EARTH ORIENTATION PARAMETERS AND STATION COORDINATES FROM SPACE GEODESY TECHNIQUES

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ABSTRACT

The orientation of the Earth in space is measured by space geodetic techniques. Each technique has its weaknesses so the best way how to get a representative solution of Earth orientation parameters is to combine all of them together using some appropriate method. There are basically two approaches, the rigorous and non-rigorous one. The method used in this paper belongs to the second category. Since 1999, when the authors Kostelecký and Pešek put basis of the combination method, the method has been modified and improved. The particular improvements are described hereafter and recent results are presented. These results of collocation station velocities are compared with the velocities published by ITRF 2005 and NUVEL-1A. The mean values of differences are 2.7 mm/y and 5.9 mm/y, respectively.

KEYWORDS: Combination method, Earth orientation parameters, tectonic model NUVEL-1A, smoothing method

1. INTRODUCTION

The body of the Earth rotates about its axis that performs nearly circular motion in the frame of inertial celestial system, with the period of about 26 000 years. This motion is called precession and it is described by theories for rotation of the flattened body. Another motion in space is called nutation whose periods, compared with precession, are much shorter, with the main period of 18.6 years.

Both motions are due to the action of the Sun, Moon and other celestial bodies. The International Astronomical Union recommends using the model of precession IAU2006 (Capitaine at al., 2003) and nutation IAU2000 (Mathews et al., 2002).

The orientation of the axis is also changing in the frame of Earth's body. This pole wandering is called polar motion and it is mainly affected by displacements of water and air masses. The largest component of this motion, Chandler's wobble, has a period of about 14 months and it corresponds to a circular motion of approximately 12 meters around the mean pole.

Apart from the previous motions, the Earth rotates around its axis with irregular velocity that is described as a time correction to the atomic time.

Then, the orientation of the Earth body is described by five parameters, EOP – Earth orientation parameters: two coordinates, x_p , y_p , of the pole with respect to the International Terrestrial Reference Frame (ITRF), proper rotation, Earth Rotation Angle (*ERA*) and two components of the celestial pole offset, dX , dY ,which denote the observed corrections to the adopted precession-nutation model.

Today, precise and actual EOP are necessarily needed for space navigation of satellites, maintaining time with Earth's rotation etc. Monitoring of EOP is provided by modern geodesy techniques: Global Position System (GPS), Very Long-Baseline Interferometry (VLBI), Satellite and Lunar Laser Ranging (SLR, LLR) and Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS).

Each technique has several analytical centers, associated to the corresponding service (in the frame of the IERS - International Earth Rotation and Reference System Service), which generates an intra-technique combined product, primarily EOP and station coordinates. The best way how to get the most representative EOP is to combine all solutions together, respecting the advantage of each technique by appropriate weighting.

There are basically two possibilities of the combination:

- The rigorous approach needs either to process the original data at a) the level of observation equations or b) to solve a new system of normal equations created from the normal equations or covariance matrices of individual techniques (e.g., Gambis et al., 2006). It yields exact solution so that efforts are made by several groups to develop the necessary procedures, but:
- The first approach is very complicated, observation equations are very complex and additional "internal" unknowns are necessary. Up to now, it has been only tested on a limited network. The "normal equation" approach is apparently simpler but even in this case the problem of applying properly all specific constraints to the new system to improve its generally lower stability remains still open.
- It is also possible to derive an appropriate solution by combining the results of individual techniques omitting co-variances, i.e. omitting interrelations between the input parameters, which are treated as independent. We use this so called "non-rigorous approach" because it yields a stable solution, if some simple constraints are applied.

The key of all combination methods are collocation stations where more than one geodetic technique observe. Results then depend on spatial distribution of the observatories and on ties between techniques used at the individual site.

The following non-rigorous method was proposed by two of the present authors (Pešek and Kostelecký, 1999). The method was tested (Pešek and Kostelecký, 2006) on one-year data measured by four space geodesy techniques (GPS, VLBI, SLR and DORIS). The results were daily EOP and monthly station coordinates. Since that time, the method has been changed several times, and described in detail elsewhere (Štefka and Pešek, 2007a; Štefka et al., 2007b and Štefka at al., 2009), where the method was applied on one-year, three-year and eight-year data, respectively.

2. NON-RIGOROUS METHOD

The basic idea of the combination is to combine station position vectors, x_c, in the celestial reference frame, where they are function of all unknowns to be solved. All techniques aspire to relate their results to the same reference frame. Nevertheless, small deflections still exist. Thus, a set of parameters of a seven-parametric transformation is derived for each technique, instead of individual station coordinates, which makes combination more stable. The equation of transformation is:

$$\mathbf{x}_C = PN(t)\mathrm{R}_3(-GTS')\mathbf{R_1}(y_P)\mathbf{R_2}(x_P)\mathbf{R}(\mathbf{p})\,\mathbf{x}_T\,, \quad (1)$$

where $PN(t)$ is precession-nutation matrix, GTS Greenwich true sidereal time, and R_i matrices of rotation around the i-th axis. Finally, x_T is station position vector in terrestrial frame and $R(p)$ is matrix of seven-parametric transformation, which is considered to be a linear function of time,

$$\mathbf{p} = \mathbf{p}(t_0) + p\,(t - t_0)$$
$$\mathbf{p}(t_0) = p_1, \ldots.., p_7\,, \text{ and } \dot{\mathbf{p}} = \dot{p}_1, \ldots.., \dot{p}_7 \quad (2)$$

When the interval of the data processed is limited we can consider the station coordinates as constant and only the constant part of equation (2) is used.

Input data for combination consists of M sets of EOP (x_P, y_P, UT1-UTC, X, $Y)_{\boldsymbol{m}}$ and corresponding station coordinates $(x_T)_{\boldsymbol{m}}$, $m = 1, \ldots, M$, as derived by analysis centers for individual techniques. Partial derivatives of equation (1) with respect to any unknown, U, gives observation equations of the form:

$$\sum_j \frac{\partial x_C}{\partial U_j} dU_j \;=\; x_C|_{obs} - x_C|_0 \;+\; r\,, \quad (3)$$

where the "observed" vector x_C is calculated from the relevant input data, $x_C|_0$ are functions of adopted a priori values of the unknowns and r is the residual.

The EOP are calculated for each epoch independently of the others so that there must be added constraints in the form of the pseudo-observations:

$$dE_i - dE_{i-1} = 0 + r_E \quad (4)$$

The pseudo-observations reduce scatter of the EOP that is caused by propagation of errors from input data to the EOP. The system, as it was introduced, is singular. To remove the singularity, a no net-rotation constraint, minimizing mutual shifts and preserving the system as a whole, has to be introduced,

$$\sum \mathbf{p}^T \mathbf{p} = \min\,, \quad (5)$$

which stabilizes calculation of the station coordinates.

The method gives the following results:

- daily values of x_p, y_p;
- daily values of (UT1-UTC);
- values of seven-parametric transformation $p(t_0)$ and eventually their time derivates, $\dot{\mathbf{p}}$, for each technique, determined once for the whole period of processing.

3. PARTICULAR STEPS OF IMPROVEMENT

Since 2006, the non-rigorous method has been improving. The improvements come from different points further described in this chapter.

3.1. THE NEW TRANSFORMATION

The transformation equation (1) used old transformation of position vectors from terrestrial to celestial system. There were different approaches to time correction and precession-nutation matrix. The new transformation (McCarthy and Petit, 2003) has the following form:

$$\mathbf{x}_C = \mathbf{Q}(t)\mathbf{R_3}(\mathrm{ERA})\mathbf{R_3}(-s^{'})\mathbf{R_1}(y_P)\mathbf{R_2}(x_P)\mathbf{R}(\mathbf{p})\,\mathbf{x}_T\,, \tag{6}$$

where $Q(t)$ is precession-nutation matrix, which is a function of the position of the celestial pole, dX, dY. ERA is a linear function of UT1 and s' shifts from ITRF x axis to the terrestrial non-rotating origin, TIO, along the intermediate equator. The matrices $R_1(y_p)$, $R_2(x_p)$, and $R(p)$ have same form as in equation (1).

3.2. VONDRAK'S SMOOTHING AND ITS IMPLEMENTATION

The smoothing method as proposed by Vondrák (Vondrák, 1969 and 1977) is finding a compromise between two contradictory requirements. The derived function should be as close to observed data as possible on one hand (F - fidelity) and, at the same time, as smooth as possible, on the other (S – smoothness). It leads to a generalized least squares condition by minimizing their combination:

$$S + \varepsilon F = \min\,, \tag{7}$$

where ε controls the smoothness. When $\varepsilon \to 0$, derived function is a second order parabola, and when $\varepsilon \to \infty$, searched curve is running through all measured points.

Implementation of Vondrák's smoothing to the non-rigorous combination consists of replacing constraints (4) by the third derivative of a third-order Lagrange polynomial $L_i(x)$,

$$L_i''' = \sum_{k=0}^{3}\left(6\prod_{j=0,j\neq k}^{3}\frac{1}{\left(x_{i+k}-x_{i+j}\right)}\right)E_{i+k}\;. \tag{8}$$

These constraints tie the values of the respective EOP, E, at four adjacent epochs instead of only two ones, and assumes that the individual values of EOP lie on a smooth curve.

3.3. USING MODIFIED CHOLESKY DECOMPOSITION

When we use the method for solving eight-year data, the system contains more than two thousand observation equations. Inversion of such system takes a lot of time. Additionally, the normal equation system has many zero-components which could bring some small computation errors during inversion. All disadvantages of solving such a system were removed by implementing the modified Cholesky decomposition as proposed by Čepek and Pytel (2005).

4. LATEST RESULTS

We used the GPS, SLR and VLBI data covering the period 2000-2008, namely GPS and VLBI data were taken from the IERS Combination Pilot Project database (*iers1.bkg.bund.de/projects/combination/intra-technique/*). For SLR, the constrained ilrsb solution was used, as published by ILRS analysis center (*ftp://cddis.gsfc.nasa.gov/pub/slr/products/pos+eop/*). The GPS and SLR data are weekly SINEX (Solution in Independent Exchange format) solutions, from which the EOP and station coordinates were extracted. VLBI data consists of *per observation* singular normal equation matrices. To solve them, the constraints have to be added to tie station coordinates to the VTRF 2005 frame (Nothnagel, 2005) with the a priori precision of 5 mm. Since GPS and SLR do not provide celestial pole offset, only the x_p, y_p, and UT1-UTC are combined.

The eight years of data were successively processed in order to obtain monthly solution of EOP and **p**. The techniques enter the adjustment with the following weights: 1.44, 0.8 and 1.0 for GPS, SLR, and VLBI, respectively. The values do not differ very much from those applied to the IERS Dynamo program (Richard et al., 2008). Thus we used our values to keep continuity with our previous analyses.

From the monthly solutions, final coordinates for each collocation station were used as a weighted mean of transformed coordinates of the techniques contributing to the particular solution. Then, biases and linear trends (velocities of stations) were computed relative to the epoch J2000, see paper Štefka et al. (2009), where comparison of EOP is also included.

The new results of the velocities are compared with ITRF 2005 (Figure 1) and with those computed from tectonic model NUVEL-1A (Figure 2).

In the first case, the differences are randomly distributed with the mean value of 2.7 mm/y. The differences exceeding the level of 5 mm/y are related to the stations whose collocation ties were not known well or were changed during observations.

In the second case, the mean value of differences (5.9 mm/y) is approximately two times bigger than in the first case. The main effect is a significant drift appearing in the differences for European plate. We can see that the differences in S direction are at the level of 5 mm/y which is connected to the fact that movement of the European plate was determined incorrectly for the tectonic model. Another reason of

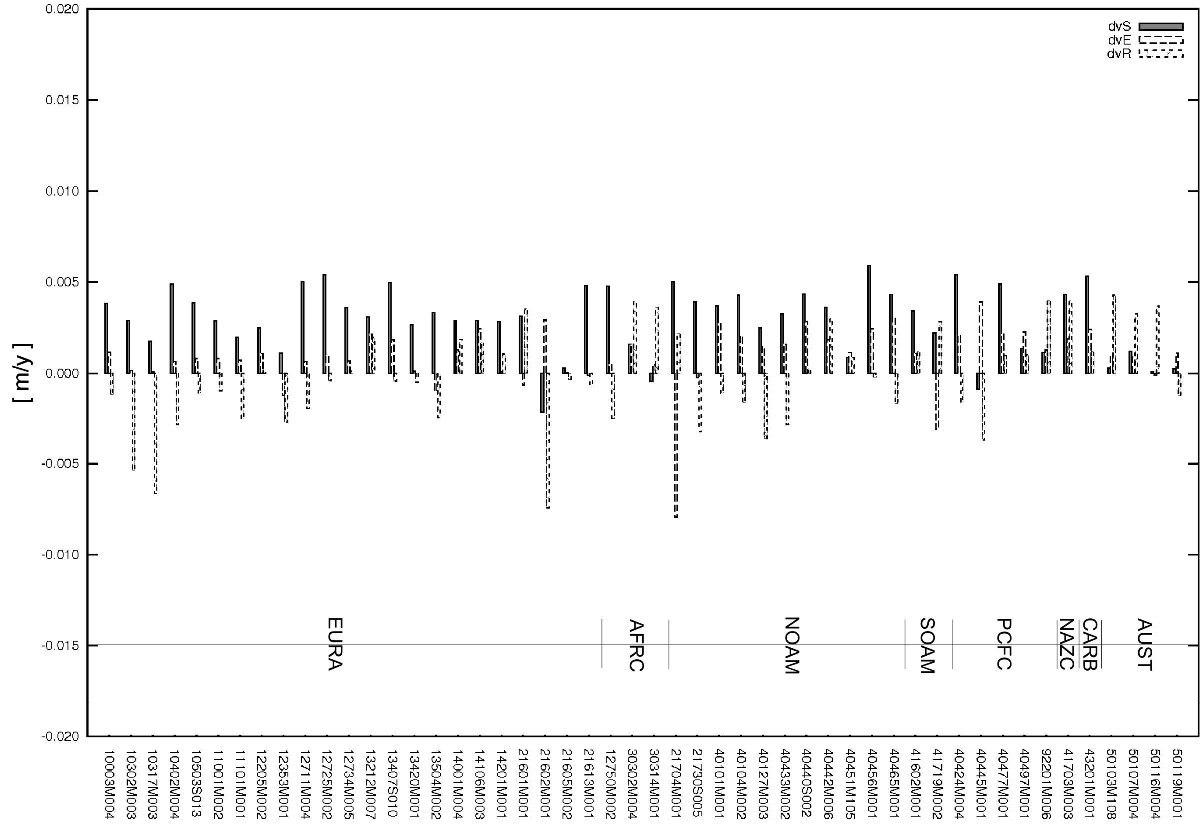

Fig. 1 Station velocities differences between the ITRF 2005 and our solutions computed from the eight-year data. All differences were transformed to a local system, where S is positive to the south, E is positive to the east and R is positive upward.

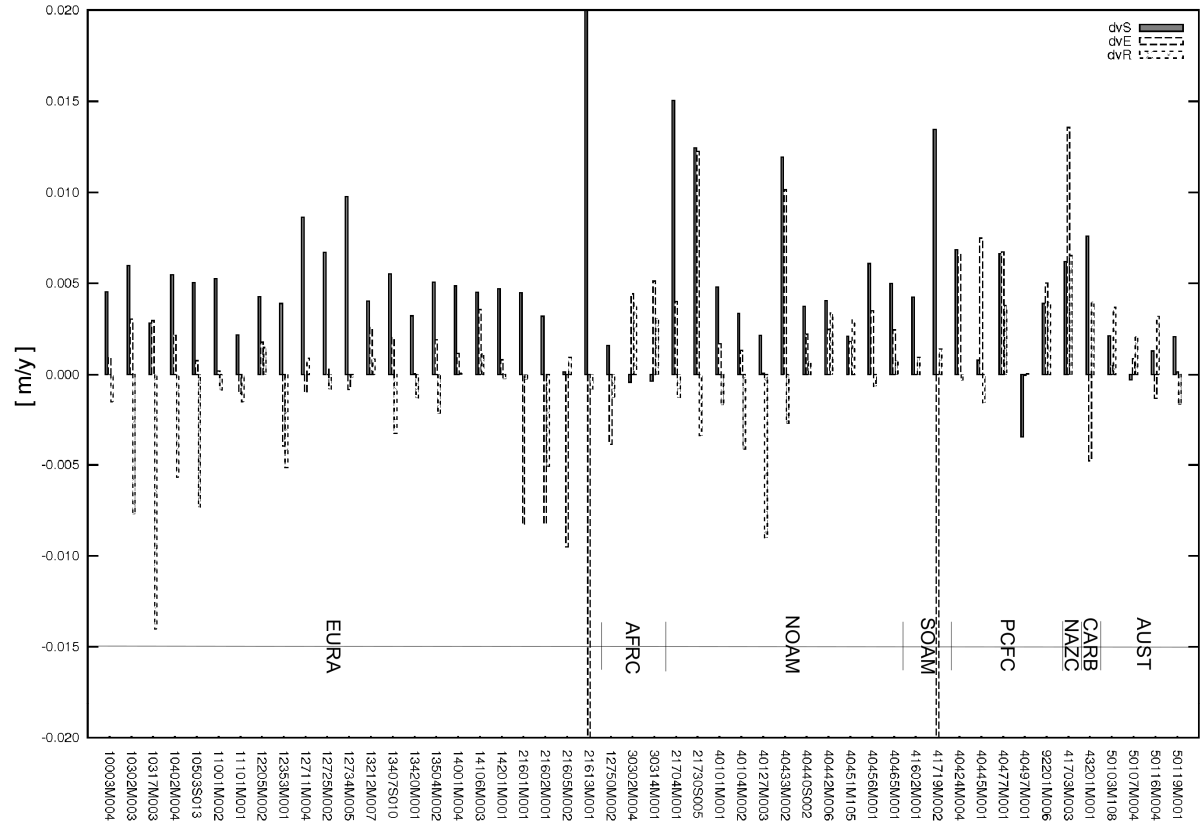

Fig. 2 Station velocities differences between our solution computed from the eight-year data and derived from tectonic model NUVEL-1A. All are shown in the local system (S, E and R) as in Figure 1.

big differences are positions of two stations (21613M001 and 41719M002) located near the boundary of the plate whose differences were not significant in the previous case.

5. CONCLUSION

Since 1999, the method of combination of results of space geodesy techniques has been improved several times. The review of particular improvements is presented by references to the appropriate papers. The most up-to-date results of collocation station velocities obtained by processing of eight-year data (measured by GPS, SLR and VLBI) are compared with results by ITRF 2005 and with velocities derived from the tectonic model NUVEL-1A. The mean values of differences are 2.7 mm/y and 5.9 mm/y for ITRF 2005 and NUVEL-1A, respectively.

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