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ANALYSIS OF VERTICAL MOVEMENTS MODELLING THROUGH VARIOUS INTERPOLATION TECHNIQUES

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ABSTRACT

The main objective of this paper is to explain how the application of various interpolation methods influence the determination of vertical crustal movements at any given point. The paper compares several methods of interpolation and verifies their suitability, including kriging, minimum curvature, nearest neighbor, natural neighbor, polynomial regression, inverse distance to a power, and triangulation with linear interpolation. The calculations show that the chosen interpolation method has significant influence on the final result of the study. Nearest neighbor method was chosen to be the best.

KEYWORDS: leveling, vertical crustal movements, interpolation

1. INTRODUCTION

The first author of this paper designed in his doctoral thesis (Kowalczyk, 2006c) a vertical crust movement model for the territory of Poland. The movements were derived from nodes of the vertical movements network (Fig. 1). Vertical movements network was created from the unadjusted height differences of first order levelling lines on the basis of third (1974-1982) and fourth (1997-2003) leveling campaigns in Poland. Unadjusted data from first (1926-1937) and second (1952-1958) levelling campaigns are in analog form stored in archives (only adjusted data is avaliable in catalogues). Therefore it was not used in this research. This network was adjusted using least squares method. As a result vertical crustal movements at nodal points were obtained. The standard deviation of vertical movements in nodal points varied from 0.03 to 0.10 mm/y distributed evenly on entire network. The calculated values of vertical crustal movements are shown in (Fig. 2). The vertical movements were referenced to the mean sea level change of the Wladyslawowo tide gauge. The changes were computed as a linear trend from annuall observations at Wladyslawowo tide gauge in years 1952 - 2001. The highest values of negative vertical movements were observed in the vicinity of Inowroclaw and Rzeszow (over - 5 mm/year). Both cities are located in the Teisseyre-Tornquist tectonic movement zone and in both areas an intensive mining activity was or is still conducted (salt mine at Inowroclaw, gas mine in the vicinity of Rzeszow and Jaroslaw). Near Warsaw the vertical crustal movements vary from -1 mm/year to -3mm/year. Most of the I order benchmarks in the territory of Poland, has vertical crustal movements varying from -1.5 mm/year to -3 mm/year. Vertical movements smaller then -3mm/year are characteristic in the areas of Elblag, Plock, Torun and Wloclawek (the border of T-T zone) and in western Poland from Wroclaw to Legnica through Zielona Gora to Gorzow Wielkopolski (border of the Czech massive). The smallest subsidence was observed in the area of eastern Bieszczady (Karpaty). No uplift movements were found. The method of collocation of least squares with Hirvonen function was applied to interpolate the vertical crustal movements (Kowalczyk, 2006a). During interpolation with collocation method the radius of points search, determined on 10 % of data, was 50 km (Kowalczyk, 2006b) .

The goal of this paper is to determine the suitability of various interpolation methods for vertical crustal movement. The computation was performed with the Surfer software, GIS software and authors own software InterVeric.

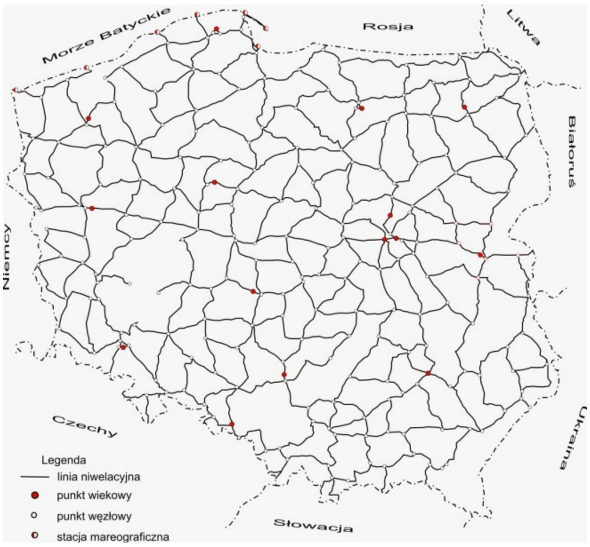


Fig. 1 Outline of vertical movements network.

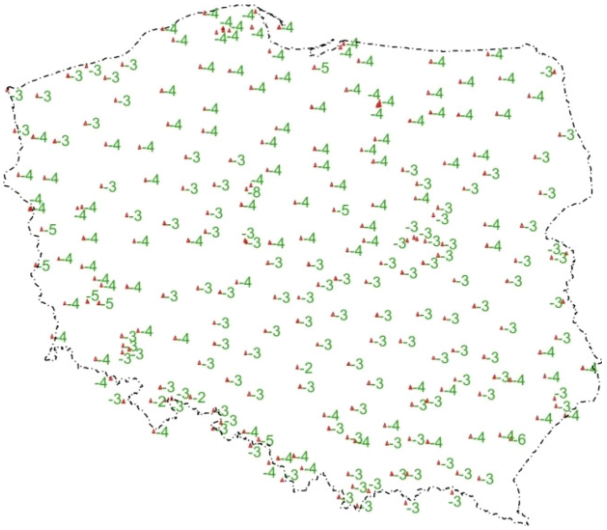


Fig. 2 Vertical movements in nodes in [mm/y] network.

2. VERTICAL MOVEMENTS INTERPOLATION USING SURFER SOFTWARE

The software enables numeric interpolation with the following methods: kriging, radial basis function multiquadric, triangulation with linear interpolation, natural neighbor, minimum curvature, nearest neighbor, inverse distance to a power, local polynomial, modified Shepards method.

Usually data used to create a model is irregular. That's why Surfer creates rectangular grid to draw isolines. The size of the grid must ensure good result of visualisation with minimal number of interpolated grid points. Usually nearest points have the biggest influence on the interpolated value. It can be changed through giving adequate weights for each point. Since in the case of vertical crustal movements it is difficult to determine confidence level without geological analysis and analysis of big cities influence (Kmiecik and Sieradzan, 1994; Wyrzykowski, 1987), all weights are assumed to be 1. The choice of interpolation method depends on data layout and their variance level. Model 2006 was created with use of 235 nodal points from double leveling on which vertical movements were calculated. Distances between nodes were about 30 km. As shown in Figure 1 the density of nodal points is not even for the whole country.

2.1. *INITIAL STUDY*

To assess the usefulness of the respective interpolation methods appropriate calculations were carried out with the Surfer 8.0 software. Default software settings were used for each method while interpolating. The change of the parameters of each interpolation method can be used to self-compare the method. To compare two or more different methods standard (default) parameters must be used. Additionall tests of each method, shows that change of parameters highlights the character of the method (eg. inverse distance to a power shows more local extremas). As a result nine interpolation results were obtained, as shown in Figure 3. These show clearly that the image of vertical movements depends, to a significant degree, on the interpolation method applied. Visual comparison shows that the: nearest neighbor, inverse square, local polynomial and Shepard's method differ significantly from other methods. A more precise assessment of the interpolation methods usefulness can be done when applying such quality criterion as interpolation error. To determine the mean square error of the selected method one should calculate the post-fir residuals.

The post-fir residuals *(ε)* was calculated using following formula:

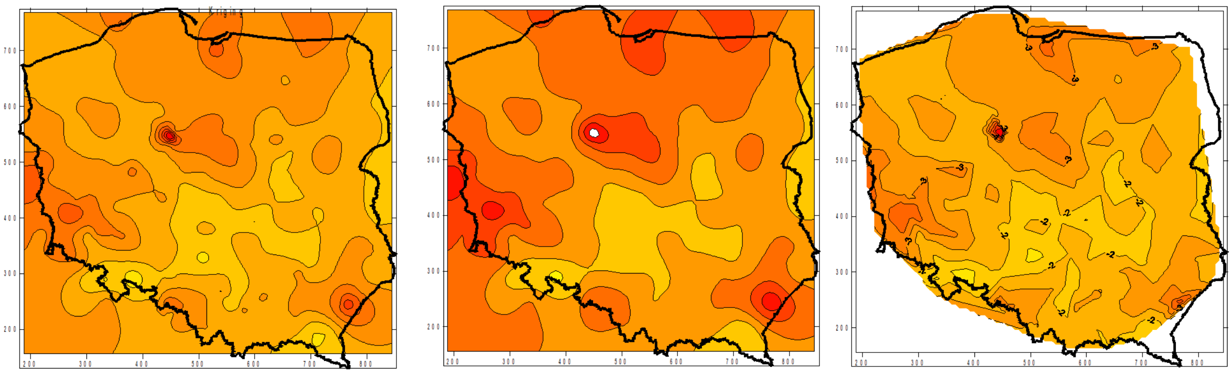
$$\varepsilon = V_{data} - V_{\text{int}} \ , \qquad (1)$$

where: V_{int} -interpolated value of movement in node,
V_{data} -movement from source data.

Interpolation error m was calculated using following formula:

$$m = \pm\sqrt{\frac{\sum \varepsilon^2}{n}} \qquad (2)$$

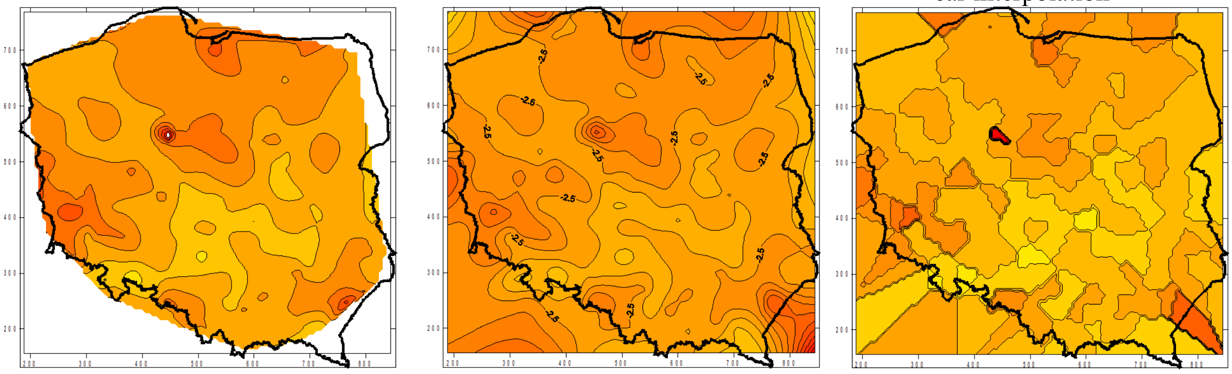
On initial assessment of the interpolation results obtained from respective methods, the model created by the modified Shepard method was discarded, due to its significant simplification. The results obtained from interpolation with the remaining



(a) Kriging

(b) Radial basis function

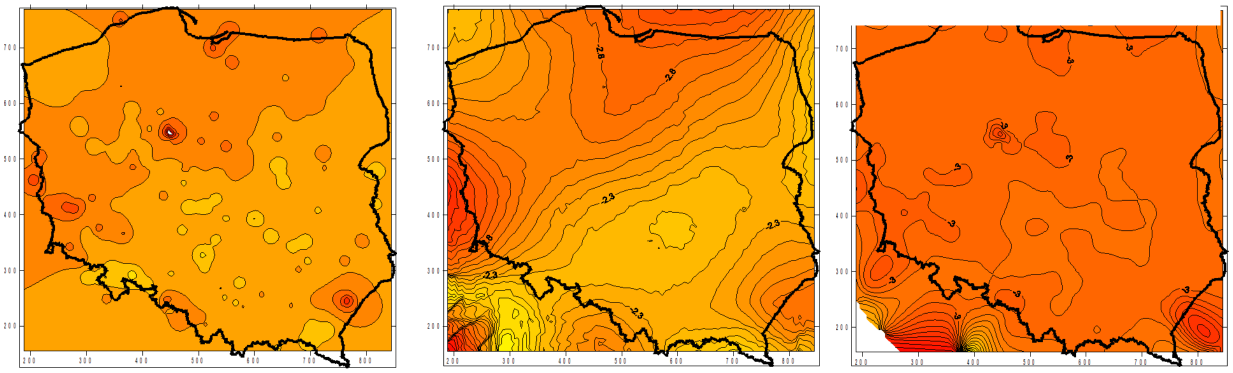
(c) Triangulation with linear interpolation



(d) Natural neighbor

(e) Minimum curvature

(f) Nearest neighbor



(g) Inverse distance to a power

(h) Local polynomial

(i) Modified Shepards method

Fig. 3 Results of vertical crustal movements interpolation in Poland obtained from different methods.

methods were qualified for further analysis. The true errors obtained are shown in Figure 4. Due to the high influence of points/nodes in Inowroclaw and Szadlowice on errors obtained, the authors discarded them in further analysis. The mean errors obtained through various interpolation methods are presented in Figure 5.

The following methods were examined:

- Kriging (El-Shejmy et al., 2005),
- The radial basis function (Hardy, 1990; Wielgosz et al., 2003),
- Triangulation with linear interpolation (Weng, 2006),
- Natural neighbor (Ledoux and Gold, 2004),
- Minimum curvature (Briggs, 1974),
- Nearest neighbor (Yang et al., 2004),
- Inverse distance to a power (El-Shejmy et al., 2005),

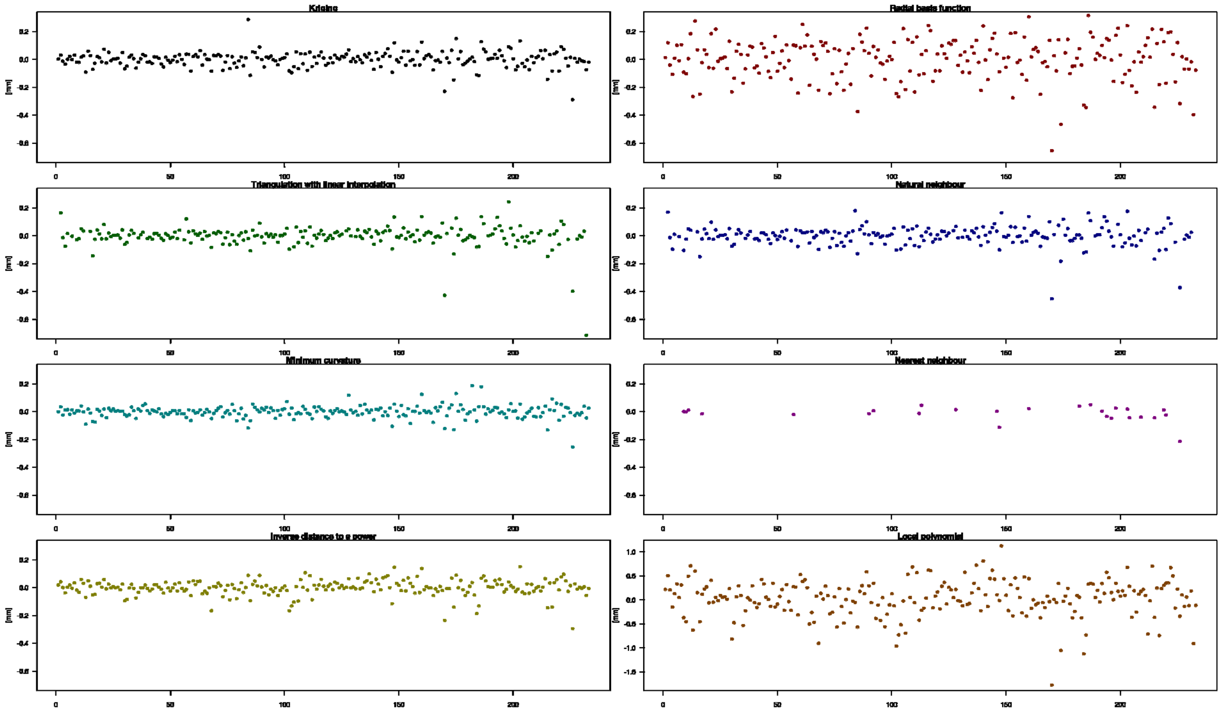


Fig. 4 Calculated residuals *e* of each interpolation method.

- Local polynomial of 10th degree (Gasca and Sauer, 2000).

2.2. *HYPOTHESIS TESTING*

Statistical hypothesis were applied to test the following conditions:

1. Does the average residuals significantly differ from zero from the statistical point of view?
2. Is the mean error significant from the statistical point of view?
3. Do the methods have equal or different variance?

A separate statistical test was used to test each condition. The following statistical tests were applied:

- T Student Test (Table 1)
- CH Square Test (Table 1)
- Test F (Table 2)

Table 1 clearly shows that from the statistical point of view, in case of the Nearest Neighbor method, the variance differs from zero significantly (T-Student Test), and is not representative as proven in CH Square Test. Table 2 shows that from the statistical point of view, variances for the following methods differs significantly: radial basis function, triangulation, and natural neighbor, as compared to the kriging method.

3. THE USE OF CHOSEN INTERPOLATION METHODS AVAILABLE IN GIS PACKAGES TO MODELLING OF VERTICAL MOVEMENTS

Some interpolation methods are available in GIS/ Image Processing packages as the tools allowing geodata input and integration into GIS-Raster modeling procedures. We have chosen two popular software packages: ENVI 4.1 of Research Systems and IDRISI32 of Clark Labs. Both packages incorporate the algorithms for interpolation of discrete elevation data for "continuous" DTM generation (Weibel and Heller, 1991). The second one includes also an algorithm for creation of an potential surface. In our approach the algorithms of DTM/DEM generation have been used for interpolation of vertical movements in chosen grid cells.

3.1. *IDRISI32 - INTERPOL FUNCTION*

Interpolates a full surface from point data. INTERPOL first requires that you specify the interpolation procedure. You may choose to interpolate a digital elevation model or to calculate a potential surface. The first option interpolates a Digital Elevation Model by means of a distance-weighted average defined by user. The second option evaluates a Potential Model. User has to input the name of the vector file containing the point data, and a name for the output image to be created. Then the distance weight exponent has to

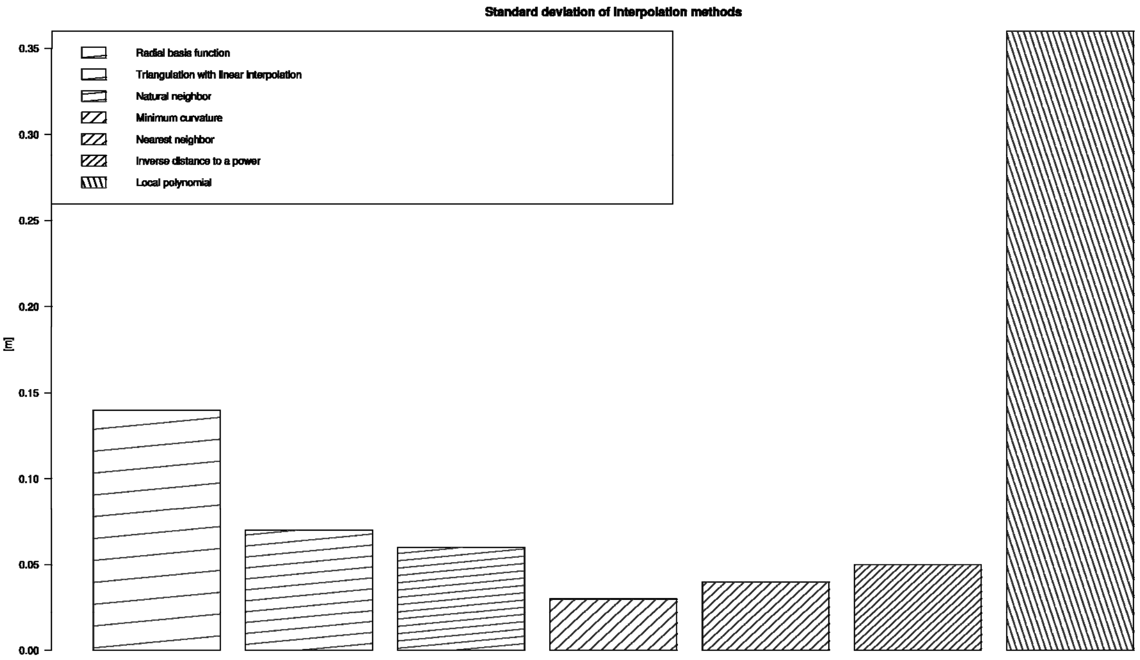


Fig. 5 Mean errors of each interpolation method.

Table 1 Results obtained from T-Student test and CH square test.

test	Kriging	Radial basis function	Triangulation	Natural neighbor	Minimum curvature	Nearest neighbor	Inverse distance to a power	Local polynomial	Theoretical value
T-student	0.09	0.51	-0.60	-0.13	-0.28	-1.31	-0.25	1.20	2.26
Ch- square	233	233	233	233	233	233	233	233	277

Table 2 Accuracy comparison, Test F.

Critical value **0.80**		Kriging	Radial basis function	Triangulation	Natural Neighbor	Minimum Curvature	Nearest Neighbor	Inverse distance to a power	Local Polynomial
	Variance of sample	*0*	*0.02*	*0.01*	*0*	*0*	*0.01*	*0*	*0.14*
Kriging	*0.00*		0.13	0.53	0.68	**1.58**	0.55	**0.98**	0.02
Radial basis function	*0.02*			**3.99**	**5.11**	**11.85**	**4.12**	**7.3**	0.17
Triangulation	*0.01*				**1.28**	**2.97**	**1.03**	**1.83**	0.04
Natural Neighbor	*0.00*					**2.32**	**1.81**	**1.43**	0.03
Minimum Curvature	*0.00*						0.35	0.62	0.01
Nearest Neighbor	*0.01*							**1.77**	0.04
Inverse distance to a power	*0.00*								0.02
Local Polynomial	*0.14*								

be used. Two (the default setting) is commonly chosen, yielding a weight equal to the reciprocal of the distance squared.

User has to indicate whether or not he wishes to limit interpolation to a 6-point search radius about each interpolated point. The default is set to use this specified radius. A minimum of 4 control points is needed to use the search radius option. With the search radius option, INTERPOL determines the value of each cell based on the values of only near-by control points. "Near-by" is determined by setting a search radius that should lead, on average, to 6 control points being found. If less than 4 control points are found, the search radius is temporarily increased until a sufficient number are found. If more than 8 control points are found, the search radius is temporarily decreased until only 4 - 8 control points are found. In addition, with a very small number of control points, it is recommended that the search radius option be disabled. If the user wishes a smoother result and has a fairly evenly distributed set of control points, he may wish to de-select this setting. This causes all control points to be used in the interpolation of each point. In our calculation the option of a 6-point search radius has been chosen in order to limit the influence of the farther, non correlated points.

3.2. *TINSURF*

TINSURF generates a raster surface image from a TIN model and the vector point file that defines the verticles of the TIN. The TIN module generates a triangulated irregular network (TIN) model (Rahman, 1994) from vector point input data. The triangulation is accomplished using either a non-constrained or constrained Delaunay triangulation. The Delaunay triangulation process is commonly used in TIN modeling and is that which is used by the IDRISI module TIN. A Delaunay triangulation is defined by three criteria: 1) a circle passing through the three points of any triangle (i.e. its circumscribe) does not contain any other data point in its interior; 2) no triangles overlap, and 3) there are no gaps in the triangulated surface. For each raster pixel in the output image, an attribute value is calculated. This calculation is based on the positions and attributes of the three vertex points of the triangular facet within which the pixel center falls and the position of the pixel center. Each pixel center will fall in only one TIN facet, but a single facet may contain several pixel center points. The quality of the generated surface will depend most significantly on the quality of the input data. The logic of interpolating pixel attribute values is as follows:

1. Solve the following set of simultaneous equations for A, B and C:

   $$H_3 = Ax_3 + By_3 + C \tag{3}$$

   Where H 1,2,3 are the attribute values (e.g. elevations) of the three triangle facet vertices and (x,y) 1,2,3 are their reference system coordinates.

2. Given A, B and C, as derived above, solve the following for H_p:

   $$H_p = Ax_p + By_p + C, \tag{4}$$

   Where H_p is the attribute of the pixel and $(x,y)_p$, is the reference system coordinate of the pixel center.

3. Assign the pixel the attribute value H_p. The algorithm proceeds on a facet-by-facet basis, so the derivation of A, B, and C in step 1 is carried out only once for all the pixels that fall within a single facet.

3.3. *ENVI - RASTERIZING POINT DATA*

Rasterize Point Data function is usefull to interpolate irregularly gridded data into a raster image. ENVI's griding function uses also Delaunay triangulation of a planar set of points. After the irregularly gridded data points are triangulated, they are interpolated to a regular grid. One can use linear or smooth quintic polynomial interpolation. It is possible to select extrapolation for grid points outside of the triangulation area. The grid points are read from an ASCII file and different input and output projections are supported. Three chosen methods have been used for interpolation of movement values in the regular raster cells of 5x5 km size. In the case of IDRISI linear interpolation the weight of 1 and the search radius of 6-point have been applied. For the purposes of more continuous surface presentation, better adapted to the human perception, the pixels have been expanded by the factor 5 after each interpolation, it means transformed to the size of 1x1 km and filtered using 3x3 window of mean filter (smoothing).

The methods of TINSURF and QUINTIC, both using the philosophy of triangulation networks do the interpolation strictly inside the network. There is a possibility of extrapolation to the image bounds but we have no justified information about the movements on the corners of the image. In order to check and compare the results of the interpolations we have used a regular grid of about 350 points (20x20) as shown in the Figure 9.

We have had 6 raster images (2 images for 3 methods filtered and non-filtered) with pixel size of 1x1 km showing interpolated values of movement for each pixel. We have compared them as shown in the Figure 10. The differences of graphs are quite clearly visible. Amongst the 6 results we have chosen the minimum and maximum of movement value regardless the method which has generated this

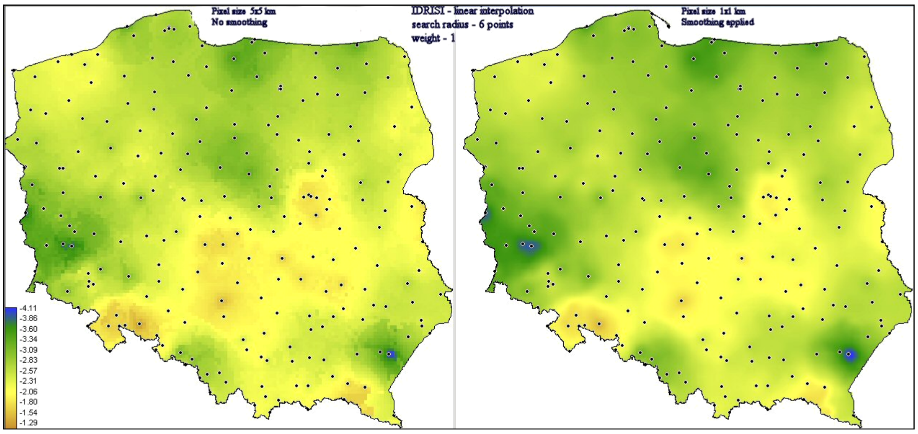


Fig. 6 Vertical movements interpolated using Idrisi linear interpolation, search radius 6 points, weight -1; without (left) and with smoothing filtering (right).

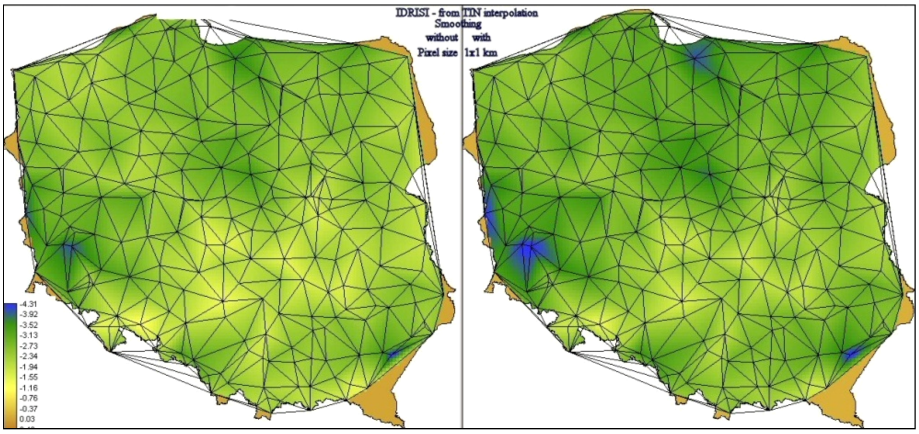


Fig. 7 Vertical movements interpolated using Idrisi "from TIN interpolation" without (left) and with smoothing filtering (right).

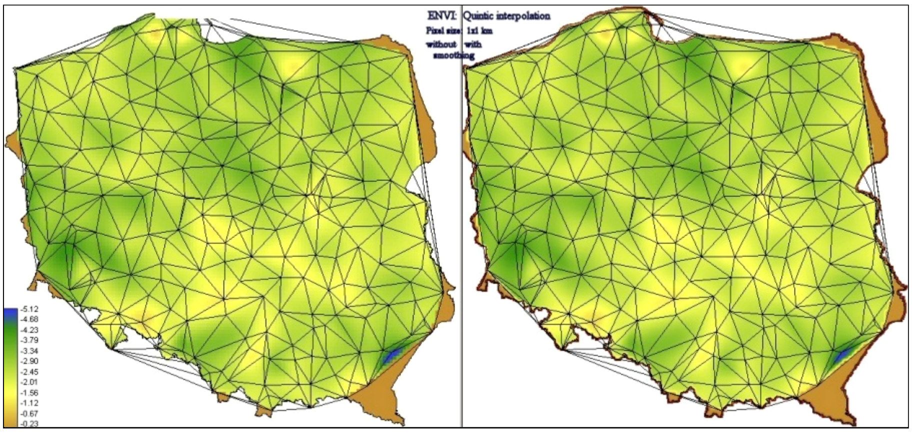


Fig. 8 Vertical movements interpolated using ENVI Quintic interpolation without (left) and with smoothing filtering (right).

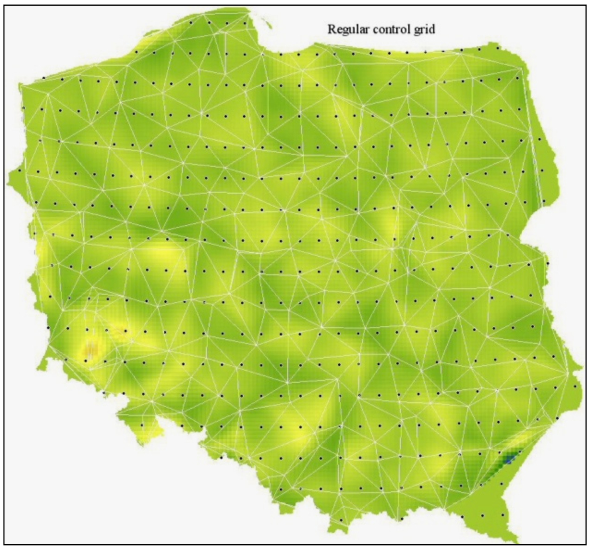


Fig. 9 Regularly distributed control points used to compare the results of interpolations.

value. It can be noted that the difference (discrepancy) between the max. and min. values is about 0.3 - 1.0 millimeter, in some cases exceeding 1.0 mm. These large anomalies are due to the fact that some control points are located outside of interpolation area for TIN and Quintic methods. We have next eliminated these points and more adequate results can be seen on the Figure. 11. The discrepancies are of order of 0.9 mm. This is confirmed in the next Figure 12 where we have shown the differences between 3 interpolation methods not affected by any improving image filtering. The 80 % of differences are in the range of +/- 0.2 mm, 10 % in the range of 0.2-0.4 mm and the last 10 % in the range of 0.4-0.6 mm. The influence of image low pass filtering, in this case - mean of 3x3 pixels, is shown in Figure 13. It is visible that such a filtering changes the initially calculated values about 0.03-0.1 mm.

4. INTERVERTIC CALCULATION OF VERTICAL MOVEMENT FOR ANY PLACE IN POLAND

Having in mind the need to obtain vertical movement for any place in Poland (bigger amount of data for movement analysis, analysis of displacements of POLREF points) and large error of reading such a movement from map (Kowalczyk, 2006a; Wyrzykowski, 1987) the decision to develop own algorithm and software was made. The existing software (despite it is rather expensive) can not interpolate vertical movement in point specified by user. Software is developed in C++ using Linux Fedora Core 6 and gcc 4.0.2 as programming platform. The software is using data from model 2006. Point in Inowroclaw was replaced with three others close points. InterVertic uses three methods to determine vertical movement:

- linear (three points)
- polynomial (10 points)
- least squares collocation method was used with local covariance function of Hirvonen (50 km radius)

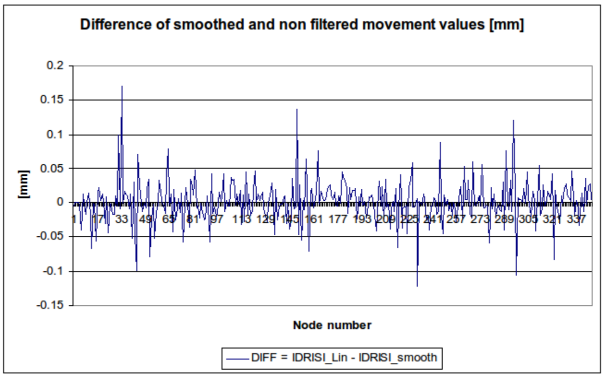


Fig. 13 The influence of image smoothing on the movement values.

29 points placed equally on the territory of Poland (about 15 km from nodes) were used for testing. Comparison of three different methods was used as a verification criteria. The results of interpolation made by InterVertic are shown in Figure 14. Polynomial and collocation method gave similar results $\delta v_{WK} = \pm 0.1 - T - 0.2$ *mm/year.* Linear method gave a little bit worse results: $\delta v_{WL} = \pm 0.5$ *mm/year,* $\delta y_{KL} = \pm 0.6$ *mm/year.* Linear method does not work well when interpolation takes place near the edge of test area. In such situation the source points are on the one side of interpolated point or are too far away. Polynomial and collocation methods were chosen for further use. They give small differences on each tested point. The difference grows with the value of movement (eg. Jarosaw $\delta v_{WK} = \pm 1.0$ *mm/year).* It is caused by the amount of points taken into account: polynomial 10, collocation 50 km radius.

5. RESULTS

1. The interpolation method which has been used has significant influence on data obtained from earlier vertical movements maps. Before using a data it is very important to know which technique of interpolation was taken to create them.
2. Three of the tested methods of interpolation from Surfer can be used: natural neighbor, triangulation of linear function and kriging. Multiquadric method gives too big errors and Minimum curvature fits source data in an affected way.
3. The present model of vertical movements should become a subject of geological analysis, which is actually under investigation.
4. Models created with GIS software approximates movements better then other methods.

5. First version of InterVertic software gives good results. In further work it can be used to condense the vertical movement network to labour more detailed kinematic model of these movements.

6. CONCLUSION

The interpolation method significantly influences the results obtained. This is crucial while obtaining data from a map, therefore beforehand one should learn which interpolation technique has been applied to prepare a map. The examination of the interpolation methods shows that all methods except the Shepard's can be used to model the vertical movements, because this method presents results in a very general way. The choice of the method depends of the characteristics one wants to emphasize. The nearest neighbor shows vertical movement in local areas. The radial basis function results in oversimplification of the model and does not include highest and lowest value points. The figure is schematic. The inverse distance to a power method perfectly depicts the extremes. Kriging, triangulation, natural neighbor give similar results, nevertheless kriging and natural neighbor present clearer graphic results. The triangulation and natural neighbor methods are limited to the area of study, while kriging reaches beyond which can cause distortion on the borderline of the analyzed area.

The interpolation errors are similar in most cases: 0.04-0.08 mm/year. In case of radial basis function and local polynomial of higher degree methods differ slightly, with corresponding values/rates of 0.15 mm/year and 0.20 mm/year. The analyses conducted show that the best method for vertical crustal movements on the territory of Poland is the natural neighbor. The interpolation does not exceed the area on which the data points are located, the figure is clear and transparent, the interpolation error did not exceed 0.07 mm/year, while the residuals vary from 0.2 to 0.1 mm/year.

REFERENCES

Briggs, I.: 1974, Machine contouring using minimum curvature. Geophysics, 39.

El-Shejmy, N., Valeo, C. and Habib, A.: 2005, Digital terrain modeling. Artech House XI.

Gasca, M. and Sauer, T.: 2000, On the history of multivariate polynomial interpolation. Journal of Computational and Applied Mathematics, 122 (1-2), 23–35.

Hardy, R.: 1990, Theory and applications of the multiquadric-biharmonic method, interpolation, natural neighbour. Computers Mathematical Applications 19 n. 8/9.

Kmiecik, J. and Sieradzan, R.: 1994, Selected problems of fundamental leveling network in Poland. Przegląd Geodezyjny 4, Warsaw, 6–7.

Kowalczyk, K.: 2006a, Modeling the vertical movements of the earth's crust with the help of the collocation method. Reports on Geodesy 6 (76), Warsaw.

Kowalczyk, K.: 2006b, New model of the vertical crustal movements in the area of Poland. Geodesy and Cartography XXXII, Vilnius.

Kowalczyk, K.: 2006c, Model of the vertical crustal movements in the area of Poland. Ph.D. thesis, University of Warmia and Mazury in Olsztyn.

Ledoux, H. and Gold, C.: 2004, An efficient natural neighbour interpolation algorithm for geoscientific modelling. In Peter F. Fisher, editor, Developments in Spatial Data Handling-11th International Symposium on Spatial Data Handling.

Rahman, A.A.: 1994, Design and evaluation of tin interpolation algorithms. EGIS Foundation.

Weibel, R. and Heller, M.: 1991, Digital terrain modelling. Geographical Information Systems 1, 269–297.

Weng, Q.: 2006, Progress in Spatial Data Handling. Springer Berlin Heidelberg.

Wielgosz, P., Grejner-Brzezinska, D. and Kasjani, I.: 2003, Regional ionosphere mapping with kriging and multiquadric methods. Journal of Global Positioning Systems, 2, No. 1.

Wyrzykowski, T.: 1987, New model of the vertical crustal movements in the area of Poland. Pr. Inst. Geod. XXXIV 1(87).

Yang, C. S., Kao, S. P., Lee, F. B. and Hung, P . S . : 2004, Twelve different interpolation methods: A case study of Surfer 8.0. In: Proceedings of the XXth ISPRS Congress, 35,. p. 778785. URL http://www.cartesia.org/geodoc/isprs2004/comm2/papers/231.pdf

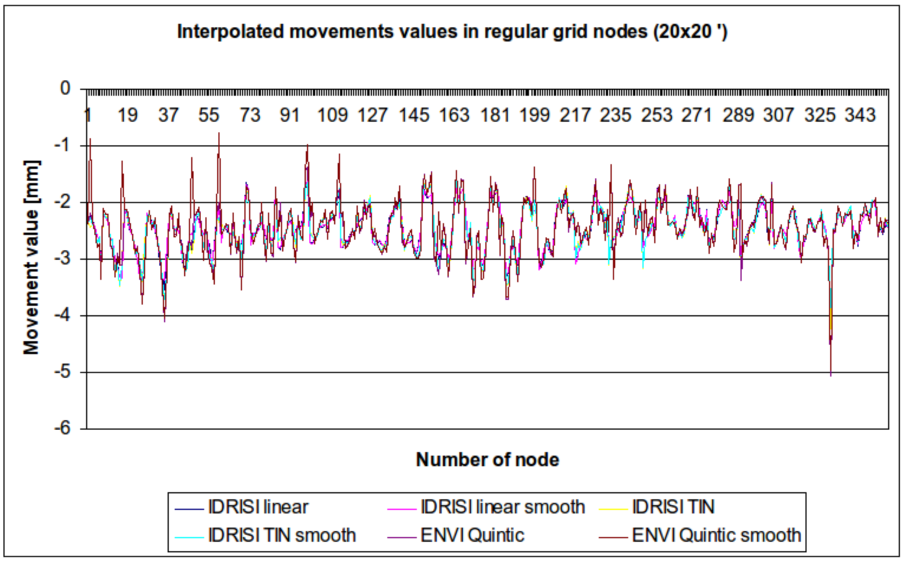


Fig. 10 The values of movements calculated for all control points.

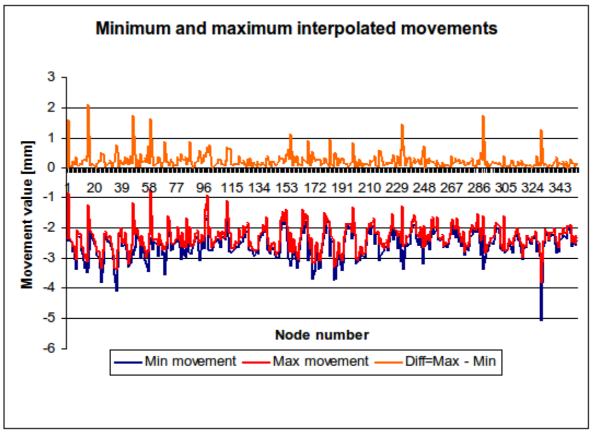


Fig. 11 The min. and max. values of movements extracted from interpolated data.

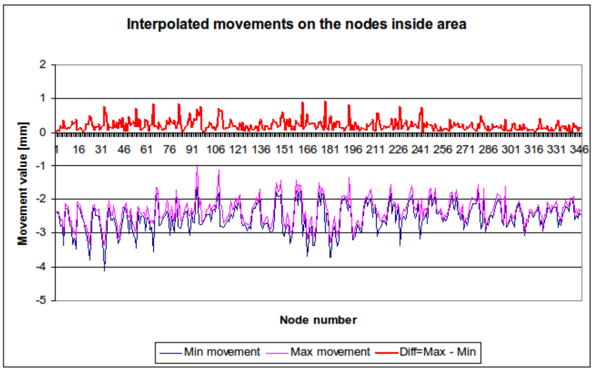


Fig. 12 The min., max. and difference values of movements extracted from interpolated

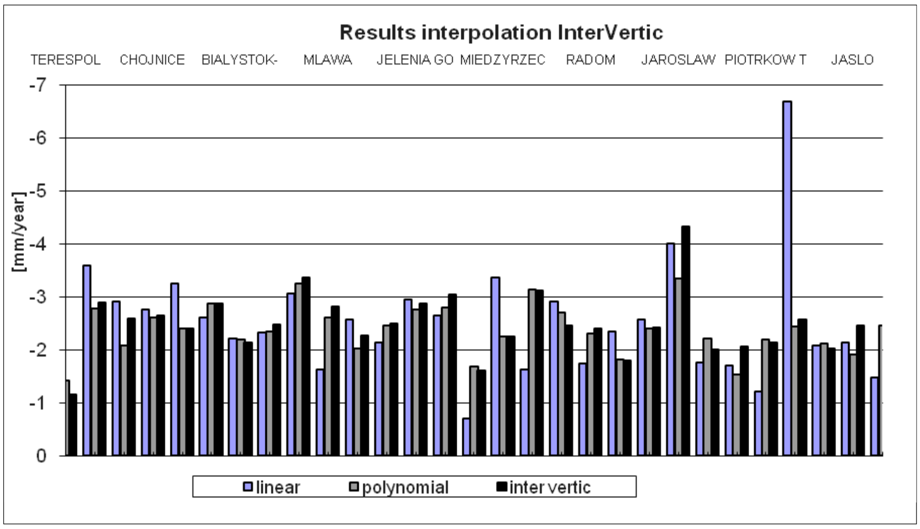


Fig. 14 Results from InterVertic.