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RECENT PROGRESS IN NON-RIGOROUS COMBINATION METHOD OF SPACE GEODETIC TECHNIQUES

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ABSTRACT
The combination method of results of different space geodetic techniques was recently improved and used to process approximately eight-year data obtained by three space geodetic techniques. The results are compared with the results obtained by the old approach of this method and finally with the solutions of ITRF 2005.

KEYWORDS: Earth orientation parameters, combination of space geodetic techniques, station coordinates, ITRF 2005

1. INTRODUCTION

The body of the Earth is changing its orientation in the space. The orientation can be defined in many ways, for example, by three Euler's angles. This way is not very practical because of quick changing of the values of those angles. The best manner is using five rotation angles, called Earth orientation parameters, EOP, which tie the Earth-fixed coordinate system ITRS to the celestial reference frame, GCRS. The EOP are two coordinates of the intermediate pole with respect to the ITRS, x_P, y_P, and the angle, which characterizes irregularity of the Earth's proper rotation, ERA, and finally, two components of the celestial pole offset, dX, dY, which denote the observed corrections to the adopted precession-nutation model (Capitaine at al., 2003 and Mathews et al., 2002).

Monitoring of EOP and station positions are provided by modern space geodetic techniques: Global Navigation Satellite System (GNSS), Very Long-Baseline Interferometry (VLBI), Satellite and Lunar Laser Ranging (SLR, LLR) and Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS). Only VLBI and LLR techniques can connect the both systems (terrestrial and celestial), consequently only these techniques provide data associated with precession and nutation.

Each technique has several analysis centers, associated to the corresponding service, which generates an intra-technique combined product, primarily EOP and station coordinates. The best way how to get the most representative EOP is to combine all solutions together, respecting the advantage of each technique by appropriate weighting.

There are basically two possibilities of the combination:

- The rigorous approach needs either to process the original data at a) the level of observation equations or b) to solve a new system of normal equations created from the normal equation or covariance matrices of individual techniques (e.g. Gambis et al., 2006). It yields an exact solution so that efforts are made by several groups to develop the necessary procedures, but:
- The first approach is very complicated, observation equations are very complex and additional "internal" unknowns are necessary. Up to now, it has been tested only on a limited network. The "normal equation" approach is apparently simpler but even in this case the problem of applying properly all specific constraints to the new system to improve its generally lower stability remains still open.
- It is also possible to derive an appropriate solution by combining the results of individual techniques omitting co-variances i.e. omitting interrelations between the input parameters, which are treated as independent. We use this so called "non-rigorous approach" because it yields a stable solution, if some simple constraints are applied.

The key of all combination methods are collocation stations where more than one geodetic technique observe. The following method was proposed by Pešek and Kostelecký (1999) and later it was tested (Pešek and Kostelecký, 2006) on one-year data measured by four space geodesy techniques.

Since that time, the method has been improved several times, and described in detail elsewhere (Štefka and Pešek, 2007; Štefka at al., 2009 and Štefka at al. 2010). The recent improvement is described in this paper.

2. NON-RIGOROUS COMBINATION METHOD

The basic idea of this method is to combine station position vectors, $\mathbf{x}_C$, in the celestial reference frame, where that are functions of all unknowns to be solved. The transformation from ITRS to GCRS (i.e. $\mathbf{x}_T \rightarrow \mathbf{x}_C$) was modified to serve our purpose:

$$\mathbf{x}_C = \mathbf{Q}(t)\mathbf{R_3}(\mathrm{ERA})\mathbf{R_3}(-s')\mathbf{R_1}(y_P)\mathbf{R_2}(x_P)\mathbf{R}(\mathbf{p})\ \mathbf{x}_T, \tag{1}$$

where $\mathbf{Q}(t)$ is the precession-nutation matrix, s' shift from ITRF x axis to the terrestrial non-rotating origin, TIO, along the intermediate equator, and x_P, y_P are coordinates of the CIP pole. $\mathbf{R_i}$ is the matrix of rotation along the i-th axis. Finally, $\mathbf{R(p)}$ is the matrix of the seven-parametric transformation, which is considered to be a linear function of time.

Since satellite techniques do not provide EOP related to precession and nutation, the method treat them as being known function of time and so only the coordinates of the pole and proper rotation are combined. Thus the input data for the combination consists of $\boldsymbol{M}$ sets of EOP (x_P, y_P, ERA, X, Y)$_{\boldsymbol{m}}$ and corresponding sets of station coordinates $(x_T)_{\boldsymbol{m}}$, m = 1, … , M, as derived by the analysis centers for the individual techniques. Values of UT1-UTC were firstly derived from integration of LOD measured by GPS and SLR with connection to UT1-UTC measured by VLBI. Secondly, they were converted into ERA using the standard formula (e.g. McCarthy, 2003).

Partial derivatives of the formula (1) with respect to any unknown, U, yields observation equations of the form:

$$\sum_j \frac{\partial x_C}{\partial U_j} dU_j \quad = \quad x_C\big|_{obs} - x_C\big|_0 \quad + \quad r \ , \tag{2}$$

where the "observed" vectors $x_C\big|_{obs}$ are calculated from the respective input data and $x_C\big|_0$ are functions of adopted *a priori* values of the unknowns and r is residual.

The EOP are calculated for each epoch independently of the others so that constraints in the form of pseudo-observations have to be added, that comes from the smoothing method (Vondrák, 1969 and 1977):

$$L_i''' = \sum_{k=0}^{3}\left(6 \prod_{j=0, j\neq k}\left(x_{i+k} - x_{j+i}\right)^{-1}\right)E_{i+k}. \tag{3}$$

These constraints tie the value of the respective EOP, E, at four adjacent epochs and assume that the individual values of EOP lie on a smooth curve. By weighting the above equation, a smoothness of the solution is controlled in order to retain as much as 99% of the signal with period greater than 5 days.

The system, as it was introduced, is singular. To remove the singularity, a no net-rotation constraint, minimizing mutual shifts and preserving the system as a whole, has to be introduced,

$$\sum p^T p = \min. \tag{4}$$

The system of observation equations and all additional constraints are solved using modified Cholesky decomposition proposed by Čepek and Pytel (2005). It solves the sparse matrix of normal equations very effectively.

3. PARTICULAR STEPS OF RECENT IMPROVEMENT

Since 2006, the non-rigorous method has been improved several times. The recent comparison of computed station coordinates with ITRF 2005 (Štefka 2010) showed relatively big differences at stations with low number of observations. This undesirable effect was originated in solving all station coordinate residuals for each technique together where stations with big number of observations had bigger contribution to solution of seven-parametric transformation then other stations.

3.1. *THE NEW TRANSFORMATION*

The transformation equation (1) was modified in order to compute directly station position vectors for each observation epoch. The part of the transformation equation related to seven-parametric transformation was removed so that the new transformation has following form:

$$\mathbf{x}_C = \mathbf{Q}(t)\mathbf{R_3}(\mathrm{ERA})\mathbf{R_3}(-s')\mathbf{R_1}(y_P)\mathbf{R_2}(x_P)\mathbf{x}_T, \tag{5}$$

where all matrices have the same form as in equation (1). The new system that is being derived from above equation (5) need to have new no net-rotation constraints, which are described in the next part of this chapter.

3.2. *THE NEW NO NET-ROTATION CONSTRAINS*

The constraint (4) was replaced by the new form of no net-rotation constraints related to Tisserant conditions:

$$\begin{aligned} &\sum_{i=1}^{N} dU_i = 0, \\ &\sum_{i=1}^{N} U_i^0 \times dU_i = 0, \\ &\sum_{i=1}^{N} \left(U_i^0\right)^T \bullet dU_i = 0. \end{aligned} \tag{6}$$

Where the capital letter, U, stands for unknown station coordinates and small superscript means *a priori* values of the unknown station coordinates. These constraints satisfy the minimization of the relative kinematic energy of the system.

The unknowns are:

- daily values of x_P, y_P,
- daily values of (UT1 – UTC),
- station positions determined for each measured epoch.

4. DATA AND NUMERICAL SOLUTION

The data cover the eight-year period 2000 to 2008 and were obtained by three types of space geodetic techniques, namely, GNSS, VLBI, and SLR. First two were taken from the IERS Combination Pilot Project database (*iers1.bkg.bund.de/projects/combination/intra-technique/*). The last one, the constrained ilrsb solution was used, as published by the ILRS analysis center (*ftp://cddis.gsfc.nasa.gov/pub/slr/products/pos+eop/*). The GNSS and SLR data were weekly SINEX (Solution in Independent Exchange format) solutions, from which the EOP and station coordinates were extracted. VLBI data consist of *per observation* singular normal equation matrices. To solve them, the constraints have to be added to tie station coordinates to the VTRF 2005 frame (Nothnagel, 2005) with the *a priori* precision of 5 mm. Since GPS and SLR do not provide celestial pole offsets, only the x_p, y_p, and UT1-UTC are combined.

The data were then used in two different ways. Firstly, the data were processed by the old approach where the system is related to equation (1) and secondly, by the new approach where the system is related to equation (5).

The techniques entered the adjustment in both cases with the following weights: 1.44, 0.8 and 1.0 for GPS, SLR, and VLBI, respectively. The values of weights were taken over from the fathers of the present method to keep continuity. Moreover, the mutual ratio of VLBI to SLR weight is nearly the same as that applied to the IERS Dynamo program (Richard et al., 2008).

Then corrected station coordinates were compared with those published by ITRF 2005 (Altamimi et al., 2007). Both comparisons are shown in Figures 1 and 2. One can see that the differences are quite smaller (approximately 0.5 cm) in the second Figure so that we can declare that the new approach gives more reasonable results that fit better to the ITRF 2005.

Since the EOP results of the two approaches are nearly similar, the differences are 0.08 mas and 0.13 ms for polar motion x_P, y_P and the time correction UT1-UTC, respectively, we compared only the new EOP solution with ITRF 2005. The comparison is depicted in Figure 3. Except a few peaks exceeding the level of 1 mas and 1.5 ms, the differences are smaller than 0.2 mas and 0.2 ms for polar motion and time correction, respectively. Those peaks are mostly related to worse results of SLR that were not removed.

5. CONCLUSION

A method for non-rigorous combination of results of different space geodetic techniques to obtain representative sets of the Earth orientation parameters and station coordinates was used to process approximately eight-year data in two different ways: firstly, the data were processed by the old approach where transformation parameters are only derived for each techniques (see Eg. 1), and secondly, by a new approach where individual station coordinates are derived for each epoch according to Eq. 5. Their comparison turned out that station coordinate results of new approach fit better to the ITRF 2005. On the other hand, the EOP comparison showed that EOP results of the two approaches are nearly similar, the differences are 0.08 mas and 0.13 ms for the polar motion x_P, y_P and the time correction UT1-UTC, respectively. All comparisons are depicted in Figures 1, 2 and 3.

We can conclude that the presented approach is better than previous one and that it can be used for testing results of regular combinations, at least at the early stages of their development.

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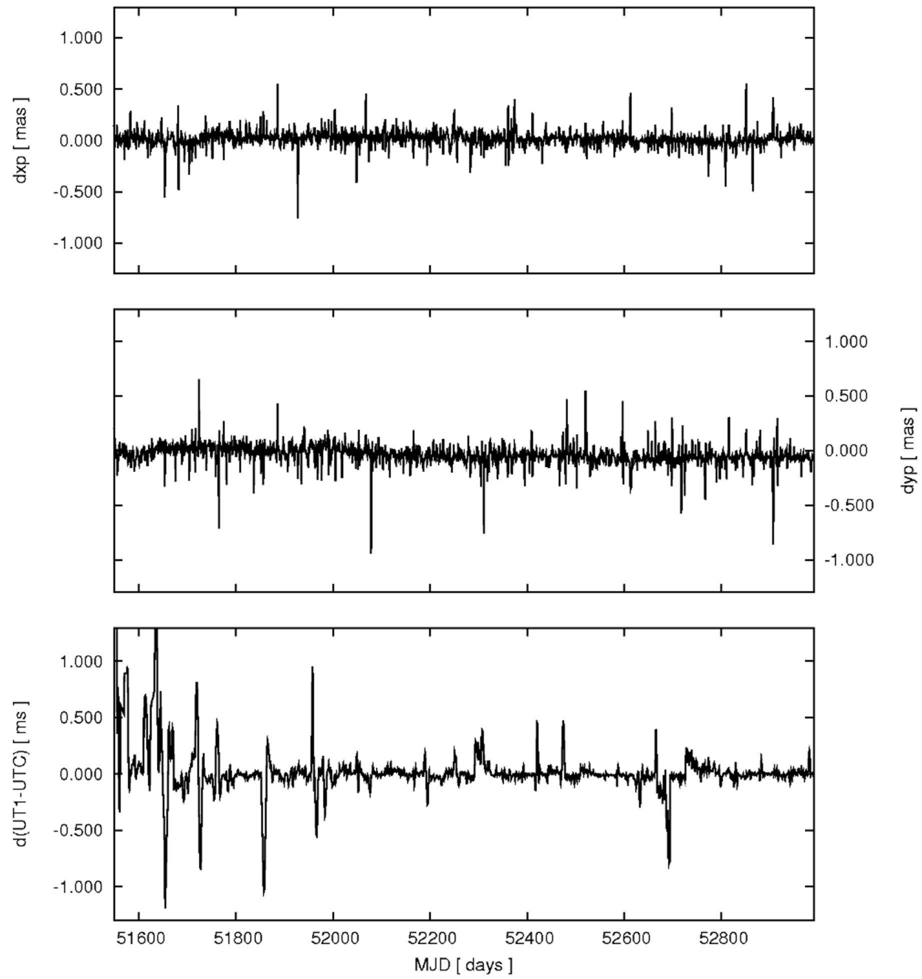

Fig. 3 The figure shows comparison of the new EOP solution with that published by ITRF 2005. The differences are 0.144 mas, 0.148 mas and 0.159 ms for the polar motion x_P (top), y_P (center) and the time correction UT1-UTC (bottom), respectively.

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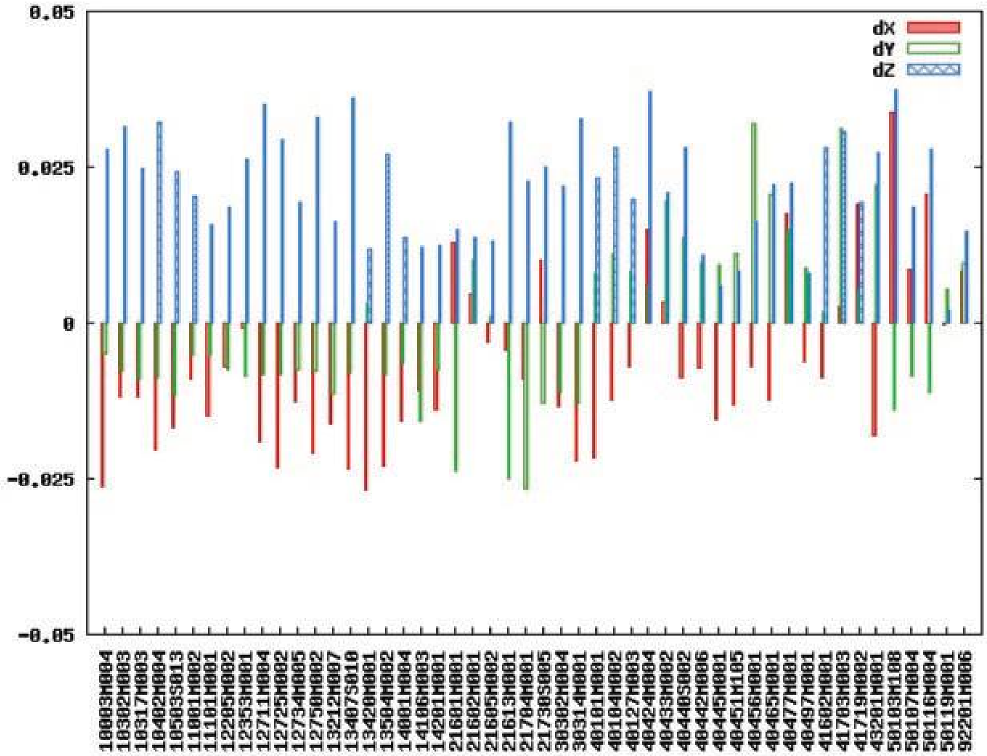

Fig. 1 Comparison of station coordinates of the old approach and ITRF 2005 at the beginning (MJD 51550) of the eight-year period. The differences are given in meters.

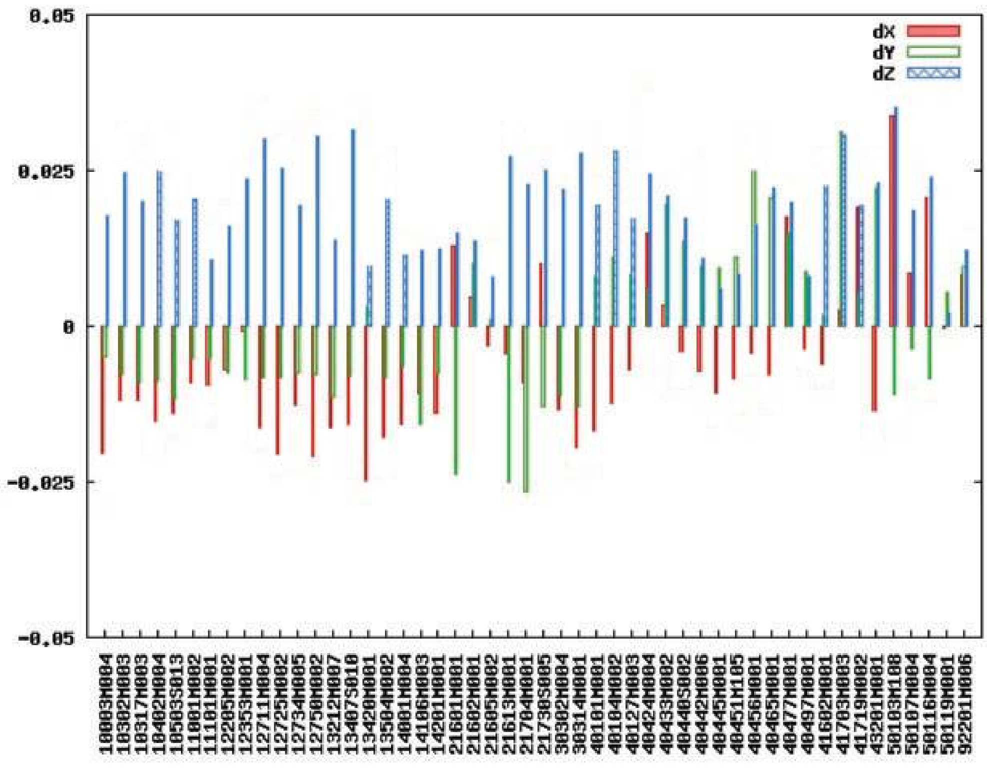

Fig. 2 Comparison of station coordinates of the new approach and ITRF 2005 at the beginning (MJD 51550) of the eight-year period. The differences are given in meters.