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TEMPERATURE EFFECTS ON ONE BAY OF ST VITUS' CATHEDRAL

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ABSTRACT

This paper focuses on the impact of changes in temperature on one bay of St Vitus' Cathedral in Prague Castle. The objective of the study is to simulate as correctly as possible the distribution of temperatures in the structure, and then to compute the thermal dilatation movements. Theoretical simulation of dilatation movements involves simulating the temperatures in the structure and then computing the displacements. Insolation and changes in air temperature around the structure are included in the temperature simulation. The computed temperature fields are used as a loading for computing the forces and deformations of one bay of St Vitus' Cathedral. The theoretical deformation values obtained by means of the 3-D finite element model were compared with the measurements. The computed surface temperatures were also confronted with the surface temperatures measured in the interior and on the exterior of the cathedral. The results obtained from the simulations correspond well with the measured surface temperatures and deformations.

KEYWORDS: historical structures, cultural heritage, Prague Castle, St Vitus' Cathedral, temperature, thermal stress, solar radiation

1. INTRODUCTION

Prague Castle (founded more than 1000 years ago) is the largest coherent castle complex in Czech Republic. It is a UNESCO World Heritage site consisting of many palaces and ecclesiastical buildings in various architectural styles. The major component of the Castle is St Vitus' Cathedral, the oldest part of which was built in the 14th century. Geodetic monitoring of this historically important building can detect early signs of structural degradation. This will help to find sources of structural building degradation, and necessary repairs can be designed effectively. The dilatation movements and irreversible deformations can be measured during long-term monitoring. The objective of this paper is to differentiate between thermal dilatation movements and irreversible deformations.

This paper focuses on theoretical computation of the dilatation movements of one bay of St Vitus' Cathedral, and on comparing the computed values with the measured displacements. Temperature is the main source of dilatation movements, and it is significantly influenced by weather conditions. A method had to be developed for making a correct theoretical analysis of the impact of temperature on the structure, and this is described in the first part of the paper.

2. THEORETICAL ANALYSIS OF HEAT TRANSFER

2.1. *FORMULATING THE PROBLEM*

This paper sets out to compare deformations obtained through geodetic monitoring of the structure with values obtained through computation. It was anticipated that the deformations are caused mainly by thermal volumetric change of the structure. For this reason, it was necessary to obtain the modeled (theoretical) distribution of the temperatures inside the structure while the geodetic measurements were being made. A numerical solution of the partial differential equation of heat transfer (1) in the Adina commercial program was used for this purpose (Baehr at al., 1998; Hens, 2007; Sarit, 2005).

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c}\nabla^2 T \qquad (1)$$

T – temperature, K
t – time, s
λ – coefficient of heat transfer, W m^{-1} K^{-1}
ρ – density, kg / m^3
c – specific heat, J kg^{-1} K^{-1}

In general, the material characteristics in equation (1) may be influenced by the impact of moisture. The problem was simplified in this analysis, and we used constant material characteristics, independent of moisture and temperature. The

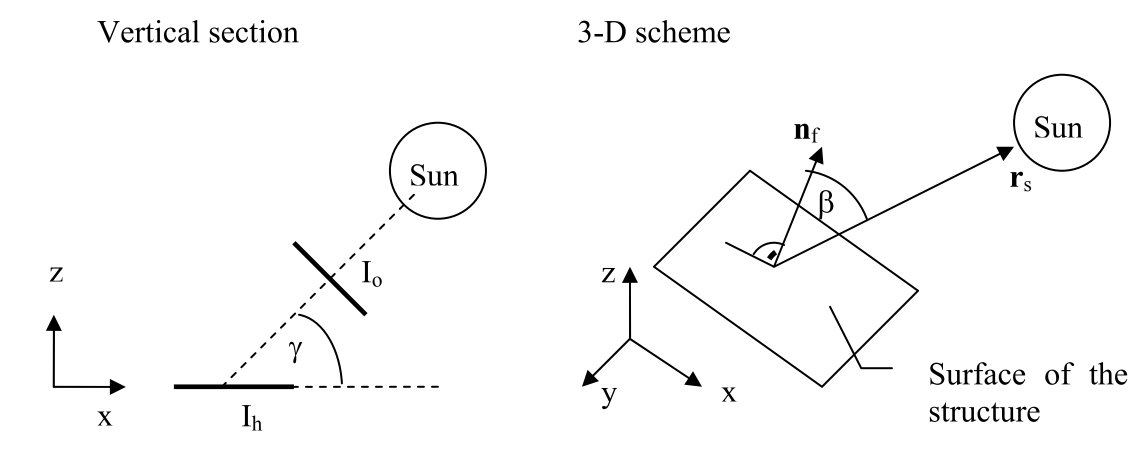

Fig. 1 Scheme for computation of the absorbed heat flux.

distribution of the temperatures in the structure may be influenced by water freezing in the masonry pores. In this case, the modeled temperatures may differ from the real values.

2.2. BOUNDARY CONDITIONS

The temperature of the structure is influenced by several factors: by the temperature and velocity of the air flow in its surroundings, by the intensity of the incident solar radiation, by evaporation of water from its surface and pores, by condensation of water vapor on its surface and pores, and by adsorption and desorption of water. Due to the large number of factors, the problem was simplified, and our model includes the following two effects: the air temperature around the structure, and the intensity of the absorbed solar radiation, i.e., the impact of insolation. The heat transfer from the air to the structure and vice versa was simulated by equation (2) (Kaminski at al., 2005; Arpaci at al., 1999).

$$q_a = \alpha\left(T_a - T_s\right) \tag{2}$$

- α heat transfer coefficient (W m^{-2} K^{-1}) - the values defined by norm ČSN 72 24 30 were used
- q_a density of the heat flux (W m^{-2}) - positive values are directed towards the structure (heating), negative values are directed away from the structure (cooling)
- T_a air temperature around the structure - the temperatures measured at the Prague-Ruzyně and Prague-Karlov meteorological stations were considered as the air temperatures outside the Cathedral. These two stations are 10 km and 3 km, respectively, away from Prague Castle. The temperatures measured by the detectors inside the cathedral were used as the air temperatures around the parts of the structure that are located in the interior.
- T_s surface temperature

The second effect included in our model is surface insolation (solar radiation). The energy transported through the atmosphere has several components: direct shortwave solar radiation, diffused shortwave solar radiation, longwave radiation emitted by the surface of the structure, and longwave radiation reflected by the atmosphere back to Earth. The total balance of the energy fluxes on the surface of the structure can be positive, null or negative. A positive balance means absorption of energy, while a negative balance means emission of energy (Adzerikho et al., 2000; Mills, 1999).

The problem was simplified, because only the intensity of global solar radiation is measured in Prague. It was considered that all solar radiation is direct radiation, which is valid especially for sunny days. The formulation of this boundary condition is described by equations (3) – (5). The principle is explained in Figure 1.

$$I_0 = \frac{I_h}{\sin\gamma} \tag{3}$$

- I_o intensity of solar radiation incident on a surface perpendicular to the direction of the solar radiation (W/m^2)
- I_h intensity of solar radiation incident on a horizontal surface: the values measured at the Praha-Karlov meteorological station were considered as the global solar radiation intensity in the Prague Castle area. The distance from Prague-Karlov to Prague Castle is approximately 3 km (W/m^2)
- γ solar altitude

$$\cos\beta = \mathbf{n}_f\,\mathbf{r}_s \tag{4}$$

- $\mathbf{n}_f$ base vector of the normal of the appropriate surface
- $\mathbf{r}_s$ base vector of the position of the Sun in the sky
- cos β the value of the scalar product between the vector of the surface normal and the vector of the position of the Sun in the sky

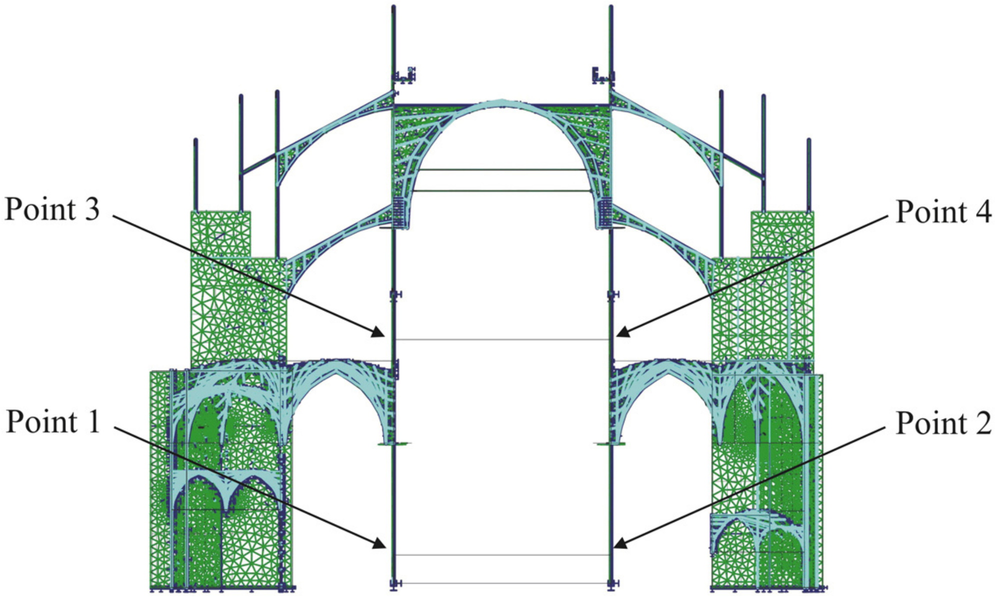

Fig. 2 Vertical cross-section of the cathedral with monitoring points.

$$q_a = \alpha \, I_0 \cos \beta \qquad (5)$$

α coefficient of the heat absorption of the surface - the values can be (0;1)

q_a intensity of the energy absorbed by the surface of the structure (W/m^2)

The boundary condition specified by equation (5) is positive. In the case of surface shadowing, it is almost equal to zero. A simplification is made in the simulation, and this condition is considered to be zero in the case of shadowing. Information on the location of the Sun in the sky (values γ and $\mathbf{r}_s$) was obtained from the Skymap pro 11 program.

2.3. INITIAL CONDITIONS

The temperature in the structure at the beginning of the analysis is the unknown variable. It is therefore necessary to simulate the temperature behavior of the structure for a specific time, so that at the end of the simulation the distribution of temperatures inside the structure is independent from the initial condition. This specific time depends on the heat inertia, i.e. the material characteristics and the dimensions of the structure. For example: a sandstone masonry wall 1 meter in thickness has heat inertia of approximately two weeks.

3. BEHAVIOR OF ONE BAY OF ST VITUS' CATHEDRAL

Geodetic monitoring of the load-bearing structure was carried out to capture early signs of possible degradation processes in St Vitus' Cathedral. The monitoring points in the structure had to be marked before the first measurements of the distances. Four points were chosen in one section of the Cathedral. The lower points were located on the nave columns, 2 metres above the floor, while the upper points were approximately 17 metres above the floor. Geodetic monitoring started on July 10th, 2000 with the first measurement of the distances between the monitoring points; this will be referred to as "the zero or basic period". The distances between the monitoring points were measured several times, and were related to the first measurement. For example: if the distances obtained from the second measurement (1st period - September 20th, 2000) are subtracted from the distances measured on July 10th, 2000 (basic period) we obtain differences which we call the horizontal and vertical displacements between the monitoring points in the "1st period", Figure 2. Geodetic measurements were made at 3-month intervals between 2000 and 2002, and twice per year after year 2002.

In order to understand the thermal dilatation behavior of St Vitus' Cathedral better, we performed a coupled thermo-mechanical analysis of one of its bays. This analysis was used for computing the theoretical deformations of one bay of the Cathedral and for comparing the results with the geodetically measured deformations. This paper sets out to answer the question "Are the measured deformations caused by cyclic changes in temperature, or by other influences?"

3.1. APPLYING TEMPERATURE LOADING

The first step in the theoretical analysis was to compute the temperatures. A theoretical formulation of the problem, together with the boundary conditions

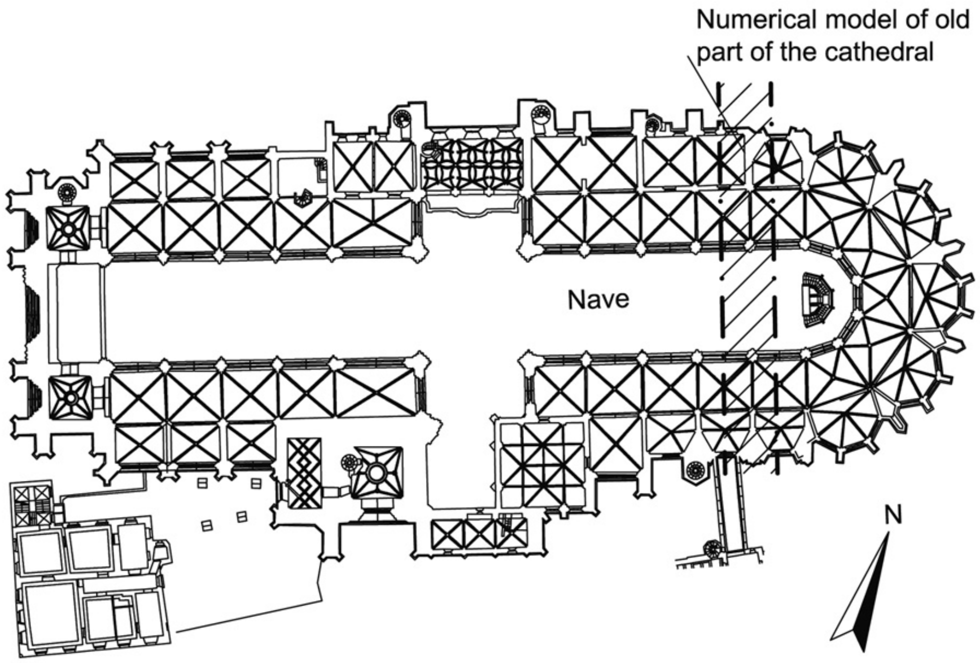

Fig. 3 Layout of St Vitus' Cathedral, and location of the numerical model.

and the initial conditions, has been given above. Only some specifics will be further dealt with below.

A 3-D finite element model of one bay of the Cathedral with four nodded elements was used to obtain the distribution of the temperatures inside the structure. The model includes all major elements of the structure, the most important of which are two bearing systems located on the northern side and on the southern side of the Cathedral, Figures 3, 4. Both of these bearing systems are exposed to the weather conditions. Only the southern system is directly exposed to solar radiation (insolation).

Six periods were simulated by this model. The word "period" is used in two slightly differing senses. Firstly, one geodetic measurement of distances is marked as "one period". As an example, the geodesists made a second measurement of the distances on September 20th, 2000, which is referred to as the 1st period. The "1st period" is used to specify the position in the order of geodetic measurements. Secondly, the word is used with reference to a time interval of approximately two weeks before the specific geodetic measurements. This time interval is important in the theoretical analysis, due to the heat inertia of the structure and the unknown distribution of the temperatures in the structure at the beginning of the analysis. As an example, the 1st period was measured on September 20th, 2000. A correct theoretical computation of the temperature distribution in the structure requires a simulation of the climatic effects from September 6th, 2000 until September 20th, 2000. The time interval from September 6th, 2000 until September 20th, 2000 is therefore marked as the 1st period in the course of the theoretical analysis. The aim of the simulation is to provide a theoretical distribution of the temperatures on September 20th, at the same time as the geodetic measurements. The expression "at the same time" does not mean that the simulation was running concurrently with the geodesists' measurements. The simulation was made later. Each period was divided into hundreds of steps (in the second sense). Each step represents a real time of 0.5 hour. Tetrahedron elements with a maximum edge length of 20-30 cm were used.

The computed surface temperatures were verified by comparing them with the measured surface temperatures. The surface temperatures were measured in two stages. The first stage was measured using an infra-thermometer on several days in June and July 2006. The second stage was measured using a thermo-camera in September 2007.

The first stage was measured using an infra-thermometer with accuracy ± 1°C. There was very hot and sunny weather in Prague at that time. The surface temperatures were measured on the exterior and in the interior. The temperatures on the exterior and in the interior were measured on each surface. As an example, the eastern surface of the main buttress of the bearing system was divided into six regions to monitor the shadowing. The temperature was measured several times in each region. The average value in each region was used as the typical temperature. The usual difference between the minimum temperature and the maximum temperature in one region was around 1-2 °C. This effect is probably caused by a combination of non-homogeneous material characteristics, shadowing, and the parameters of the ambient space.

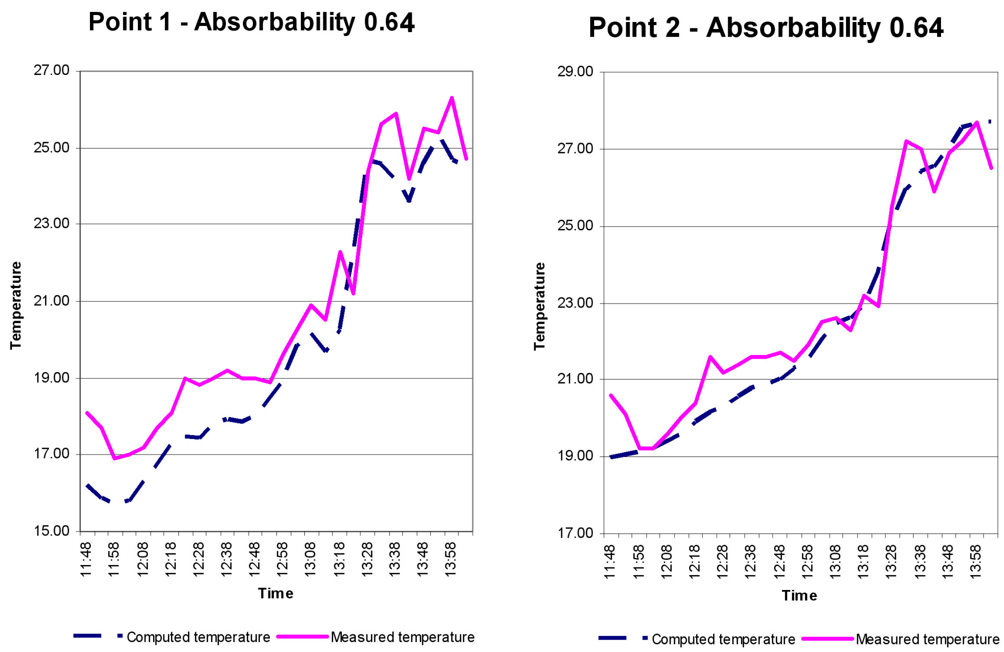

Fig. 5 History of the computed and measured temperatures at two points - September 13th, 2007.

The differences in the surface temperatures on the exterior were significantly higher than in the interior. The maximum measured surface temperature on the vertical surface of the southern exterior was 40°C, while the temperature of the vertical surface of the shadowed northern exterior was 24 °C. The maximum difference in the surface temperatures in the interior of the Cathedral was around 2 °C. The changes in the surface temperatures in the interior are significantly slower than on the exterior.

The computed surface temperatures corresponded best with the measured values when the coefficient of the heat absorption in the theoretical analysis was 0.64, and the following input data were used:

- temperature data from the Prague-Ruzyne meteorological station as the air temperature around the structure on the northern side of the Cathedral
- temperature data from the Prague-Karlov meteorological station as the air temperature around the structure on the southern side of the Cathedral

The mean-root-square error of the measured and computed surface temperature was 1.59 °C on the southern side of the cathedral, and 1.17 °C on the northern side.

The second stage verifying the temperature model was carried out on September 13th, 2007. The upper parts of the southern bearing system were scanned using a thermo-camera. The absolute accuracy of the camera is ± 1 °C (Fig. 4). Two monitoring points located on the upper surface of the upper flying buttress were chosen from the whole Figure 4. The history of the computed and measured temperatures at these two points is plotted in the chart in Figure 5.

The results obtained from computation correspond well with the measurements when similar parameters as in the first stage were used as the input data: coefficient of heat absorption 0.64 and temperatures from the Prague-Karlov meteorological station. The maximum difference between the measured and computed temperatures was less than 2 °C, see Figure 5.

The temperature model corresponds well with reality, and can be used for computing the thermal displacements of the structure. It should be noted that the verification was carried out only during sunny weather and partly cloudy weather, with no precipitation. No verification under other climatic conditions has yet been carried out (Beran, 2008; Beran et al., 2007).

The material characteristics used in the temperature analysis were:

$\lambda = 5040 / 3600 = 1.4$ W.m^{-1}.K^{-1} Coefficient of heat transfer

$\rho = 2600$ kg / m^3 Density

$c = 840$ J. kg^{-1} Specific heat capacity

$\alpha = 54000 / 3600 = 15$ W . K^{-1}. m^{-2} Coefficient of heat transfer on the exterior

3.2. STATIC MODEL

The second step in the numerical analysis was to compute the deformations and stresses in a part of the structure. The temperature fields were the loading of the static model. The geometry of the numerical model was the same in both steps of the analysis. The linear elastic material model was used in the second step of the analysis.

St Vitus' Cathedral consists of a transept, a nave and two lower side bodies flanked by chapels. The transversal pressures of the nave vault and the transversal reactions of the nave roof are captured by the exterior bearing system. The Cathedral contains 10 bays. A typical bay includes two columns, vaults, walls and two bearing systems.

The static finite element model of one bay of the cathedral has several kinds of boundary conditions. The following boundary conditions were applied in the planes of symmetry: the rotations around the x-axis and around the z-axis were fixed; the displacements in direction of the y-axis were fixed by some springs. The stiffness of the springs was obtained by means of a simplified 3-D numerical model of the old part of the Cathedral. The 3-D model was loaded by the forces, and the deformations were computed. The stiffness of the springs was obtained by dividing the loading force by the computed displacement. A detailed description of the computation of the stiffness of the springs can be found in (Beran et al., 2009). The displacements in the direction of the z, x and y axis were fixed in the bottom border of the 3-D model. The material characteristics used in the stress analysis were:

E = 15 GPa	Young's modulus
ν = 0.2	Poisson's ratio
α = 0.000 011 74	Coefficient of thermal expansion of sandstone

3.3. COMPARISON OF THE COMPUTED AND MEASURED DEFORMATIONS

The main objective of this study is to verify whether the changes in the horizontal displacements of the columns obtained from geodetic monitoring were caused by temperature changes. For this purpose, the measured displacements obtained from the geodesists were compared with the structure displacements obtained from the theoretical analysis.

All geodetic measurements are related to the basic measurement when the deformations of the structure are considered to be equal to zero. A theoretical simulation of the thermal dilatation displacement fields was carried out for six different periods corresponding with the six times when geodetic measurements were made in the Cathedral. The initial condition, i.e. the temperature at the beginning of the thermal simulation, was considered to be 10 °C. If the temperature of the structure is equal to 10 °C, the theoretical values of the displacements in the structure are considered to be equal to zero.

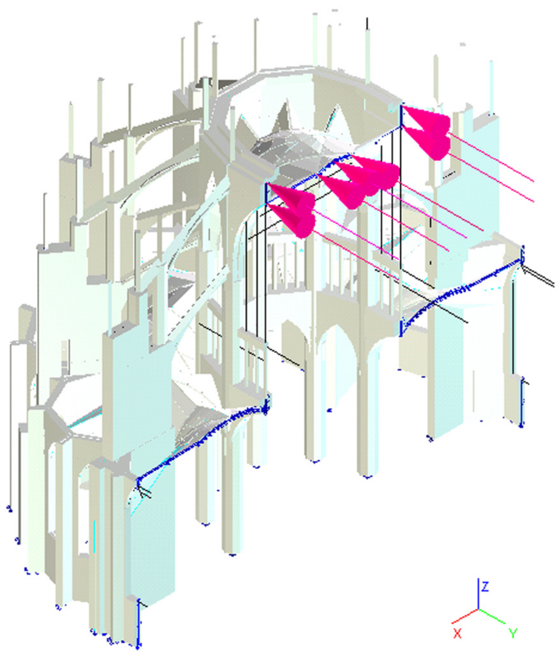

Fig. 6 3-D simplified numerical model consisting of 2-D and 1-D elements - Feat 2000 program.

The computed displacements between the monitoring points in an interval of two days before the first geodetic measurement of the displacements are displayed in the graph in Figure 9. The geodetic measurements were usually run at about 6 pm.

The daily amplitude of the horizontal displacement between the monitoring points is higher on the northern side than on the southern side. The displacements caused by temperature are of two kinds: the first of these is uniform heating (cooling) of the structure, due to an increase (decrease) in the air temperature; the second type of effect is due to unequal heating of the upper surfaces of the flying buttresses of the southern bearing system, due to solar radiation. These two effects are combined on the southern side of the Cathedral. Uniform heating causes a positive displacement - a displacement between the monitoring points in the direction towards the longitudinal plane of the cathedral symmetry. Thorough unequal heating of the structure due to solar radiation causes negative displacements between monitoring points, i.e. a displacement from the longitudinal plane of cathedral symmetry. The impact of solar radiation is negligible on the northern side of the cathedral, because most of the surfaces are shadowed by the roof of the nave almost throughout the year, Figures 7, 8.

The graph in Figure 10 shows the differences between the displacements obtained from the numerical model and from the geodetic measurements. The graph in Figure 11 shows the displacement trend on the northern nave column. The displacements obtained from the model correspond well with the displacements obtained from the

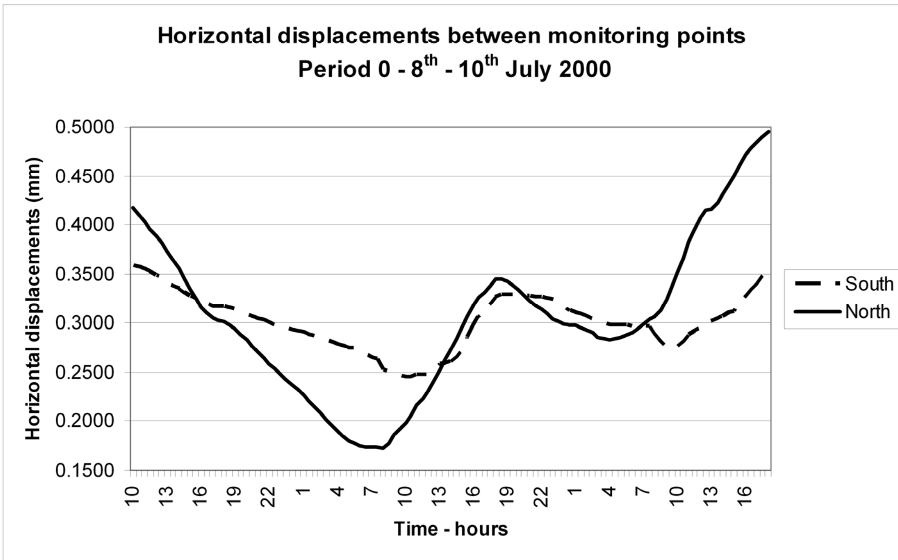

Fig. 9 Horizontal displacements between the monitoring points obtained from the numerical model, basic period: July 2000.

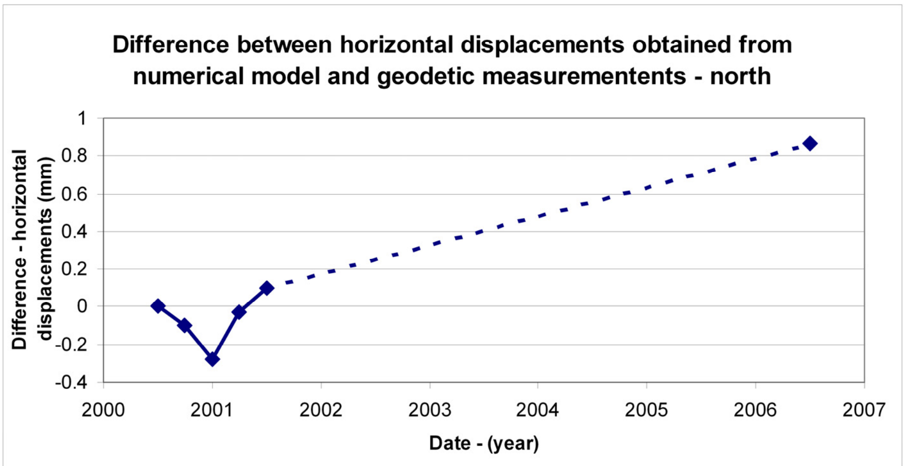

Fig. 10 Difference between horizontal displacements obtained from the numerical model and from the geodetic measurements on the northern column of the analyzed span of St Vitus' Cathedral.

geodetic measurements during periods 1 - 4, which were measured in 2000 and in 2001. However, the diference between the displacements is almost 0.9 mm during the fifth period (2006), which is a more than 3 times greater difference than in the other periods. The character of the deformation is also different. This may be due to the following effects:

Firstly, it is by the influence of moisture, which is not included in this method. The heat inertia of the most robust part of the bearing system is about 2 weeks, but the moisture inertia of the same part of the structure is at least several months. Thus changes in moisture are slower. Moisture affects the final change in the displacements between particular measurements only when the climatic conditions influencing the moisture content are different for a very long time. As an example, we might assume that the impact of the moisture content is not significant for the fifth period, because the measurements for the fifth and basic periods were made in the same month of the year. Therefore the moisture content in the structure should be similar.

A second possible effect is a gradual increase in irreversible deformations, which may be due to cyclic thermal and moisture strain.

The significantly higher difference between the theoretical and measured displacements may have been caused by a horizontal movement of the soil. Such a movement was recently detected in the geologists' drill hole located in the foundations of the northern wall of the cathedral in the bay under analysis here. The detected displacement in the

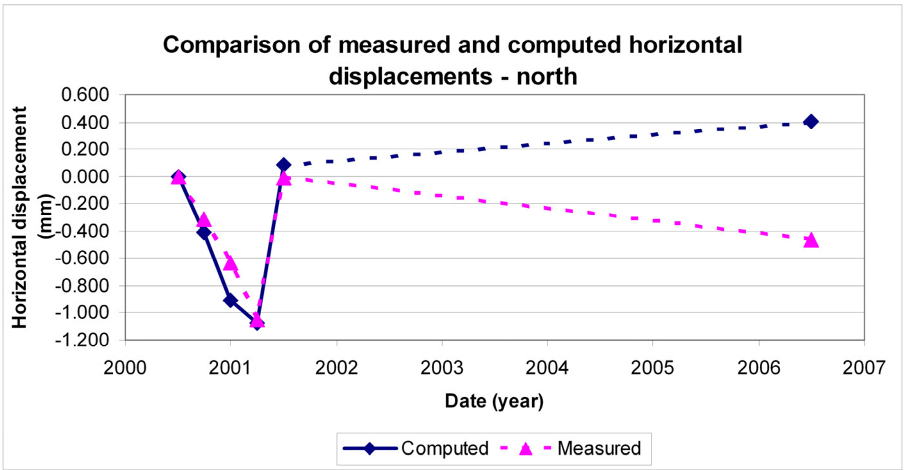

Fig. 11 Horizontal displacements on the northern column of St Vitus' Cathedral, obtained from measurements and from the theoretical model.

geologists' drill has a value of 2-3 mm. Of course, a single value does not enable us to draw conclusions. However, in the course of long term monitoring of the structures, and with the use of an improved theoretical method, early signs of degradation processes might be detected.

4. CONCLUSIONS

The thermo-mechanical analysis of the Cathedral bearing system carried out using the 3-D numerical finite element model has proved that the structure is periodically under temperature strain. The bearing system is bent and twisted, together with the connected structures. The method presented in this paper could help to detect the onset of irreversible degradation processes in the structure by comparing the theoretical displacements with the measured displacements. The method is only under development, but in the future it could differentiate between displacements caused by temperature and those caused by other effects. The method is nondestructive, and if used in long-term monitoring of the structure it could draw attention to possible problems of the building.

Horizontal displacements between the monitoring points were detected during the first year of monitoring the columns of St Vitus' Cathedral. These displacements are caused mainly by thermal volumetric changes in the structure. The trend of good agreement between the measurements and the model was, however, broken in the most recent compared period, in 2006. The presumed reason is a horizontal movement of the soil, which was recently detected in the geologists' drill through the foundation of the northern Cathedral wall in the part of the structure under analysis here. The horizontal movement of the geologists' drill has a value of approximately 2-3 mm, which does not now endanger the load bearing structure of the Cathedral. No new cracks or other failures originating in the structure were detected.

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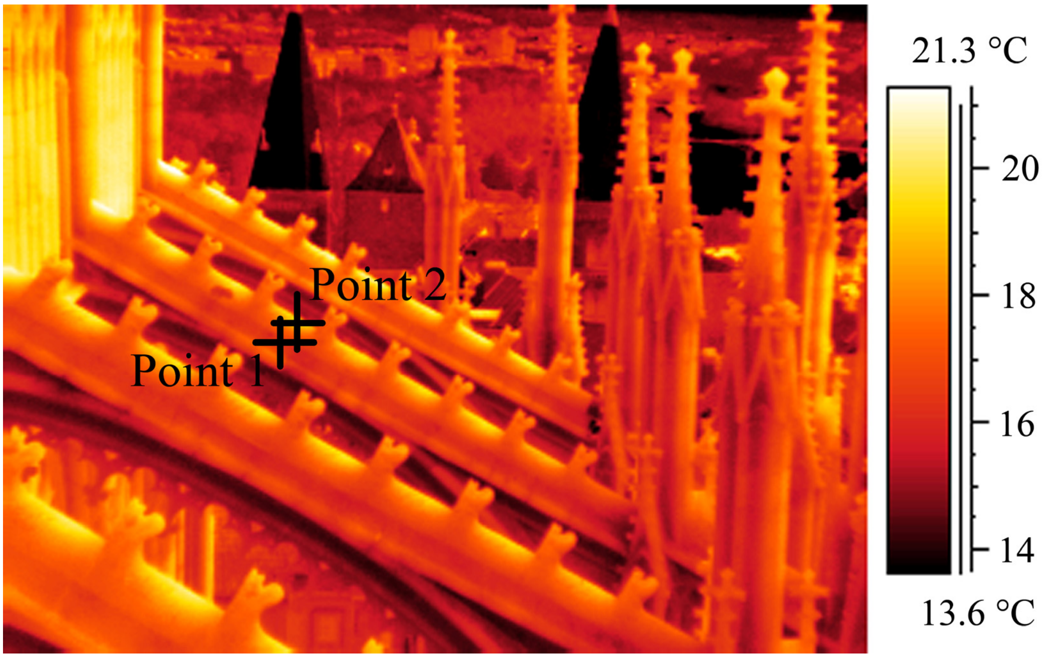

Fig. 4 Thermovision - September 13th, 2007 11:42 a.m.

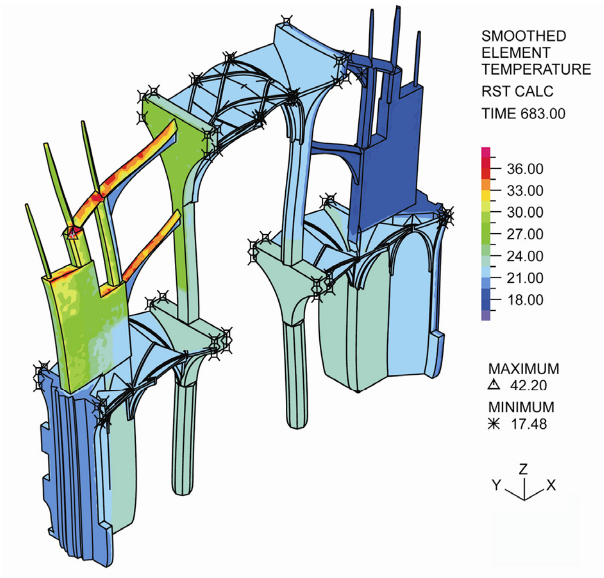

Fig. 7 Numerical model of one span of the Cathedral - Distribution of temperatures at 11.00 am on July 2nd, 2006.

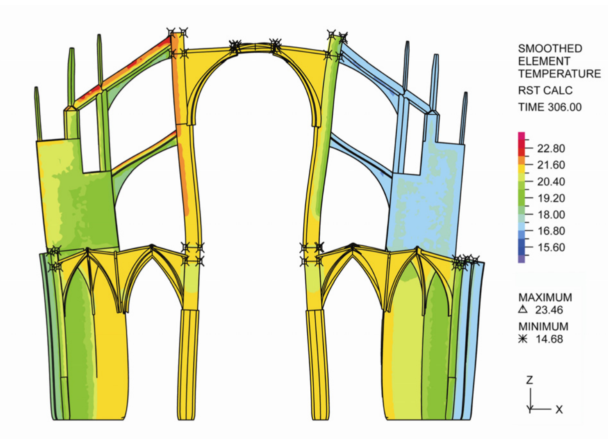

Fig. 8 Temperature field and deformations of the structure, period 4. At 6 pm on July 2nd, 2001.