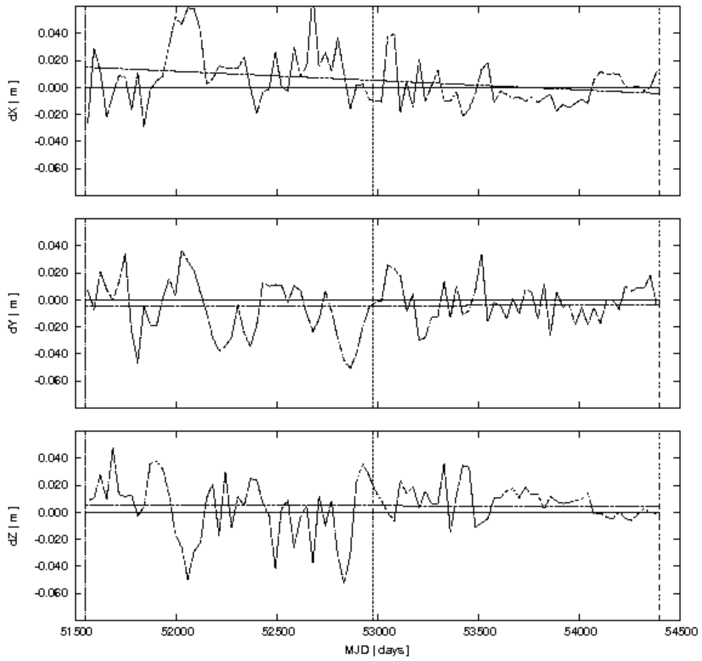

Fig. 4 Detailed view of coordinate differences from ITRF 2005 during the whole eight-year period, of the station 12205. Positions obtained from individual monthly solutions (dotted line) are approximated as a sum of J2000 position and linear drift (dash-dotted), and referred to ITRF 2005 positions (full line). Vertical lines correspond to epochs in Figure 3.

solution is based on much bigger amount of observations covering longer period of time. Thus, the two solutions can be treated as independent and their good agreement considered as a check of our results.

5. CONCLUSIONS

A method for non-rigorous combination of results of different space geodesy techniques to obtain representative sets of the Earth orientation parameters and station coordinates was used to process approximately eight-year data in two different ways (monthly solutions and a common solution of all data in one step). Their comparison turned out that the common solution is very similar to the monthly solutions except a few peaks due to worse station distribution. Hence, the monthly solution was used to compare EOP and transformed station position vectors with ITRF 2005.

The comparison of EOP yields rms of mutual differences 0.148 mas, 0.154 mas and 0.162 ms for the polar motion x_P, y_P and UT1–UTC, respectively. Comparison of the station coordinates is displayed in Fig. 3 for epochs 51150, 52980 and 54400, which correspond to the beginning, middle and the end of the analyzed period. It can be clearly seen that our solution of station coordinates fits with ITRF 2005 fairly well.

We can conclude that the presented method of non-regular combinations or results of space geodesy techniques is suitable for testing results of regular combinations, at least at the early stages of their development.

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