STUDY OF ATMOSPHERIC AND OCEANIC EXCITATIONS IN THE MOTION OF EARTH'S SPIN AXIS IN SPACE

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ABSTRACT
In this review paper we study the atmospheric and oceanic effects in nutation. It is a continuation and summary of our previous studies that we made during the last five years or so. We use slightly modified methods and apply them to the most recent data (both atmospheric/oceanic excitation functions and combined solution of celestial pole offsets by International VLBI Service for Geodesy and Astrometry - IVS). We find that the atmospheric and oceanic excitations provide significant changes in nutation, mostly with annual and semi-annual periods. The numerical integration of Brzeziński’s broadband Liouville equations yields Free Core Nutation (FCN) that is consistent with VLBI-based observed values. The analysis of VLBI observations shows small quasi-periodic fluctuations of the period and quality factor of retrograde FCN, ranging between 429.8 to 430.5 days and 17000 to 22000, respectively. To this end, we use resonant effects in several dominant forced nutation terms to calculate the period and quality factor of FCN in running 6-year intervals. Numerically integrated geophysical excitations are removed from the observed celestial pole offsets, and the remaining part is used again to derive the period and quality factor of FCN in running intervals. Our conclusion is that the observed quasi-periodic variations of both parameters are not caused by these geophysical excitations, but another source should be searched for.

KEYWORDS: Earth orientation, nutation, geophysical excitation, VLBI

1. INTRODUCTION
There are five Earth Orientation Parameters (EOP) that describe the full orientation of the Earth in space: two components of polar motion (fixing the position of the spin axis in the body), the angle of proper rotation around the spin axis, and two components of the celestial pole offsets (describing the misalignment of the spin axis with its position in space given by a precession-nutation model). In this study, we concentrate on the latter two. Thanks to a very accurate present IAU models of nutation (IAU2000 – Mathews et al., 2002) and precession (IAU2006 – Capitaine et al., 2003), these values are very small, typically around 0.1 milliarcsecond (mas), their dominant part being the retrograde Free Core Nutation (FCN) with the period of about 430 days. This motion is due to a strong resonance in a near-diurnal part of the spectrum (in terrestrial frame) in celestial motion of the Earth’s spin axis, caused by the flattened fluid outer core of the Earth. This resonance, together with a partial viscosity of the mantle, leads to a significant modification of amplitudes and phases of the forced (by external torques exerted by the Moon, Sun and planets) nutation terms, with respect to the solution calculated for a rigid Earth model. The largest influence is observed for the retrograde annual term that is closest to the resonance.

Atmospheric and oceanic excitations have dominant effects in polar motion and proper rotation of the Earth, with significant seasonal periods in terrestrial frame. Their near-diurnal part, which is reflected into long periods in celestial frame, is rather weak. However, these small excitations are amplified by a resonance so that they become detectable with the recent observational techniques. Availability of atmospheric and oceanic excitation functions, defined by Barnes et al. (1983), with 6-hour resolution (that appeared more than ten years ago) enabled recently to study these effects in detail (see, e.g., Bizouard et al., 1998; Bizouard, 1999; Yseboodt et al., 2002; Brzeziński et al., 2002; Lambert, 2006; Vondrák and Ron, 2006 a, b, 2007, and 2008, or Vondrák, 2009). These studies show that the effect is most significant in annual and semi-annual terms of nutation, and is of the order of a hundred microarcseconds (µas). The first part of the present study is devoted to this problem.

The value of the FCN period, used to derive the older model of nutation IAU1980, was equal to about 460 days (Wahr, 1981). It corresponded to the case where core is in hydrostatic equilibrium. The observations by Very Long-Baseline Interferometry (VLBI), so far the only observational technique capable of observing a celestial motion of the Earth
spin axis with a sufficient accuracy, revealed that the period is somewhat smaller, only about 430 days (Roosbeek et al., 1999; Hinderer et al., 2000; Mathews et al., 2002; Vondrák and Ron, 2005; Vondrák et al., 2005). This corresponds to the core’s flattening about 4 per cent larger than the hydrostatic equilibrium value (Herring et al., 1986; Gwinn et al., 1986). The question if the period \( P \) and quality factor \( Q \) of FCN are stable in time was recently addressed, e.g., by Lambert and Dehant (2007). They concluded that the resonant period is stable within less than half a day, but the differences in approach of different analysis VLBI centers have an impact of the same order of magnitude. They also found that the contribution of the atmosphere is negligible. We addressed the same problem recently (Vondrák and Ron, 2009) and found that all IVS analysis centers yield similar quasi-periodic changes that do not disappear when atmospheric and oceanic contributions are removed. In addition, different models of atmosphere/ocean forcing result in systematically different results. Here, we re-visit the problem of stability of \( P \) and \( Q \) and study the role of atmospheric/oceanic excitations in more detail.

2. THE DATA

In this work, we use the following data:

- Since different IVS analysis centers yield very similar results (Vondrák and Ron, 2009), we use the values of celestial pole offsets (CPO) only from the most recent IVS combined solution (Schlüter and Behrend, 2007) \texttt{ivs09q3X.eops}, covering the interval 1984.1–2009.7. The data in the form \( dX, dY \) are given in unequally spaced intervals, 1–7 days long, sometimes with large outliers. Therefore, we first cleaned the data by removing CPO values exceeding in absolute value 1 mas, and then interpolated them to regular 3-day intervals, by using a weak smoothing and cubic spline function.

- Atmospheric and oceanic angular momentum functions are now available from different sources. Since the most important part of excitation is in near-diurnal part of the spectrum (in terrestrial frame), we need the data sampled with sub-diurnal interval. We use the following data, available in 6-hour intervals directly from the authors:
  - Atmospheric angular momentum excitation functions (pressure + wind terms):
    - NCEP/NCAR re-analysis, 1983.0–2009.5 (Salstein, 2005);
    - ERA, 1979.0–2009.0 (Thomas et al., 2006; Dobslaw and Thomas, 2007, driven by re-analysis atmospheric model before 2001.0, by operational model afterwards);
  - Oceanic angular momentum excitation functions (matter + motion terms):
    - ECCO model, 1993.0–2009.7 (Gross et al., 2005);
    - OMCT model, 1979.0–2009.0 (Thomas et al., 2006; Dobslaw and Thomas, 2007, driven by re-analysis atmospheric model before 2001.0, by operational model afterwards).

These time series of \( \chi \) (complex numbers) are given in terrestrial frame, so we have to transform them into the celestial (non-rotating) frame values \( \chi' \). To do so, we first removed a constant part, which would lead to a big diurnal signal (with the period equal to a sidereal day) in celestial frame. Then a relatively simple formula in complex form \( \chi' = -\chi e^{i\phi} \) is applied, where \( \phi \) is the Greenwich sidereal time. The near-diurnal variations in terrestrial frame become long-periodic in celestial frame. Because we are interested in only long-periodic motion, that is comparable to nutation frequencies, we further apply the smoothing (Vondrák, 1977) to remove all periods shorter than 10 days. It is also necessary to say that the ECCO model is forced by the NCEP/NCAR data, so these two should be used together; a similar argument holds also for OMCT/ERA combination.

3. OBSERVED CELESTIAL POLE OFFSETS AND THEIR EXCITATION

The observed celestial pole offsets in mas, brought to regular 3-day intervals, are depicted in Figure 1. Both components display a clear quasi-periodic character, the accuracy of the results are evidently improving in time. The FFT spectral analysis, shown in Figure 2 reveals several dominant peaks. The largest one corresponds to the retrograde FCN; its double peak is the sign of the fact that both amplitude and phase of this motion are unstable. Significant are also long-periodic motions with periods of about 14 and 28 years. These are probably caused by mismodeled long-periodic nutation, but surely not by geophysical fluids (see below). Smaller peaks at prograde and retrograde annual (and to some extent also at semi-annual and terannual) periods reflect small differences from the adopted nutation model. These can be caused either by deficiency of the nutation model itself, or by additional geophysical excitations.

The spectra of atmospheric and oceanic angular momentum excitation functions are depicted in Figures 3 through 6. If we compare the first two, showing the atmospheric excitations from NCEP and ERA, there are practically no difference. The most prominent peaks are at prograde annual and semi-annual periods, less visible are retrograde excitations. Wind terms are much larger than the pressure ones. The situation is different for oceanic excitations, displayed in Figures 5 and 6. The peaks are smaller than for the atmosphere, and the differences between matter (equivalent to pressure) and motion (equivalent to wind) terms are almost negligible. OMCT peaks are
Fig. 1 Combined IVS celestial pole offsets, cleaned and interpolated to 3-day intervals.

Fig. 2 Spectrum of IVS celestial pole offsets.

Fig. 3 Spectrum of NCEP atmospheric excitations in celestial frame.
Fig. 4  Spectrum of ERA atmospheric excitations in celestial frame.

Fig. 5  Spectrum of ECCO oceanic excitations in celestial frame.

Fig. 6  Spectrum of OMCT oceanic excitations in celestial frame.
Table 1 Atmospheric and oceanic contribution in nutation [μas], calculated by convolution with Brzeziński transfer function (1).

<table>
<thead>
<tr>
<th>Excitation</th>
<th>Annual (l')</th>
<th>Semi-annual (2F–2D+2Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>prograde Re</td>
<td>prograde Im</td>
</tr>
<tr>
<td>NCEP+ECCO p</td>
<td>–62.0</td>
<td>–96.9</td>
</tr>
<tr>
<td>w</td>
<td>3.5</td>
<td>–8.5</td>
</tr>
<tr>
<td>sum</td>
<td>–58.5</td>
<td>–78.4</td>
</tr>
<tr>
<td>ERA+OMCT p</td>
<td>–141.2</td>
<td>–46.8</td>
</tr>
<tr>
<td>w</td>
<td>10.9</td>
<td>–6.1</td>
</tr>
<tr>
<td>sum</td>
<td>–130.3</td>
<td>–52.9</td>
</tr>
</tbody>
</table>

Two additional sets of data (NCEP+ECCO) and (ERA+OMCT) are shown in Figures 7 (NCEP+ECCO) and 8 (ERA+OMCT). The matter and motion terms are shown separately to demonstrate the dominance of the matter terms. Comparison of both graphs reveals that the amplitudes are significantly higher in the latter case, due to a larger oceanic OMCT contribution.

We can use Brzeziński transfer function (1) also to estimate the atmospheric and oceanic contribution to individual nutation terms, by estimating the most significant terms of the excitation by the least-squares method and then multiplying them by \( T(σ) \). We can only argue which arguments to take for the dominant annual and semi-annual terms. In principle, the atmospheric and oceanic excitation should be connected with changing seasons, i.e., with the tropical year. On the other hand, for practical reasons, they should be identified with the known dominant station terms, in order to facilitate their mutual combination. Here we take the nutation arguments \( l' \) (with the period 365.26d), and \( 2F–2D+2Ω \) (with the period 182.62d) that are both very close to the tropical year and its half. To convert the estimated cosine/sine terms in \( X, Y \) into prograde/retrograde terms, we use the formula

\[
C^+ = \frac{1}{2} [S_x - C_y + i(C_x + S_y)],
\]

\[
C^- = \frac{1}{2} [-S_x - C_y + i(C_x - S_y)].
\]

The results, based on data span 1993.0–2009.5 (NCEP+ECCO) and 1979.0–2009.0 (ERA+OMCT), are shown in Table 1.

Another possibility of how to compare the excitations with the observed celestial pole offsets is to numerically integrate the Brzeziński broad-band Liouville equations. In complex form they read as

\[
P - i(σ_c + σ_f) \bar{P} - σ_c σ_f P = -σ_c \left( σ_f \left( \chi_p' + \chi_w' \right) + σ_c \left( a_p \chi_p' + a_w \chi_w' \right) \right) + i \left( (1 + a_p) \chi_p' + (1 + a_w) \chi_w' \right)
\]

where \( P = dX + idY \) is the excited motion of Earth's spin axis in celestial frame.
The numerical integration of Eq. (3) is done by the fourth-order Runge-Kutta method with 6-hour step. We use the procedure \( \text{rk4} \) from Numerical Recipes (Press et al., 1992) that we modified to our purpose by rewriting it into the complex form. To obtain two first-order equations, instead of a second-order one given by Eq. (3), we use the substitution

\[ y_1 = P, \quad y_2 = \dot{P} - i\sigma'_c P, \]

which leads to differential equations for two complex functions \( y_1, y_2 \):

\[
\begin{align*}
y_1' &= 1\sigma'_c y_1 + y_2 \\
y_2' &= 1\sigma'_c y_2 - \sigma_c \left( \frac{\sigma_f'}{\sigma_f} \left( \chi'_p + \chi'_w \right) + +\sigma_c \left( a_p\chi'_p + a_w\chi'_w \right) + \right. \\
&\left. +i \left( (1 + a_p)\chi'_p + (1 + a_w)\chi'_w \right) \right)
\end{align*}
\]

The solution generally yields two free damped circular motions: a rapid prograde Chandler wobble and a slow retrograde FCN with celestial frequencies \( \sigma'_c \) and \( \sigma'_f \), respectively. To integrate the system (4), we need to choose the initial values, that is two complex constants, defining the amplitudes and phases of both free motions. We are not interested in rapid (nearly diurnal) Chandlerian motion, so we choose only one, pole position at initial epoch \( P_0 \); its first derivative is constrained so that the Chandlerian amplitude disappears. This is easily assured by choosing the values \( y_1(0) = P_0, \quad y_2(0) = i(\sigma'_f - \sigma'_c)P_0 \). The final choice of \( P_0 \) is made by repeating the integration with different values \( P_0 \) until the fit of the integrated motion to VLBI observations reaches a minimum. The results of these integrations are displayed in Figure 9 (for NCEP+ECCO) and 10 (for ERA+OMCT) as full black lines, in both cases shown together with VLBI-based celestial pole offsets (gray points). Even if we have the data in a longer interval for ERA+OMCT, we use the interval of only 1993.0–2009.0 to facilitate the comparison of both integrations. Annual and semi-annual parts were removed from both series, and also long-periodic component from IVS, so that they represent practically only the FCN. Both figures show that the integration follows more or less the same pattern as the observations; the rms fit is slightly better for NCEP+ECCO. The amplitudes seem to be too high for ERA+OMCT. The integrated series can be used to estimate the annual and semi-annual sine/cosine terms. Using formula (2) we arrive again to prograde and retrograde nutations that are displayed in Table 2. It is necessary to say that the whole intervals common for IVS and geophysical fluids are used here, i.e., 1993.0–2009.5 for NCEP+ECCO and 1984.1–2009.0 for ERA+OMCT. Theoretically, the results should be identical with those shown in Table 1 (rows in bold).

### Table 2: Atmospheric and oceanic contribution in nutation [\( \mu \)as], computed from numerically integrated broadband Liouville equations (3), and comparison with IVS-based values.

<table>
<thead>
<tr>
<th>excitation</th>
<th>Annual (( l' ))</th>
<th>Semi-annual (( 2F−2D+2\Omega ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Re</td>
<td>Im</td>
</tr>
<tr>
<td>NCEP+ECCO</td>
<td>−58.6</td>
<td>87.8</td>
</tr>
<tr>
<td>ERA+OMCT</td>
<td>−126.7</td>
<td>97.6</td>
</tr>
<tr>
<td>IVS</td>
<td>−22.5</td>
<td>104.7</td>
</tr>
</tbody>
</table>
Fig. 8 Spectrum of atmospheric and oceanic excitation (ERA + OMCT), convoluted with Brzeziński transfer function.

last row, the estimated nutation corrections from IVS solution, in which we also considered that the IAU2000 model of nutation implicitly contains empirical, so called Sun-synchronous correction to prograde annual term, that is equal to \((-0.0104+0.1082i)\) mas. This value was added to the term estimated from IVS, to be directly comparable with the atmospheric contribution.

The amplitudes are mostly similar, but there are larger differences in some cases, e.g., for retrograde annual term or prograde semi-annual term.

4. RESONANCES IN EARTH ROTATION AND ESTIMATION OF $P$, $Q$

The resonances in Earth orientation are most completely described by Mathews et al. (2002). They derived so called Mathews-Herring-Buffet (MHB) transfer function in the form

$$T(\sigma) = \frac{e_R - \sigma}{e_R + 1} N_0 \left[ 1 + (1 + \sigma) \left( Q_0 + \sum_{j=1}^{4} \frac{Q_j}{\sigma - s_j} \right) \right],$$

(5)

which expresses the ratio of the non-rigid amplitude of a forced nutation term with terrestrial frequency $\sigma$ (in cycles per sidereal day – cpsd) to its rigid Earth value. Here $e_R$ denotes the dynamical ellipticity of the rigid Earth used to compute the rigid solution, $N_0$, $Q_j$ are complex “strength” parameters, and $s_j$ are four complex resonance frequencies corresponding to Chandler Wobble (CW, with terrestrial period of about 435 days), retrograde FCN (with celestial period of about 430 days), Prograde Free Core Nutation (PFCN, with celestial period of about 1020 days) and Inner Core Wobble (ICW, with terrestrial period of about 2400 days), respectively. In our study, only $s_2 \approx -1.0023$ cpsd (FCN frequency) is important since it is close to the frequencies of all nutation terms and, at the same time, the corresponding coefficient $Q_2 \approx -4.89 \times 10^{-5}$ is two orders of magnitude larger than $Q_1 \approx -2.96 \times 10^{-4}$, corresponding to PFCN. Equation (5) was used, together with a rigid-Earth solution by Souchay et al. (1999) to derive the presently adopted model of nutation IAU2000.

MHB transfer function (5) can be used to derive the parameters on its right-hand-side, provided the value $T(\sigma)$ is known for several different frequencies $\sigma$ from the observations (for more details see, e.g., Vondrak et al., 2005). Here we use the amplitudes and phases of five dominant nutation terms (with periods 365.26, 182.62, 121.75, 27.55 and 13.66 days) as determined by IVS, both for positive and negative frequencies. We estimate them in six-year running intervals, using the weights computed from VLBI formal standard errors. To account for possible longer term variations, we insert also a bias and a linear trend to the least-squares fit, and, to avoid possible aliasing, we also estimated a term with FCN period. Dividing the estimated nutation amplitudes by their rigid-Earth values by Souchay et al. (1999) we get the values $T$. It is necessary to note that MHB solution also contains a relatively simple model of atmospheric excitation in the form of so called Sun-synchronous correction (see also above). It is a prograde annual term with the amplitude of about 100 μas, obtained as an empirical correction removing obvious residuals at this frequency. We removed this correction from VLBI-based celestial pole offsets before determining the values $T$. Thus, we obtain ten different complex values of $T$ (i.e., twenty different “observables”). These, in turn, were used to estimate the complex frequency $s_2$ in a weighted least-squares solution, in six-year moving intervals; all remaining parameters of Eq. (5) were fixed to MHB values. Once we have the complex values of resonance frequency $s_2$, we can compute the period and quality factor from the simple equations

$$P = 0.99727/[\text{Re}(s_2) + 1], \quad Q = -\text{Re}(s_2)/2\text{Im}(s_2)$$

(6)

where $P$ is given in solar days.
Fig. 9  Numerically integrated (NCEP+ECCO – full line) and observed (IVS – points) celestial pole offsets, with the signal only near FCN left.

Fig. 10  Numerically integrated (ERA+OMCT – full line) and observed (IVS – points) celestial pole offsets, with the signal only near FCN left.

The results for $P$ and $Q$ values are graphically depicted in Figures 11 and 12, together with their error bars. The uncertainties are computed from the dispersion among different values of $T$ for different nutation frequencies. Full lines in both figures represent the variations based on only IVS observations and an assumption that the geophysical excitation is modeled by MHB Sun-synchronous correction. One question automatically arises: Can the observed variations of $P$ and $Q$ be caused by the influence of the atmosphere and oceans that can significantly perturb estimates of the forced nutation amplitudes? To verify this possibility, we use the time series of that part of nutation that is due to geophysical excitations, obtained in preceding section. We remove these integrated values from VLBI observations, instead of MHB Sun-synchronous correction, and repeated the same estimation of $P$, $Q$ in six-year running intervals. The plots of these results are added to Figures 11 and 12, from which we see that using the real atmospheric and oceanic excitations instead of MHB model does not improve the temporal stability of the results. Different models of atmosphere/ocean yield results that differ significantly, and also error bars are larger. From this we also see how much the atmospheric and oceanic excitations at near-diurnal band from different sources diverge. It is in agreement with the findings by Yseboodt et al. (2002) that different agencies providing atmospheric excitations differ significantly at near-diurnal band. In addition to this, the average values of $P$ and $Q$ seem to be systematically different if real geophysical excitations are used instead of MHB Sun-synchronous correction.

5. CONCLUSIONS

Forced nutations due to excitation by the atmosphere and ocean are significant, especially at annual and semi-annual periods, but different models give different results. Slightly better agreement with VLBI-based celestial pole offsets is obtained for atmospheric and oceanic excitation NCEP+ECCO, if compared with ERA+OMCT (see Figs. 9 and 10). The quasi-periodic variations of estimated period $P$ and quality factor $Q$ of Free Core Nutation keep within the limit of 429.8 – 430.5 solar days in case in the period,
and 17000 – 22000 in case of the quality factor. The observed variations exceed the formal uncertainty of the results, so they seem to be real. If the geophysically excited nutations are used instead of MHB Sun-synchronous correction in $P$ and $Q$ estimation, these variations are even larger. A systematically longer period of FCN is obtained for both models used, and systematic growth of quality factor is observed for ERA+OMCT excitations. The temporal variations of $P$, $Q$ are thus heavily dependent on the removed AAM/OAM effect from different sources. We conclude that the current AAM and OAM data, at the present state of their modeling, are not capable of explaining the observed variations of $P$ and $Q$. Other processes (as e.g., fluctuations of the core flattening caused by mantle convection at core-mantle boundary) are probably responsible for this effect.

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