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GEOLOGICAL CONDITIONS AND LOCAL CHANGES OF VERTICAL DEFLECTIONS

Zbigniew SZCZERBOWSKI*, Piotr BANASIK and Jacek KUDRYS

AGH University of Science and Technology, Faculty of Mining Surveying and Environmental Engineering, Mickiewicz Avenue 30, 30-059 Cracow, Poland
**Corresponding author's e-mail:* szczerbo@uci.agh.edu.pl

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ABSTRACT
The paper presents local changes of plumb line direction in the area of Inowrocław, central Poland. The changes were determined by geodetic (GPS, leveling) and gravimetric surveys. The presented conclusions point out that increasing resolution of a precise quasigeoid model in areas with variable distribution of rock mass is necessary. The final results of the paper are suggestions concerning a practical approach applied to obtain proper values of plumb line deflections that can be useful for further detail research for making more precise local models of the quasigeoid.

KEYWORDS: salt tectonics, gravitational effect, leveling, plumb line direction

1. INTRODUCTION

The problem of vertical deflection is a matter of debate. That is a consequence of applications of GPS technique in determinations of normal heights in engineering practice. Transformation between height systems involves utilization of a quasigeoid model for a given surveyed area. The "quality" of a model can be concerned as an agreement between modeled and measured values. The requirements define accuracy: they can be different for research purposes (e.g. discovery and description of the Earth's geometry) and different in engineering practice. Hence, the performance of any model needs to be validated on local scale. The misleading, occasionally used, term "precision" is a category beyond from survey standards as erroneously used to mean measurement accuracy (International Vocabulary of Metrology, 2008). Verification of a quasigeoid model (and vertical deflections evaluated from that model), e.g. provision of objective evidence that a given item fulfils specified requirements, involves another measurement procedure as presented in the paper gravimetric modeling.

The presented study demonstrates a role of geological conditions (varying density of rock mass) for vertical deflections on a local scale. So, the local changes of vertical deflections are discussed on an example of a quite small and flat area, featuring a distinctive gravity anomaly induced by a salt dome. The presented paper is a continuation of articles published in this journal before, where some geodynamical aspects of the Inowrocław salt dome were discussed:

- geodetic, recent activity of the salt structure observed by geodetic methods (Szczerbowski, 2007),
- geological, geomorphostructural evidence of its uplift history or structural features, such as: relief of the structure, its boundaries, etc. (Szczerbowski, 2007).

The presented study is a combination of geological and geodetic aspects in geodynamic research by analyzing gravity anomaly of the area and a parameter describing this phenomenon – vertical deflection by means of a GPS/leveling method.

2. THE SHORT OUTLINE OF THE AREA

The problem of accuracy of regional quasigeoid model is usually confined to the effect of "topographical mass" – a geomorphological feature resulting from elevation changes. In local-scale studies of accurate quasigeoid models for areas of a few square kilometers, it is more sensible to consider effects of diversity of rock mass distribution. It means that in areas of flat terrain, where changes in altitude do not exceed a few meters, significant changes of vertical deflection can occur as a result of gravity anomalies induced by rock mass distribution or ground water level variations and other reasons. Inowrocław is an example of such area, where a gravimetric model is essential for understanding the nature of evaluated vertical deflection changes occurring at a relatively short distance. Such an approach requires introduction to geological conditions of an area. In the case of Inowrocław, the main element controlling local geological setting is a salt dome. Some aspects of Inowrocław geology were presented in previous issues of this journal (Szczerbowski, 2007, 2010a).

The area has been studied by the AGH-UST as a geodynamic test area, where geodetic and geophysical surveys are carried out. This field laboratory was established for investigation of some of the scientific and engineering problems related to geodynamics (activity of the salt structure, hazards resulting from subsurface erosion). Some results of several campaigns were already discussed by Szczerbowski (2004, 2007, 2009a, 2009b, 2010a).

The most important facts about geology were obtained thanks to mining activity that was carried out for more than 100 years in this area. Specific geological conditions led to mass distribution and gravity anomaly, the amplitude of which in this relatively small area, a few square kilometers large, amounts to 7 mGal. The Bouguer anomaly evaluated on the base of gravimetric surveys (Łąka, 1981) is presented in Figure 1. As far as geodetic constraints are concerned, the area can be characterized by values of vertical deflections. The vertical deflection components were evaluated for the control point 3403 (Fig. 1), which position is determined by coordinates: φ=52°45'45.90228″, λ=18°15'13.59211″, h=123.369 m. Values of the components for this control point included in the Polish national network (POLREF), determined within the Leveling Geoid 2001 model (Pażus et al., 2002), are as follows: NS component of vertical deflection ξ = 3.6″ and EW component of vertical deflection η = 3.8″. These values can be considered as referential for evaluated and modeled changes of vertical deflection components, which will be discussed in further chapters.

3. MODELING OF LOCAL CHANGES IN THE DIRECTION OF PLUMB LINE

Deflection of the vertical is a term having a geodetic and gravimetric context. Hence, the modeled values should be verified by another approach, like the presented gravimetric modeling of the plumb line.

In geodetic studies there is a definition of vertical deflection commonly used:

difference between the true plumb line and its theoretical vertical (normal) direction on a global ellipsoid (Kamela, 1955).

Gravimetric modeling based on recognized geology of the area was carried out to determine gravitational effects of geological structures and gravity changes. The salt deposit as an anomalous body was approximated by a group of rectangular prisms what facilitated calculations of the gravitational effect (Szczerbowski, 2010b). The effect induced by density contrast Δρ (approximately -0.25 Mg/m^3) is a cause of gravity changes and changes in direction of plumb line at Inowrocław. Finally, the modeled values of gravity changes (especially horizontal components $-\Delta g_X$, Δg_Y) and changes in the direction of plumb line (with north-south component $\Delta\xi$ and east-west component $\Delta\eta$) were derived in a local coordinate system using formula obtained from trigonometric function combining a vector and its components (Sas, 1990):

$$\Delta\xi_{XP} = \frac{\Delta g_{XP}}{g} \cdot \rho'' \quad \text{and} \quad \Delta\eta_{YP} = \frac{\Delta g_{YP}}{g} \cdot \rho'' \tag{1}$$

where:

Δg_{XP}, Δg_{YP} – horizontal components of gravitational effect induced by anomalous body,

g – average value of gravity = 981000 mGal,

ρ'' – the conversion factor between arcseconds and radians, which is approximately equal to the number of arc seconds per radian =206265″.

The total changes in the area amount to 1.5". The maximum values of the components are located just above the boundary of the salt dome – the vicinity of GPS9 and GPS13 points, the minimum ones are placed outside the deposit or above the centre of its mass – the vicinity of GPS10 and GPS6 points (Fig. 1). The parameters of the model and gravitational effect were verified by means of an empirical approach – the results of gravimetric measurements (Łąka, 1981; Szczerbowski, 2010b). Assuming that the distribution of gravity anomaly determined by measurements is a true model, the discrepancies were analyzed and uncertainty of the model was evaluated.

The value of residual variance (s_e^2) was calculated by the following formula:

$$s_e^2 = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n-2} \tag{2}$$

where:

y_i – input variable, true value obtained in the ith observation,

$\hat{y}_i$ – predicted value of response variable,

n – the number of observations (calculation points) taken into account in the analysis.

The parameter s_e^2 provides information about agreement between modeled and empirical data. It represents a measure of dispersion of empirical data about values derived from the model. For determination of average discrepancy between empirical and modeled data, standard residuals were analyzed as well.

$$s_e = \sqrt{s_e^2} \tag{3}$$

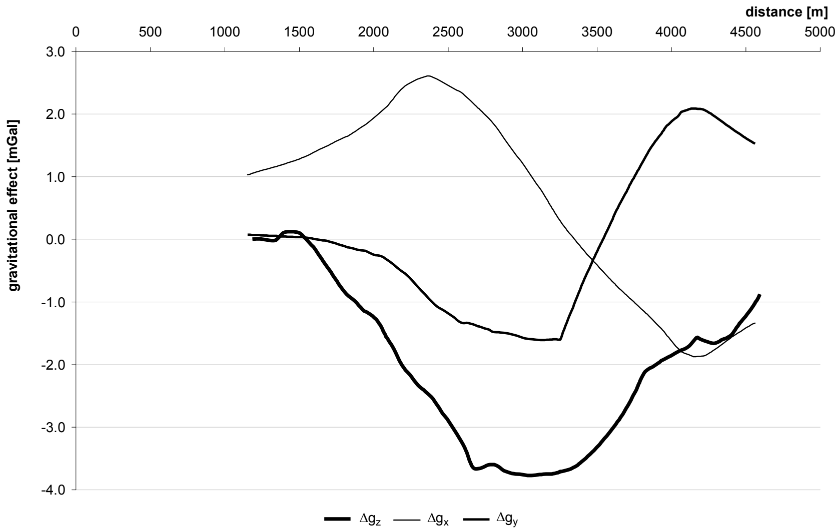

Fig. 2 The distributions of modeled components of gravitational effect along the S-N line.

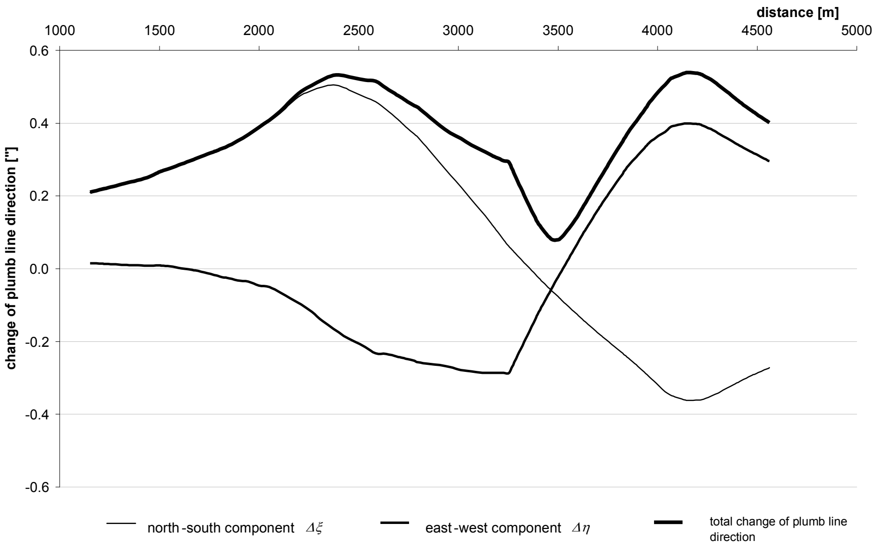

Fig. 3 The distributions of total value of vertical deflection and its components along the S-N line.

The obtained s_e value amounted to 0.5 mGal. Referring this value to relations between gravity changes and angular changes of plumb line (see formula 1), the obtained uncertainty of the modeled values of the vertical changes amounts to 0.1″. More detailed information about the modeling, uncertainties and distributions of the changes of the vertical can be found in Szczerbowski (2010b). The presented results of statistical analysis are essential for reliability of investigation involving relations between the measured (GPS and leveling) and modeled (gravimetrically) vertical deflections.

The final result of the modeling are distributions of components of gravity anomaly (Δg_{ZP}, Δg_{XP}, Δg_{YP}) induced by the salt dome and of horizontal components of the plumb line changes in direction ($\Delta\xi$,$\Delta\eta$).The distributions are presented in Figure 2 and Figure 3.

Evaluation of the vertical deflections is possible by the use of geodetic methods, such as the presented GPS/leveling approach (Hofmann-Wellenhof and Moritz, 2005):

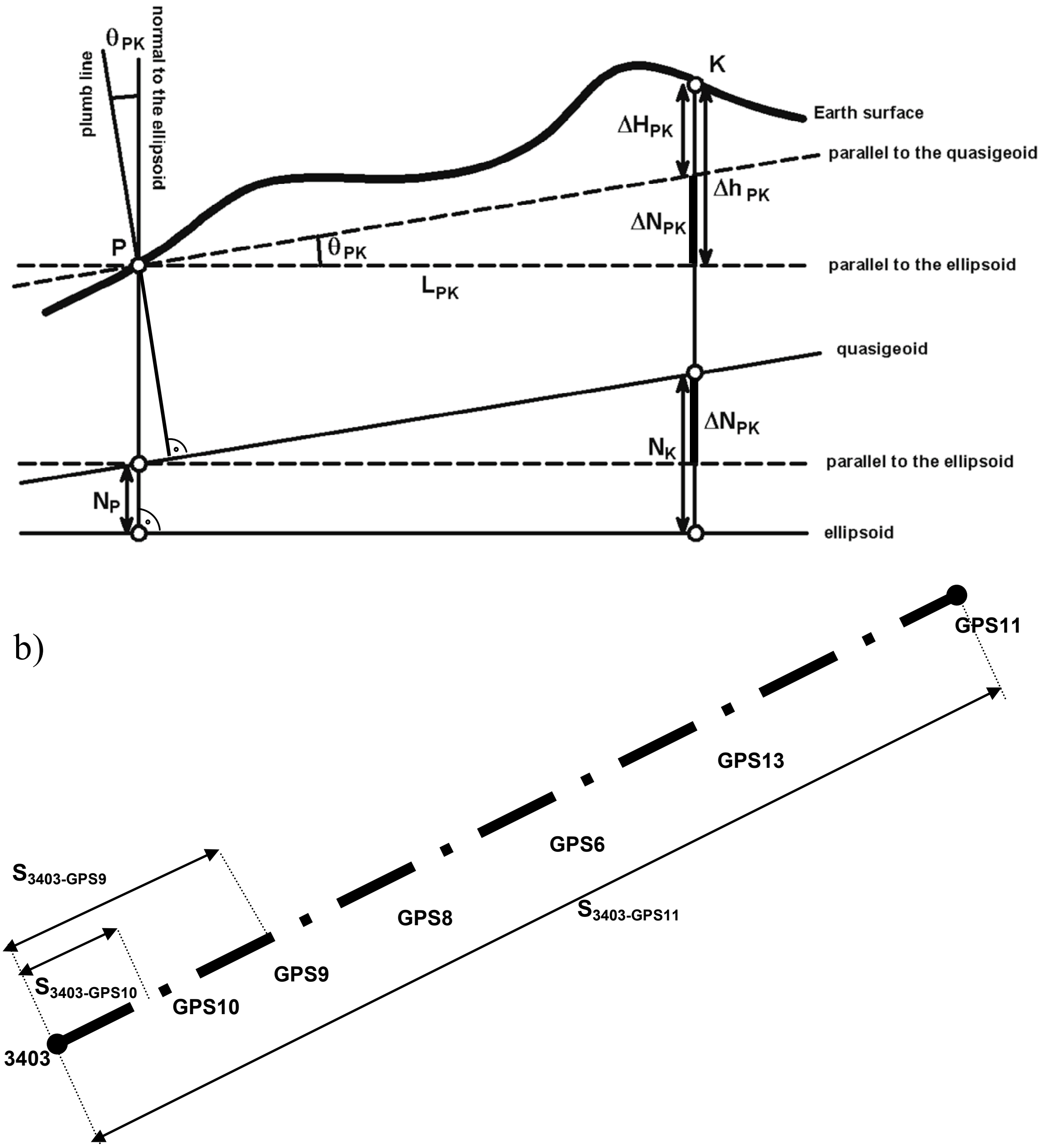

Fig. 4 Principle of the deflection of vertical determination (a) and segments of the S-N line (b).

4. THE EVALUATION OF VERTICAL DEFLECTIONS BY MEANS OF A GPS/LEVELING METHOD ALONG THE S-N LINE

The GPS/leveling method was used to determine the deflections of the vertical. The principle of the method is shown in the Figure 4. It shows the Earth surface, quasigeoid and ellipsoid in the vertical plane of azimuth determined by two points P and K. Vertical deflection θ_{PK} is the slope of the quasigeoid to the surface of the ellipsoid. Due small lengths of a segments and the points' heights curvature of the plumb line can be omitted.

GPS and leveling surveys were carried out in the area mostly for a detection of terrain surface displacements. The results were applied in analysis of vertical deflection. In the presented geodetic approach, they can be evaluated for segments bounded by control points (Fig. 4). A group of points included in the Inowrocław geodynamic network represents a longitudinal line (S-N). The analyzed S-N profile line is approximately South-North oriented and it is located over the longer axis of the salt dome (Fig. 1). It is approximately 4 km long.

Considering that the value of uncertainty of evaluated vertical deflection (VD) is inversely

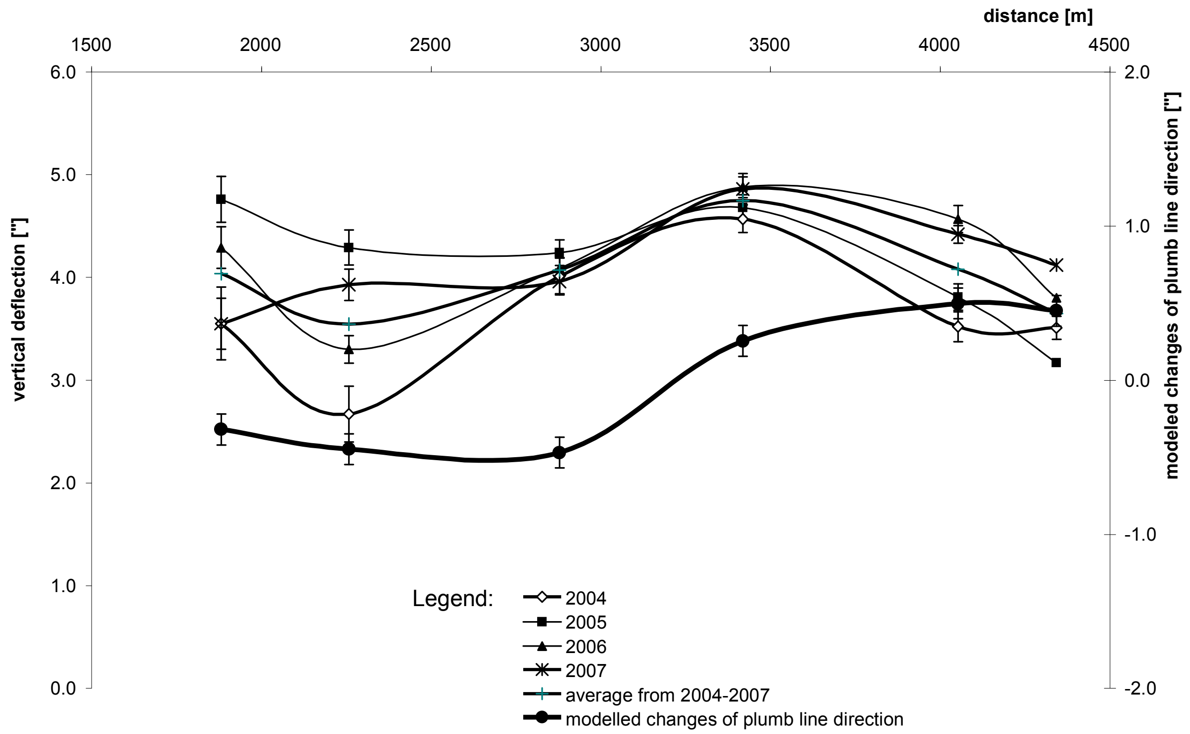

Fig. 5 The vertical deflection based on GPS/levelling surveys in 2004-2007 with uncertainties and modelled and changes of plumb line direction along the part of S-N line.

proportional to the length of the segment, it is more reliable analyze longer segments. The calculations were carried out for the control points of the S-N line extending nearly longitudinally from 3403 to GPS11 control points.

According to the relation (Hofmann-Wellenhof and Moritz, 2005):

$$N = h - H \tag{4}$$

in the P and K points the quasigeoid undulations N_P and N_K were determined based on GPS satellite measurements (h) and precise leveling (H) results (Fig. 4). In the presented study GRS 1980 ellipsoid was applied. Based on Helmert's formula (Hofmann-Wellenhof and Moritz, 2005):

$$\theta = -\Delta\mathrm{N} \cdot \mathrm{L}^{-1} \tag{5}$$

where: L – distance between points of bounding a segment [m],
ΔN – quasigeoid undulation difference [m].

It is possible to make many combinations of various segments the S-N line bounded by two points of the S-N line (3403, GPS10, GPS9…GPS11). So, within the presented study the total vertical deflection (θ) for each successive segment of the line (3403-GPS10, 3403-GPS9, 3403-GPS8, 3403-GPS6, 3403-GPS13, 3403-GPS11) were examined (Fig. 4). Analyzing vertical deflections evaluated for the segments, differences between segment's azimuth should be considered. However, the changes of azimuths are less than 10 degrees and do not involve significant changes of θ values, notably having their orientations nearly along one of the calculated component.

All analyzed segments are bounded by the 3403 control point, so it makes the length of the following segment successively longer and longer (Fig. 4). Various combinations of end points bounding the analyzed segments make it possible to evaluate VD values combinations, e.g. adjoining segments, any segment with different starting point, etc. :

$$\theta_{3403-GPS10} = \frac{\Delta\mathrm{N}_{3403-GPS10}}{\mathrm{s}_{3403-GPS10}} \qquad \theta_{3403\text{-GPS9}} = \frac{\Delta\mathrm{N}_{3403-GPS9}}{\mathrm{s}_{3403-GPS9}}$$
$$\ldots \quad \theta_{3403\text{-GPS11}} = \frac{\Delta\mathrm{N}_{3403-GPS11}}{\mathrm{s}_{3403-GPS11}}$$
$$\theta_{GPS10-GPS9} = \frac{\Delta\mathrm{N}_{GPS10-GPS9}}{\mathrm{s}_{GPS10-GPS9}} \qquad \theta_{\text{GPS9-GPS8}} = \frac{\Delta\mathrm{N}_{GPS9-GPS8}}{\mathrm{s}_{GPS9=GPS8}}$$
$$\ldots \quad \theta_{\text{GPS13-GPS11}} = \frac{\Delta\mathrm{N}_{\text{GPS13-GPS11}}}{\mathrm{s}_{\text{GPS13-GPS11}}} \tag{6}$$

Nevertheless, smaller length of a segment makes higher value of the uncertainty of evaluated θ that represents that segment accordingly to the propagation of uncertainty formula:

Table 1 Values of vertical deflections (VD) and uncertainties evaluated for particular segments of the S-N profile.

segment	VD (θ) in years [″] 2004	2005	2006	2007
3403_GPS10	3.55	4.76	4.29	3.55
3403_GPS9	2.67	4.29	3.3	3.93
3403_GPS8	4.01	4.24	4.09	3.96
3403_GPS6	4.57	4.68	4.88	4.86
3403_GPS13	3.52	3.81	4.57	4.42
3403_GPS11	3.51	3.17	3.8	4.120

segment	Uncertainty in years[″] 2004	2005	2006	2007
3403_GPS10	0.36	0.30	0.30	0.34
3403_GPS9	0.27	0.22	0.20	0.25
3403_GPS8	0.18	0.17	0.13	0.15
3403_GPS6	0.13	0.12	0.10	0.12
3403_GPS13	0.15	0.12	0.10	0.15
3403_GPS11	0.11	0.13	0.13	0.085

$$u(\theta) = \sqrt{\left[\frac{1}{L} \cdot u(\Delta N)\right]^2 + \left[\Delta N / L^2 \cdot u(L)\right]^2} \quad (7)$$

where:

u(θ)) – uncertainty of vertical deflection (θ) evaluated for a segment,

u(L) – uncertainty of evaluated length of a segment,

u(ΔN) – uncertainty of evaluated separation between quasigeoid and reference ellipsoid for a section:

$$u\Delta N = \sqrt{\left(u\left(N_{3403}\right)\right)^2 + \left(u\left(N_i\right)\right)^2} \quad (8)$$

where:

$$u(N_i) = \sqrt{\left(u\left(h_i\right)\right)^2 + \left(u\left(H_i\right)\right)^2} \quad (9)$$

where:

u(h_i) – ucertainty of a control point height determined by GPS (usually 2-3 mm),

u(H_i) – uncertainty of a control point normal height determined by leveling (1-2 mm).

The average value of determined values of vertical deflection uncertainty evaluated for surveys carried in 2004-2007 did not exceed 0.18″. That value can be considered as internal error and it is nearly equal to uncertainty of modeled changes of the plumb line directions in the area. The values of determined VD and their uncertainties are presented in Table 1.

The results obtained for individual years demonstrated a certain similarity of θ distribution (Fig. 5), what suggests reliable values of assumed uncertainties of measurements. The vertical deflections evaluated by GPS/levelling method was compared with theoretical changes of the plumb line direction resulting from gravimetric model. In this case, the evaluated distribution - as it was mentioned before - was compared with results of gravimetric measurements. Because of a sinusoidal form of the presented distributions, comparative characteristics requires analysis including both oscillations and amplitudes.

5. DISCUSSION

Peak-to-peak amplitude depicts changeability of the plumb line in the study area. Its value evaluated by GPS/leveling method exceeds 1″ (Fig. 5). Peak-to-peak amplitude of the modeled plumb line varies - 0.5″÷0.5″. Differences between geodetic, "segmental" approach, where values are attributed to a segment and gravimetric method must be considered. The character of geodetic data makes a problem of comparative presentation in a graphical form. This difference between the methods is a cause of displacement of peaks in the presented distributions: peaks in a line representing VD values evaluated by geodetic method are shifted horizontally to the starting point (3403).

The observed repeatability of θ distributions in individual years is a confirmation of reliability provided by analysis of uncertainty. The highest discrepancy between the distributions was denoted for the 3403-GPS10 segment. That is the shortest segment what involves of the segment and as a consequence from uncertainty of the evaluated θ value. The best agreement is observed for the 3403-GPS6 and 3403-GPS11 sections (Table 1). Generally, results obtained with the procedure demonstrate accordance with calculations based on gravimetric model.

Precision requirements involve long distance sections to minimize errors of determined deflections of the vertical. The presented values of evaluated uncertainties (for long distance sections) can be regarded as satisfying. While the requirements involve increasing of the distance, the precise assignment of position of the determined vertical deflections is more difficult. The long distance sections (over 1 km) do not provide precise localizations of the determined vertical deflections

6. CONCLUSIONS

The presented results show that considerable values of the total changes of vertical deflection (θ) in a small and flat area are possible. The main cause of the vertical deflection is a gravitational effect of the salt deposit as a local disturbance in mass distribution. Total changes amount to nearly 1.7″ at a distance of 4 km (Fig. 5). According to formulas (1), (4) and (5) in the presented area omission of the disturbances in a local quasigeoid model can cause a maximum error of up to 11 mm in normal heights evaluated by GPS technique.

The problem of precise modeling of vertical deflection can be important in studies of precise leveling by GPS technique, like those used in geodynamic studies. It concerns especially areas with high values of gravity gradient that are usually observed in areas of geodynamic networks. In central Europe, most of such networks are located in mountainous terrains, areas of mineral resources, often affected by mining or boundaries of tectonic provinces (Banasik et al., 2005; Cacoń et al., 2007; Schenková et al., 2007; Jamroz, 2008).

Usually, in geodetic studies the cause of changes of vertical deflection is considered in small areas as a result of vertically oriented factors, such as changes in heights of the considered profile line. The presented approach extends the meaning of the problem of vertical deflections pointing to significance of variability of rock mass distribution on a local scale as an important factor determining orientation of the plumb line. Mass disturbances (mass deficiency or mass excess) affect this orientation as a topographical effect usually considered in studies of quasigeoid modeling.

The presented results suggest to comprise geological environment as an evident factor like elevation or topography that control the plumb line changes. A combination of modeling of gravity vector and vertical deflections based on GPS/leveling surveys seems to be important for the estimation of the external accuracy of quasigeoid models.

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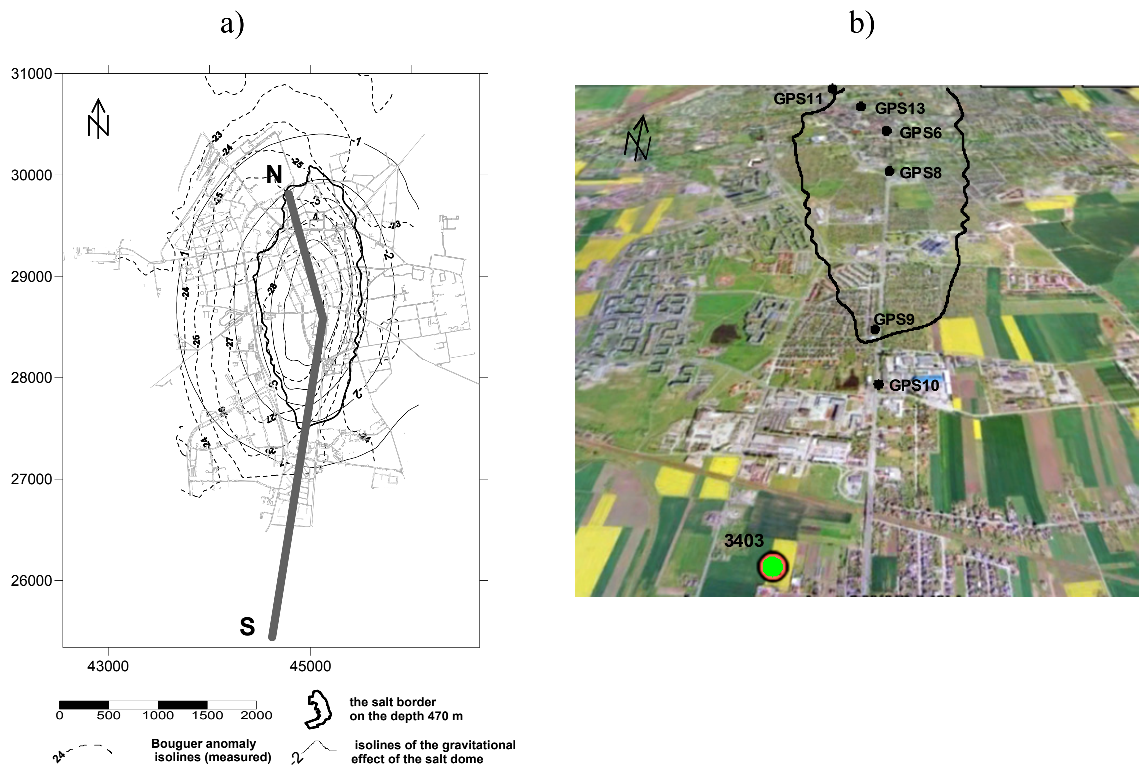

Fig. 1 Inowrocław – localization of the studied area with the S-N profile line. Distribution of the modelled gravitational effect of the salt dome [mGal] with the Bouger anomalny [mGal] evaluetd from survey data (according to Łąka, 1980) –a) and aerial view (Google) illustrating localization of control points of the S-N line –b).