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INFLUENCE OF TEMPERATURE CHANGES ON THE VLADISLAV HALL VAULT

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ABSTRACT
The vault in Vladislav Hall is a structural masterpiece of great historical value. Its structural analysis revealed that the permanent load exerted by its self weight is uniformly transmitted into the vaults and ribs without any potential crack appearance. The topical issue, however, is its response to temperature changes with respect to actual effects. Computations show that temperature changes may cause problems.

KEYWORDS: vault, rib, rod, hall, temperature, loading

INTRODUCTION

Vladislav Hall is a secular Gothic structure with Renaissance windows. The start of its construction dates back to 1493. The works were completed around 1502. The vault designed by Benedikt Ried had most likely suffered partial collapse during the construction. This assumption is based on a letter written by the later builder of a small hall vaulting whose vault had also collapsed and who put his failure in the context of the collapse of the Ried's vault.

In the Middle Ages, the Hall served for knights' tournaments, for representative purposes and later for holding markets and commercial meetings. In 1541, the roof truss was damaged by a big fire, and its part collapsed onto the vault. The vault itself resisted the impact, but the ribs were torn away from the vault at several points. The upper walls have preserved a reddish shade caused by the big fire to the present day. During the repair, the vault was covered by reinforcing mortar and the majority of bays were clamped by additional pull rods.

Vladislav Hall went through several repairs in the past. The first reconstruction was after the big fire in 1541, and the last one in 1920. Now, it is in good condition and only small microfissures and inactive cracks can be found there. A problem appeared during the election of the President of the Czech Republic, which was held in the Hall in January, when the cracks reactivated and plaster fell down. The investigation of the origin of the failure revealed a problem in the heating system used. The vault and the ribs were warmed up by hot air very quickly, and the temperature pattern through the vault was strongly non-uniform.

A similar problem with temperature changes was detected during summer months when the vault had different temperature values on the upper and lower surfaces.

Individual parts of the structure have different thermal inertia values – the vault has a larger thermal inertia than the ribs – and this also affects the temperature distribution through the vault.

DESCRIPTION OF THE STRUCTURE

The vault is composed of five bays with a total length of 62 m and a width of 16 m. The hall's height is 13 m. The ground plan with associated sections is in Figure 1. Each bay is vaulted by a rib vault of roughly spherical shape as is seen in Figure 2. The ribs are of stone and were evidently erected first; the brickwork vault was then vaulted into them. The dimensions of individual bays slightly differ. The upper part of the vault is not connected with the wall, see Figure 3. The brick wall transfers the load exerted by the roof truss straight onto the pillars eliminating thus the moment load at the vault crown.

The vault thickness is variable and not known for most of the area. According to the estimate based on partial surveys performed, the average vault thickness is 250 mm. The thickness of the supporting walls is 900 mm reaching 1500 mm at the points of pillars. The walls are lightened by windows and doors. The foundation of the pillars varies – on the south side Romanesque walls are erected, while in the north part the pillars pass through arcades, and the walls are terminated on the arcade vaults. The vaults are clamped by pull rods at two levels. It is not exactly known when these were mounted. Some of them were

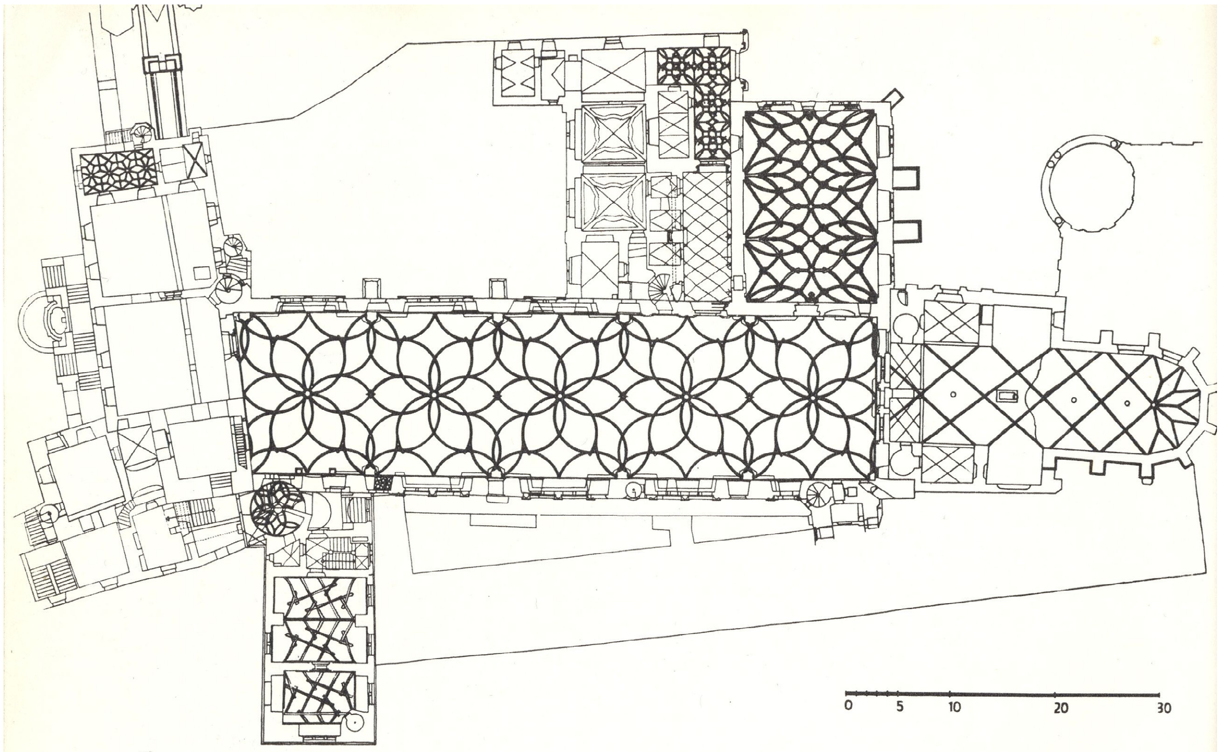

Fig. 1 Ground plan of Vladislav Hall.

most likely applied after the alleged vault collapse, and others were added after the fire. In terms of the structural function, the most efficient is the bottom Gothic iron pull rod with dimensions of 30/60 mm passing through the vault into the pillars.

The structure was built up of three parts – stone ribs, brick vaults and iron pull rods.

We may assume that the structural role of individual parts underwent changes in the history. We may say that in the beginning – with respect to the usual construction method when the ribs were built up before the vault – the groins had a supporting function. During the time, however, the vaults took over some part of loading.

This is also confirmed by the results of earlier computations (Fajman et al., 2009) where half of the load is taken over by the ribs and the second half by the vault.

From the results of computations, low-level participation of the pull rods in the linear vault action may also be derived.

It is evident that the rods increased the load bearing resistance of the structure.

The connection between the groins and the vault is provided by steel anchors, see Figure 4 (Chotěbor and Měchurka, 2007).

COMPUTATIONAL MODEL

The cross-section of the hall with one gore has been computed. The symmetry boundaries separated

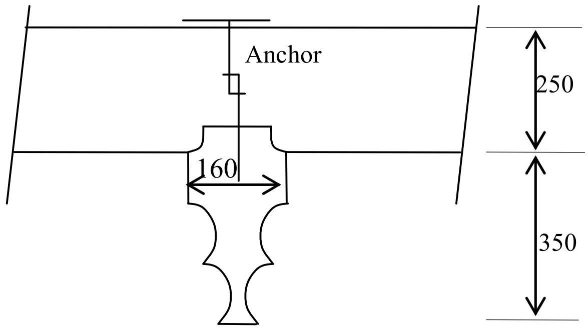

Fig. 4 The rib-vault connection

the surrounding parts of the Hall. The walls are weakened by a lot of windows and door holes; the shape is shown in Figure 5. This model was established to obtain the distribution of forces through three parts of the structure: the vault, the rib and the rod.

The following assumptions were considered during the creation of the model:

- The wall is modelled by thin-walled elements and is elastically supported at the foot. The support accounts for different foundation conditions – the south side stands on the edge of a weathered arenaceous marl slope where k = 3 MN/m/m, the north side of the pillar is founded on arenaceous

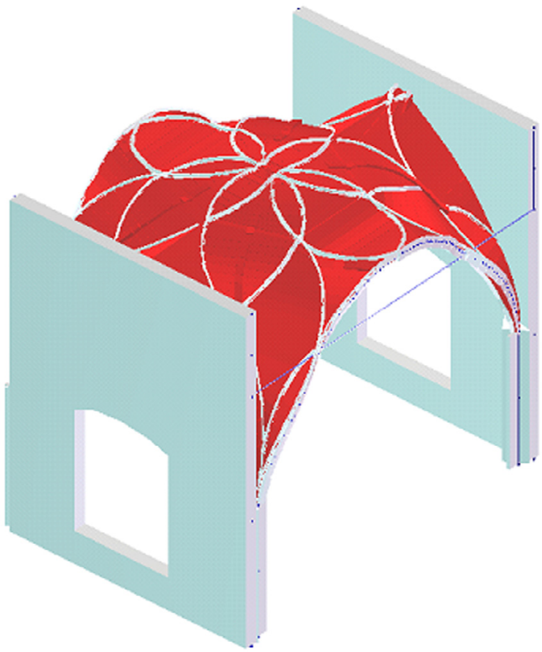

Fig. 5 Shape of the structure – numerical model of the Vladislav Hall.

marl subgrade with k =10 MN/m/m, and the wall stands on the arcade vault with k=1 MN/m/m.

- The pillars are recessed inside against the wall. This enhances the wall stability where vertical forces exerted by the vault cause a reverse moment at the wall foot than horizontal forces.
- The ribs are formed by bar elements with an area of 0.055 m^2 and their axes are oriented away from the vault in the direction into the hall.
- The joint vault and rib are assumed to be rigid in the computational model. It means that the model is stiffer than the real structure and it results in smaller displacements and larger forces due to temperature changes.
- The vault crown in not connected with the wall.

The material characteristics could not be determined with due accuracy as no tests or other interventions into the structure were allowed. Finally, they were selected with a view to the standard and to recommendations presented in (Doležel and Konvalinka, 2005) as follows (Fajman et al., 2007):

- sandstone with the tangent modulus of elasticity E = 12 GPa, Poisson's ratio ν = 0.2, the coefficient of thermal expansion α = 0.00001, the unit weight γ = 22 kN/m^3
- brick masonry with the tangent modulus of elasticity E = 3.15 GPa, Poisson's ratio ν = 0.2, the coefficient of thermal expansion α = 0.00001, the unit weight γ = 19 kN/m^3
- steel with the tangent modulus of elasticity E = 210 GPa, Poisson's ratio ν = 0.25, the coefficient of thermal expansion α = 0.000012, the unit weight γ = 78 kN/m^3

The self weight of the brick masonry vault is 1250 kN, of the stone ribs 400 kN and the walls about 4050kN.

Due to the fact that material inputs could only be estimated, only a simplified computation was performed under the conditions of linear material behaviour, while geometric non-linearity was limited to large displacements.

The FEAT computer program is based on the FEM analysis with the mesh composed of triangular plane elements with three drilling degrees of freedom (Fajman, 2002) and three plate bending degrees of freedom.

INFLUENCE OF TEMPERATURE LOADING ON THE REDISTRIBUTION OF FORCES IN THE VAULT

Temperature loading is a very frequent cause of failures of historical structures. In the Prague Castle complex, its effects are investigated on four prominent monuments – Vladislav Hall, St. Vitus Cathedral (Beran and Maca, 2006), St. George Basilica and Belvedere Royal Summer Palace.

The determination of the effects of temperature changes on the Vladislav Hall vault was based on the experience of the sudden temperature change in winter. During the presidential election, the structure was exposed to a thermal shock and cracks started to appear on the vault.

The second unpleasant situation occurs in summer months when the vault extrados is exposed to intensive warming from the roof truss area.

TEMPERATURE LOADING – WINTER DAYS WITH FAST HEATING

There are different temperature values in the ribs and in the vault. The temperature values were estimated. At this time, the Hall is heated very slowly before the presidential election and the structures are uniformly warmed. We chose an extreme situation when only the ribs were warmed.

The presented temperature loading produces tension in the whole vault (Fig. 6). The magnitudes of the main compression stress are 70 kPa in straight parts on the top of the vault, and 680 kPa near the stud hole. The associated moment is 5 kNm/m and 2 kNm/m respectively. The ribs are subjected to compression axial forces N_{max} of 40 kN. The bending moments in the ribs are small, M_{max} equals 5 kNm. The existence of compression forces in the pull rods means that the pull rods do not participate in the load transfer. The marginal values of the stresses and forces is given in Table 1.

Table 1 Summary of max. and min. stresses and forces in the Vladislav Hall vault.

Model	Bending in vault (kNm/m)	Stress in vault (kPa)	M in ribs (kNm)	N in ribs (kN)	N In pull (kN)
Self weight	-3 up	-400/ 115	1	-23	37
	7 down	-750	8	-165	
Temperature in winter	5	680/ -70	1	-40	(-10)
	-2	300/ 250	5	-40	
Temperature in summer	-4	-50/ 0	0.5	27	15
	3	-200/ 150	1	5	

Report (Procházka et al., 2009). Due to the fact that these were only several one-day measurements performed in different years, the results could not be considered as representative. The heat conduction analysis defined the heat inertia of the structure which is approximately one day. After that, a one-day temperature pattern in the vault slab and the rib was calculated. The subsequent structural computation was made using the biggest temperature difference that was linearized (Fig. 7). The temperature difference between the vault slab surfaces was 6 °C, and between individual parts it was 3 °C.

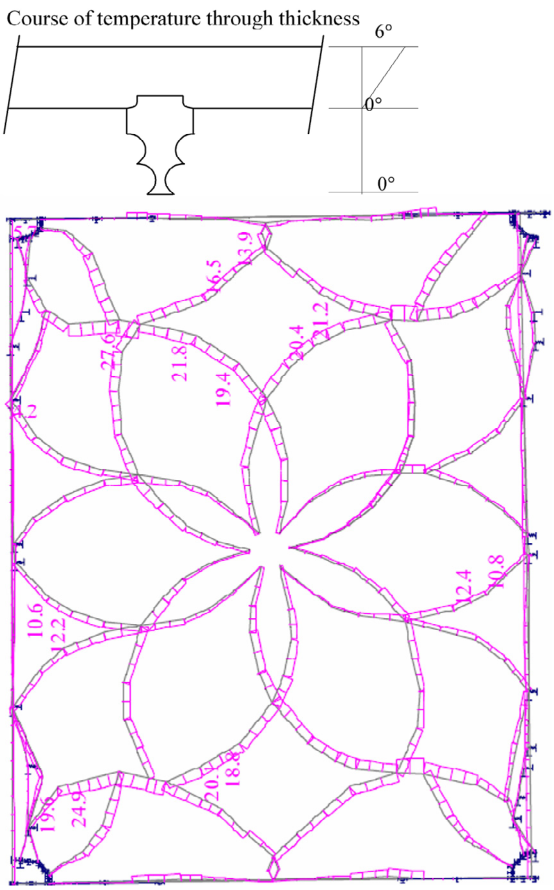

Fig. 7 Forces in the ribs of the Vladislav Hall vault.

TEMPERATURE LOADING – SUMMER DAYS

There are different temperature values on the top and the bottom surface of the vault.

The computation was performed using only an estimated temperature gradient. The temperature values available had been in-situ measured under the vault and over the vault in the loft space, see the Temperature loading produces bending moments and compression in the whole vault. The magnitudes of the main compression stress are 100 kPa in straight parts on the top of the vault and 200 kPa near the stud hole. The associated moment is 4 kNm/m and 3 kNm/m respectively. The ribs are subjected to tension axial forces N_{max} of 27 kN (Fig. 7). The bending moments in the ribs are very small, M_{max} equals 1 kNm. The tension forces in the pull rods are 15 kN and the stress is 50 MPa. It is interesting that this temperature loading simulates the situation when the ribs lost their load bearing function.

The results show that the vault is under strong tension from the temperature difference of 10 °C between the ribs and the vault. Actual conditions in the past could have been even worse - the temperature of heating air was about 60 °C. This seems to be the main reason of the origin of cracks. The summary of the stresses and forces is given in Table 1.

CONCLUSIONS

The computation of loading by temperature changes characterizing fast warming of the hall interior showed that prominent tensile stresses arise in the vault. The difference of 10 °C between the rib and the vault slab causes tensile stresses which result in the rise of microfissures appearing either in the vault or between the ribs and the vault. The results contributed to a change in the heating method during the presidential election. Now, the alternative of slow temperature growth during 7 days has been adopted. This reduces the temperature differences in the structure to 3 °C.

The temperature loading due to the summer vault warming from the upper space leads to the rise of tensile stresses in the ribs. The warming of the vault extrados by 6 °C causes tensile forces in the ribs comparable to loading by self weight. For this reason, intensive ventilation of the loft space on summer days is recommended.

The facts above imply that temperature loading causes prominent tensile stresses in individual parts of the structure. It must, however, be realized that only a physically linear computation has been performed where the crack appearance and ductility of the joint between the rib and the vault were not considered. According to in-situ observations, big temperature changes caused cracking and thus decreased rigidity of the structure. This resulted in reduced peak stress values. Despite these drawbacks of the computation, it has been confirmed that the appearance of cracks in the vault was due to a significant change in temperature.

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Fig. 2 Inside space of the Hall.

Fig. 3 Vault extrados.

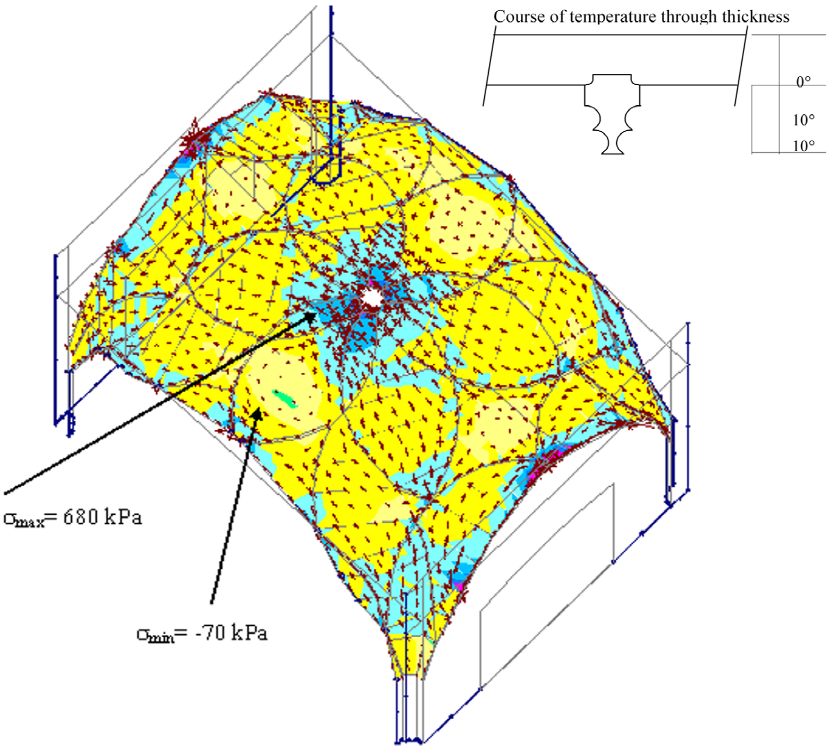

Fig. 6 Axial stress in the vault subjected to fast heating.