Acta Geodyn. Geomater., Vol. 9, No. 3 (167), 283–290, 2012

DETERMINATION OF INTERMEDIATE ORBIT AND POSITION OF GLONASS SATELLITES BASED ON THE GENERALIZED PROBLEM OF TWO FIXED CENTERS

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(*Received* February 2012*, accepted* July 2012)

ABSTRACT
The mathematical model and algorithms for calculating the position of GLONASS satellites by means of their broadcast ephemeris is presented in the paper. The algorithms are based on the generalized problem of two fixed centers. One of the advantages of the analytical solution obtained from the generalized problem of two fixed centers is the fact that it embraces perturbations of all orders, from the second and also partly from the third zonal harmonics (Aksenov, 1969). GLONASS broadcast ephemeris – provided every 30 minutes – contain satellite position and velocities in the Earth fixed coordinate system PZ-90.02 (ICD, 2008), and acceleration due to luni-solar attraction. The GLONASS Interface Control Document recommends that a fourth order Runge-Kutta integration algorithm shall be applied. In the Department of Geomatics (AGH UST) a computer program has been established for fitting position and velocity of GLONASS satellites using their broadcast ephemeris. Intermediate GLONASS satellite orbits are calculated considering also the second and third zonal harmonics in the gravitational potential of the Earth. In this paper results of the analytical integration of the equation of the motion of the GLONASS satellites compared to the numerical solution are provided.

KEYWORDS: GLONASS, orbit determination, problem of two fixed centers

1. INTRODUCTION

The problem of two fixed centers is a special case of the restricted three – body problem. The two fixed centers problem is well known in classical celestial mechanics: two fixed centers with masses m_1 and m_2, attract some massless particle, moving in their field according to Newton's laws. The integrability of this problem was the first time proved by Euler, by means of the separation of variables. A review of publications on the problem of two fixed centers, including its generalizations and applications are presented in (Lukyanov et al., 2005). A good example how to implement the model of the generalized problem of two fixed centers for geodetic applications is given in (Aksenov, 1969).

It is well known that the force function in the generalized problem of two fixed centers approximates the potential of the earth's gravitation within the square of the earth flattening (Aksenov, 1969; Lukyanov et al., 2005). It is therefore appropriate to adopt intermediate orbits for earth satellites obtained by solving the generalized problem of two fixed centers (Demin, 1970; Aksenov, 1977). The solution of the generalized problem of two fixed centers in a geocentric coordinate system may be written in the form (Aksenov, 1977, p. 49) :

$$\begin{aligned} x &= \sqrt{\left(\xi^2 + c^2\right)\left(1-\eta^2\right)}\ \cos w, \\ y &= \sqrt{\left(\xi^2 + c^2\right)\left(1-\eta^2\right)}\ \sin w, \\ z &= c\sigma + \xi\eta. \end{aligned} \tag{1}$$

The fundamental plane of this system is the equator and the principal axis x points towards the vernal equinox. The rectangular coordinates x, y, z are associated with the spheroidal oblate coordinates ξ, η, and w (Aksenov, 1977, p. 49; Escobal, 1965, p. 146). The oblate spheroidal coordinate system finds substantial use in the theory of satellite motion based on the problem of two fixed centers. The coordinates ξ, η, and w are orthogonal: that is, the coordinate surfaces are an ellipsoid (ξ = const.), a hyperboloid (η = const.) and a plane (w = const.), which intersect one another at right angles (Demin, 1970; Aksenov, 1977). The constants c and σ are specified by the conditions (Aksenov et al., 1963).

$$c = a_e \cdot \sqrt{J_2 - \left(\frac{J_3}{2 \cdot J_2}\right)^2}, \tag{2}$$

$$\sigma = \frac{J_3}{2 \cdot J_2} \cdot \frac{1}{\sqrt{J_2 - \left(\frac{J_3}{2 \cdot J_2}\right)^2}} = \frac{J_3}{2 \cdot J_2} \cdot \frac{a_e}{c}, \tag{3}$$

where J_2, J_3 are the second and the third gravitational zonal harmonics of the Earth, respectively, and a_e is the equatorial radius of the reference ellipsoid. Assuming WGS-84, a_e=6378137.0 m, J_2 =1.082626684·10^{-3}, and J_3=-2.53265649·10^{-6} (NIMA Technical Report, TR8350.2, 2000), we obtain: c = 209.728939 km, and σ = -0.035571596.

2. CALCULATION OF THE INTERMEDIATE ORBIT ELLIPTIC ELEMENTS IN THE GENERALIZED PROBLEM OF TWO FIXED CENTERS

The intermediate satellite orbit in the generalized problem of two fixed centers is most simply described by the elements: *a, e, i*, ω, Ω, *M* which for c = 0 and σ = 0 coincide with the corresponding Keplerian elements (Aksenov, 1977).

The position and velocity components of GLONASS satellites, taken from the navigation message, are re-computed from ECEF (Earth Centered, Earth Fixed) Greenwich coordinate system (PZ-90.02) to an absolute coordinate system.

We assume that at the initial moment $t = t_0$, the values of the geocentric rectangular coordinates in an absolute system x_0, y_0, z_0 as well as of their derivatives with respect to time $\dot{x}_0, \dot{y}_0, \dot{z}_0$, are known. The presented algorithms are in general based on the book of Aksenov, 1977.

First, from the following formulae three constants: α_1 (the energy integral), α_2 (the area integral), and α_3 are determined (Aksenov, 1977).

$$2\alpha_1 = V_0^2 - \frac{2GM(\xi_0 - c\sigma\eta_0)}{J_0},$$

$$\alpha_2^2 = r_0^2 V_0^2 - r_0'^2 - c^2 \dot{z}_0^2 + Q_0, \tag{4}$$

$$\alpha_3 = x_0 \dot{y}_0 - y_0 \dot{x}_0.$$

where
GM = 398600.4418 km^3/s^2

$$V_0^2 = \dot{x}_0^2 + \dot{y}_0^2 + \dot{z}_0^2,$$

$$r_0^2 = x_0^2 + y_0^2 + (z_0 - c\sigma)^2,$$

$$r_0' = x_0 \dot{x}_0 + y_0 \dot{y} + (z_0 - c\sigma)\dot{z}_0,$$

$$J_0 = \xi_0^2 + c^2 \eta_0^2,$$

$$Q_0 = \frac{2GM\xi_0\eta_0\left(c^2\eta_0 + c\sigma\xi_0\right)}{J_0}.$$

From formulas (1) we find that

$$\xi_0^2 = \frac{r_0^2 - c^2}{2}\left[1 + \sqrt{1 + \frac{4c^2(z_0 - c\sigma)^2}{\left(r_0^2 - c^2\right)^2}}\right], \tag{5}$$

$$\eta_0 = \frac{z_0 - c\sigma}{\xi_0}, \quad tg\ w_0 = \frac{y_0}{x_0}. \tag{6}$$

From (4) we obtain the following constants $\hat{a}$, $\hat{e}$, $\hat{s}$.

$$\hat{a} = -\frac{GM}{2\alpha_1}, \quad \hat{e}^2 = 1 + \frac{2\alpha_1\alpha_2^2}{(GM)^2}, \quad \hat{s}^2 = 1 - \frac{\alpha_3^2}{\alpha_2^2} \tag{7}$$

The elements $\hat{a}$, $\hat{e}$ and $\hat{s} = \sin i$ are analogous to the major semi-axis, the eccentricity and the sine inclination of the elliptical orbit, respectively. When $c = \sigma = 0$, they coincide. The parameters *a*, *e*, *s* can be determined by the method of successive approximations at the level of small quantities of order ε^4.

$$\begin{aligned} a_{i+1} &= \hat{a}\left\{1 - \varepsilon_i^2\left(1 - e_i^2\right)\left(1 - s_i^2\right) + \varepsilon_i^4 s_i^2\left(1 - s_i^2\right)\left(1 - e_i^2\right)\left(3 + e_i^2\right)\right\} \\ 1 - e_{i+1}^2 &= \left(1 - \hat{e}^2\right)\left\{1 - \varepsilon_i^2\left(1 + 3e_i^2\right)\left(1 - s_i^2\right) + 2\varepsilon_i^4\left(1 - s_i^2\right)\right. \\ &\quad \left.\left[1 + e_i^2\left(4 + 2s_i^2\right) + e_i^4\left(3 - 2s_i^2\right)\right]\right\} \\ 1 - s_{i+1}^2 &= \left(1 - \hat{s}^2\right)\left\{1 + \varepsilon_i^2 s_i^2\left(1 - e_i^2\right) - \right. \\ &\quad \left. - \varepsilon_i^4 s_i^2\left(1 - e_i^2\right)\left[\left(3 - 2s_i^2\right) + e_i^2\left(1 + 4s_i^2\right)\right] - \varepsilon_i^2\sigma^2\left(1 - 7s_i^2\right)\right\} \end{aligned} \tag{8}$$

where the small dimensionless parameter ε_i is defined as follows:

$$\varepsilon_i = \frac{c}{a_i\left(1 - e_i^2\right)}, \ i = 0,1,2,3...$$

For GPS and GLONASS satellites that parameter is less then 1/100.

Substituting $\varepsilon = 0$ we find approximate values for *a*, *e*, *s*, which we then adopt as first approximation. The process of approximation ends in case the following conditions: $\left|a_{i+1} - a_i\right| \langle\ 10^{-7}$, $\left|e_{i+1} - e_i\right| \langle\ 10^{-10}$, $\left|s_{i+1} - s_i\right| \langle\ 10^{-10}$, are fulfilled. Practically the third approximation will be sufficiently accurate.

3. DETERMINING THE ANGULAR ELEMENTS ω_0, Ω_0 AND M_0 OF A SATELLITE ORBIT

First, according to (Aksenov, 1977, p. 108,109) the auxiliary variables ψ_0 and Θ_0 are computed according to the following formulae:

$$\cos\psi_0 = \frac{a(1 - e\bar{e}) - \xi_0}{\xi_0 e - a(\bar{e} - e)}, \tag{9}$$

$$\sin\psi_0 = \frac{ae\left(1-\bar{e}^2\right)J_0\dot{\xi}_0}{\sigma_2\left[\xi_0\bar{e} - a(\bar{e}-e)\right]^2\sqrt{1-k_2^2\left(1-\cos^2\psi_0\right)}},$$

in which

$$\bar{e} = e\left\{\begin{array}{l}1+\varepsilon^2\left(1-e^2\right)\left(1-2s^2\right)+\varepsilon^4\left(1-e^2\right)\cdot \\ \cdot\left[\left(3-16s^2+14s^4\right)-2e^2\left(1-s^2\right)^2\right]\end{array}\right\}, \tag{10}$$

$$\sigma_2 =$$
$$= \sqrt{GMa\left(1-e^2\right)}\left\{1-\frac{\varepsilon^2}{2}\left(3-4s^2-e^2\right)-\right.$$
$$\left.-\frac{\varepsilon^4}{8}\left[\begin{array}{l}\left(8-72s^2+64s^4\right)+ \\ e^2\left(2-40s^2+48s^4\right)+e^4\end{array}\right]\right\},$$

$$\dot{\xi}_0 = \frac{ae\sigma_2\left(1-\bar{e}^2\right)\sin\psi_0\sqrt{1-k_2^2\sin^2\psi_0}}{J_0\left(1+\bar{e}\cos\psi_0\right)^2},$$

$$k_2^2 = \varepsilon^2 e^2\left\{s^2-\varepsilon^2\left(1-10s^2+11s^4+e^2s^4\right)\right\}.$$

For Θ_0 we have

$$\sin\Theta_0 = \frac{\eta_0-\gamma}{s-\eta_0 d},$$

$$\cos\Theta_0 = \frac{(s-\gamma d)J_0\dot{\eta}_0}{\sigma_1(s-\eta_0 d)^2\sqrt{1-k_1^2\sin^2\Theta_0}}, \tag{11}$$

where

$$\sigma_1 =$$
$$\sqrt{GMa\left(1-e^2\right)}\left\{1+\frac{\varepsilon^2}{2}\left(1-s^2\right)\left(3+e^2\right)+\frac{\varepsilon^2\sigma^2}{2}\left(6-7s^2\right)-\right.$$
$$\left.-\frac{\varepsilon^4}{8}\left(1-s^2\right)\left[\left(9+11s^2\right)+e^2\left(6+34s^4\right)+e^4\left(1+3s^2\right)\right]\right\},$$

$$\dot{\eta}_0 = \frac{(s-\gamma d)\sigma_1\cos\Theta_0\sqrt{1-k_1^2\sin^2\Theta_0}}{J_0\left(1+d\sin\Theta_0\right)^2}, \tag{12}$$

$$k_1^2 = \varepsilon^2 s^2\left\{1+\sigma^2-e^2-4\varepsilon^2\left(1-s^2\right)\left(1-e^2\right)\right\},$$

$$d = \varepsilon\sigma s\left\{1-\varepsilon^2\left[\left(5-6s^2\right)-e^2\left(1-2s^2\right)\right]\right\},$$

$$\gamma = -\varepsilon\sigma\left\{1-2s^2-\varepsilon^2\left[\left(3-12s^2+10s^4\right)+e^2\left(1-2s^4\right)\right]\right\}.$$

With $\varepsilon = 0$, $\sigma = 0$, the angle ψ_0 coincides with the true anomaly and Θ_0 with the argument of latitude of a Keplerian orbit. Next we calculate the orbital elements ω_0, Ω_0, and M_0 according to the following formulae:

$$\omega_0 = \Theta_0-(1+\nu)\psi_0-\frac{k_1^2}{8}\left(1+\frac{k_1^2}{2}\right)\sin 2\Theta_0+$$
$$+\frac{k_2^2}{8}\left(1+\nu+\frac{k_2^2}{2}\right)\sin 2\psi_0, \tag{13}$$

where

$$\nu = \frac{\varepsilon}{4}\left(1+\sigma^2\right)\left(12-15s^2\right)+$$
$$+\frac{\varepsilon^4}{64}\left[288-1296s^2+1035s^4-e^2\left(144+288s^2-510s^4\right)\right]. \tag{14}$$

$$\Omega_0 = w_0 - arctg\left(\frac{\cos i\sin\Theta_0+\beta}{\cos\Theta_0}\right)-\mu\psi_0-\mu_1\sin\psi_0-$$
$$-\mu_2\sin 2\psi_0-\mu_3\sin 3\psi_0-\mu_1'\cos\left(\psi_0+\omega^{(0)}\right), \tag{15}$$

where

$$\mu = -\frac{3}{2}\alpha\left\{\varepsilon^2\left(1+\sigma^2\right)+\frac{\varepsilon^4}{8}\left(6-17s^2-24e^2s^2\right)\right\},$$

$$\mu_1 = -2\varepsilon^2\alpha e\left\{1+\frac{\varepsilon^2}{8}\left[\left(4-28s^2\right)-e^2\left(6+7s^2\right)\right]\right\},$$

$$\mu_2 = -\frac{\varepsilon^2\alpha e^2}{4}\left\{1-\frac{\varepsilon^2}{4}\left[\left(22+s^2\right)+e^2\left(2+s^2\right)\right]\right\}, \tag{16}$$

$$\mu_3 = \frac{\varepsilon^4\alpha e3}{4}\left(2-s^2\right),$$

$$\mu_1' = \varepsilon^3\sigma\alpha s\left(1-e^2\right),$$

$$\omega^{(0)} = \nu\psi_0+\omega_0. \tag{17}$$

Next, from the following formula we obtain the eccentric anomaly E_0.

$$tg\frac{E_0}{2} = \sqrt{\frac{1-\bar{e}}{1+\bar{e}}}\,tg\frac{\psi_0}{2}, \tag{18}$$

Finally we determine mean anomaly M_0.

$$M_0 = E_0-e_0^*\sin E_0-\lambda\psi_0+\lambda_1\sin\psi_0+$$
$$+\lambda_2\sin 2\psi_0+\lambda_1'\cos\left(\psi_0+\omega^{(0)}\right)+\lambda_2'\sin 2\left(\psi_0+\omega^{(0)}\right), \tag{19}$$

where the coefficients λ, λ_1, λ_2, λ_1', λ_2' and e^* are defined as follows (Aksenov, 1977, p. 90):

$$\lambda = -\frac{3}{16}\varepsilon^4\left(1-e^2\right)^{3/2}\left(8-32s^2+25s^4\right),$$

$$\lambda_1 = -\frac{1}{4}\varepsilon^4 s^2 e\left(4-5s^2\right)\left(1-e^2\right)^{3/2},$$

$$\lambda_2 = \frac{3}{32}\varepsilon^4 s^4 e^4 \left(1-e^2\right)^{3/2}, \tag{20}$$

$$\lambda_1' = \frac{\varepsilon^3\sigma}{2}\, s\, \left(4-5s^2\right)\left(1-e^2\right)^{3/2},$$

$$\lambda_2' = -\frac{\varepsilon^2}{4}\, s^2\left(1-e^2\right)^{3/2}\left\{1-\frac{\varepsilon^2}{4}\left[\left(12-13s^2\right)-e^2\left(4-5s^2\right)\right]\right\},$$

$$e^* = e\left\{1-\varepsilon^2\left(1-e^2\right)\left(1-s^2\right)+\varepsilon^4 s^2\left(1-e^2\right)\left(3+e^2\right)\right\}.$$

With $\varepsilon = 0$, $\sigma = 0$, the element ω_0 coincides with the argument of perigee, Ω_0 is the longitude of the ascending node and M_0 is the mean anomaly of the Keplerian orbit at epoch t_0.

4. ALGORITHM OF CALCULATION OF A SATELLITE POSITION AT INSTANT *t*

The position of the satellite in space is defined by the six known intermediate orbital elements a, e, $s = \sin i$, ω_0, Ω_0 and M_0.

$$M = n_0\left(t-t_0\right)+M_0, \tag{21}$$

$$n_0 = \frac{\left(-2\alpha_1\right)^{3/2}}{GM}. \tag{22}$$

The parameters $\psi, \omega\, and\, E$ are calculated by the technique of successive approximations:

$$tg\,\frac{\psi_{i+1}}{2} = \sqrt{\frac{1+\bar{e}}{1-\bar{e}}}\; tg\,\frac{E_i}{2}, \tag{24}$$

$$\omega_{i+1} = \nu\psi_{i+1}+\omega_0. \tag{25}$$

$$\begin{aligned} E_{i+1} = {} & M + e^*\sin E_i + \lambda\psi_{i+1} - \lambda_1\sin\psi_{i+1} - \lambda_2\sin 2 \\ & - \lambda_2'\sin 2\left(\psi_{i+1}+\omega_{i+1}\right) - \lambda_1'\cos\left(\psi_{i+1}+\omega_{i+1}\right), \\ & i = 1,2,3,.... \end{aligned}$$

A reasonable initial value for E_0 is M. The process of approximation ends when the following condition is fulfilled: $\left|E_{i+1}-E_i\right| \langle 10^{-10}$. Usually 3-4 iterations are sufficient.

The Cartesian coordinates are computed (at instant t) using the following formulae (Aksenov, 1977, p. 93):

$$\begin{aligned} x &= \rho\left(\cos\Theta\cos\hat{\Omega} - \alpha\sin\Theta\sin\hat{\Omega} - \beta\sin\hat{\Omega}\right), \\ y &= \rho\left(\cos\Theta\cos\hat{\Omega} + \alpha\sin\Theta\cos\hat{\Omega} + \beta\cos\hat{\Omega}\right), \\ z &= c\sigma + \rho'\left(s\cdot\sin\Theta + \gamma\right), \end{aligned} \tag{27}$$

where

$$\xi = a\left(1-e\cos E\right),$$

$$\begin{aligned} \Theta = {} & \psi + \omega + \frac{k_1^2}{8}\left(1+\frac{k_1^2}{2}\right)\sin 2\left(\psi+\omega\right) - \\ & -\frac{k_2^2}{8}\left(1+\nu+\frac{k_2^2}{2}\right)\sin 2\psi + \frac{3}{256}k_2^4\sin 4\psi + \\ & +\frac{k_1^4}{256}\sin 4\left(\psi+\omega\right) - \frac{k_1^2k_2^2}{32}\sin 2\psi\cos 2\left(\psi+\omega\right), \end{aligned}$$

$$\begin{aligned} \hat{\Omega} = {} & \mu\psi + \Omega_0 + \mu_1\sin\psi + \mu_2\sin 2\psi + \mu_3\sin 3\psi + \\ & + \mu_1'\cos\left(\psi+\omega\right) + \mu_2'\sin 2\left(\psi+\omega\right), \end{aligned} \tag{28}$$

$$\rho = \frac{\sqrt{\left(1-\varepsilon^2\sigma^2\right)\left(\xi^2+c^2\right)}}{1+d\sin\Theta},$$

$$\rho' = \frac{\xi}{1+d\sin\Theta},$$

$$\alpha = \cos i,$$

$$\beta = 2\varepsilon\sigma\alpha s\left\{1-\varepsilon^2\left(4-5s^2+e^2s^2\right)\right\}.$$

Table 1 Positions, velocities and luni-solar accelerations of the GLONASS satellite GLN20, in the PZ-90.02 reference frame, from broadcast ephemeris file, 17 October 2011. Epoch t_0 and t_1.

words [units]	$t_0 = 11^h\, 45^m\, 00.0^s$ UTC	$t_1 = 12^h\, 15^m\, 00.0^s$ UTC
X [km]	12317.93408200	10391.4926758
Y [km]	-2245.13232422	3032.69384766
Z [km]	22212.8173828	23096.2607422
$\dot{X}$ [km/s]	-1.25356674194	-0.864193916321
$\dot{Y}$ [km/s]	2.77420043945	3.04328060150
$\dot{Z}$ [km/s]	0.980820655823	-0.00542259216309
$\ddot{X}_{LS}$ [km/s^2]	$-0.931322574616\cdot 10^{-9}$	0.0
$\ddot{Y}_{LS}$ [km/s^2]	0.0	0.00
$\ddot{Z}_{LS}$ [km/s^2]	$-0.186264514923\cdot 10^{-8}$	$-0.279396772385\cdot 10^{-8}$

Table 2 Calculation intermediate orbital elements of the GLONASS satellite GLN20 at epoch 17 October 2011, t_0=11^h 45^m 00.0^s UTC.

item	words [units]			words [units]	
1	$\boldsymbol{x}_0$ [km]	**11881.413366**	30	σ_2 [km^2/s]	100834.261470836
2	$\boldsymbol{y}_0$ [km]	**-3950.207035**	31	$\dot{\xi}_0$ [km/s]	0.0049314043
3	z_0 [km]	**22212.817383**	32	k_2^2	0.00000000009
4	$\dot{\boldsymbol{x}}_0$ [km/s]	**-0.564121**	33	ψ_0 [rad]	1.4863757692
5	$\dot{\boldsymbol{y}}_0$ [km/s]	**3.788976**	34	σ_1 [km^2/s]	100835.150279399
6	$\dot{z}_0$ [km/s]	**0.980821**	35	$\dot{\eta}_0$ [rad/s]	0.0000382876
7	V_0^2 [km^2/s^2]	15.6365838761	36	k_1^2	0.0000554971
8	r_0^2 [km^2]	650512863.357182	37	d	-0.0002648328
9	ξ_0^2 [km^2]	650502263.311214	38	γ	-0.0001871300
10	η_0 [rad]	0.8712144653	39	Θ_0 [rad]	1.2946375300
11	r_0' [km^2/s]	124.3079598242	40	$\boldsymbol{\omega}_0$ [rad]	**6.0914509486**
12	J_0 [km^2]	650535649.531330	41	ν	-0.0000050595
13	Q_0 [km^4/s^2]	-4137728.33716742	42	w_0	5.9622124440
14	α_1 [km^2/s^2]	-7.8132471441	43	μ	-0.0000430857
15	α_2^2 [km^4/s^2]	10167603454.3809	44	μ_1	-0.0000000718
16	α_3 [km^2/s]	42789.9984002332	45	μ_2	-0.00000000001
17	$\hat{\alpha}$ [km]	25507.988833	46	μ_3	0.0
18	$\hat{e}^2$	-0.000010606	47	μ_0'	-0.0000000076
19	$\hat{s}^2$	0.8199198099	48	$\omega^{(0)}$ [rad]	6.0914434283
20	ε (i= 0)	0.0082220048	49	Ω_0 [rad]	**4.9805303799**
21	a (i=1) [km]	25507.67852	50	E_0 [rad]	1.4851283455
22	e^2 (i=1)	0.0000015658	51	λ	0.0000000012
23	s^2 (i=1)	0.8199097563	52	λ_1	0.0000000009
24	ε (i=1)	0.0082222049	53	λ_2	0.0
25	$\boldsymbol{a}$ **(i=4) [km]**	**25507.678491**	54	λ_1'	0.0000000009
26	$\boldsymbol{e}^2$ **(i=4)**	**0.0000015675**	55	λ_2'	-0.0000138571
27	$\boldsymbol{s}^2$ **(i=4)**	**0.819909756**	56	$e*$	0.0012519867
28	ε (i=4)	0.0082222049	57	$\boldsymbol{M}_0$ **[rad]**	**1.4838736773**
29	$\bar{e}$	0.0012519478	58	n_0 [s^{-1}]	0.0001549725

5. NUMERICAL TESTS

Knowing the satellite position and velocity at epoch t_0 (Table 1) from broadcast ephemeris the intermediate orbital parameters of the GLONASS satellite GLN20 are computed. Detailed numerical examples of these computations are given in Table 2. Firstly, the coordinates X, Y, Z and velocity vector components (Table 1) at epoch t_0 are transformed from the ECEF coordinate frame (PZ-90.02) to an absolute coordinate system x_0, y_0, z_0, $\dot{x}_0, \dot{y}_0, \dot{z}_0$, (Table 2, 1÷6) using the formulae given in GLONASS ICD 2008. Table 2 contains all the values of the computed parameters in agreement with the formulae and designations given in section 2 and 3. Computed orbital parameters are highlighted in bold. The algorithm of the computation of satellite positions, described in section 4, is illustrated in detail in Table 3 and Table 4.

Table 3 Luni-solar accelerations in an absolute coordinate system and orbital parameters of the GLONASS satellite GLN20 at epoch t_0 and t_1, 17 October 2011.

item	words [units]	$t_0 = 11^h\,45^m\,00.0^s$ UTC	$t_1 = 12^h\,15^m\,00.0^s$ UTC
1	$\ddot{x}_{LS0}$ [km/s^2]	$-9.2212106887\cdot 10^{-10}$	0.0
2	$\ddot{y}_{LS0}$ [km/s^2]	$1.3059277289\cdot 10^{-10}$	0.0
3	$\ddot{z}_{LS0}$ [km/s^2]	$-1.8626451492\cdot 10^{-9}$	$-2.7939677239\cdot 10^{-9}$
4	a (i=4) [km]	25507.678491	25507.66778
5	e^2 (i=4)	0.0000015675	0.0000015649
6	s^2 (i=4)	0.8199097560	0.8199097528
7	ω_0 [rad]	6.0914509486	6.0911021308
8	Ω_0 [rad]	4.9805303799	4.9805303167
9	M_0 [rad]	1.4838736773	1.7631751942

Table 4 Calculation of position of the GLONASS satellite GLN20 at epoch $t = 12^h\,00^m\,00.0^s$, 17 October 2011.

item	words [units]	$t = t_0 + 15^m$	$t = t_1 - 15^m$
1	M [rad]	1.6233489021	1.6236998816
2	ψ (i=1) [rad]	1.6245990807	1.6249490022
3	ω (i=1) [rad]	6.0914427289	6.0910939094
4	E (i=1) [rad]	1.6246029369	1.6249528586
5	ψ (i=4) [rad]	1.6258529142	1.6262017771
6	ω (i=4) [rad]	6.0914427226	6.0910939031
7	E (i=4) [rad]	1.6246028200	1.6249527413
8	ξ [km]	25509.396008	25509.395025
9	Θ [rad]	1.4341122025	1.4341122460
10	$\hat{\Omega}$ [rad]	4.9804602561	4.9804601779
11	ρ [km]	25516.951759	25516.950778
12	ρ' [km]	25516.09048	25516.089499
13	α	0.4243704089	0.4243704127
14	β	-0.0002247771	-0.0002247772
18	x [km]	11259.895951	11259.895712
19	y [km]	-512.795156	-512.794967
20	z [km]	22876.805241	22876.804074

Moreover, on the base of broadcast ephemeris the intermediate orbital elements of satellite GLN5 are computed for various epochs between 1:45 to 5:15 (March 18, 2011) at 30 minutes interval. Based on these orbital elements forward (for 15 minutes) and backward (for 15 minutes) positions of GLN5 satellite are computed. The difference between forward and backward positions are given in (Fig. 1). In the next numerical example, on the base of known orbital elements of GLN5 satellite obtained at epoch t_i forward (for 30 minutes, t_i + 30) and backward (for 30 minutes, t_i - 30) satellite positions are calculated, and then the results were compared with the corresponding coordinates given in the broadcast ephemeris. Differences between these computations are shown in (Fig. 2). Worth mentioning is that from

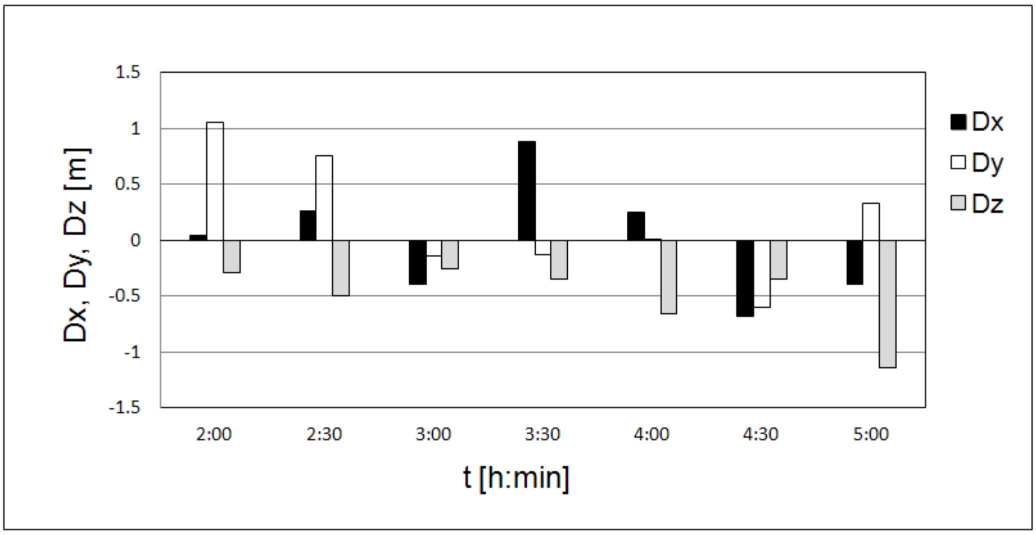

Fig. 1 Differences Dx, Dy, Dz, between the coordinates of satellite GLN5 obtained from reference time points $1^h 45^m$ to $5^h 15^m$ on March 18, 2011, forward for 15 minutes and backward for 15 minutes.

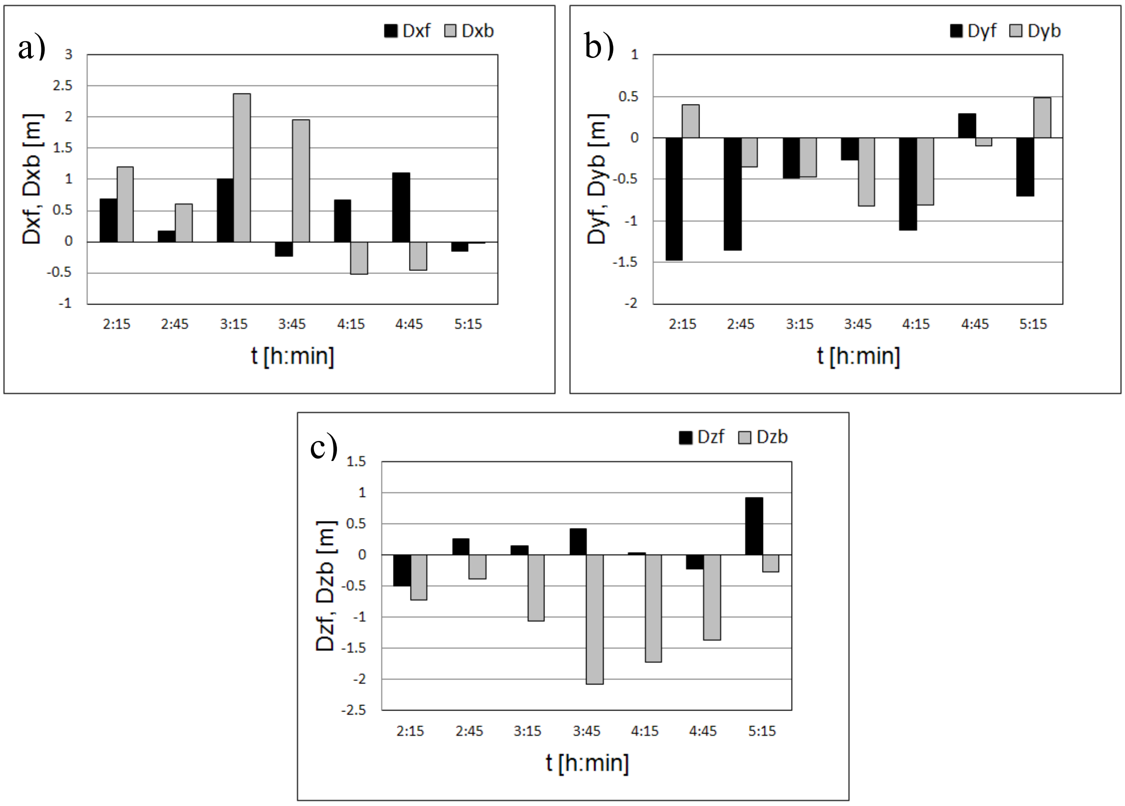

Fig. 2 Differences between the coordinates of GLN 5 satellite given in navigation file and its values obtained from forward (..f) and backward (..b) processing with ±30 min time interval, on March 18, 2011.

21 coordinate differences (as shown in Fig. 2) only two differences exceed 2 m.

6. CONCLUSIONS

The major aim of this paper was to illustrate a procedure for computing orbital intermediate elements from the GLONASS broadcast ephemeris. The paper concentrates on the practical issues of implementing the model of the generalized problem of two fixed centers for computing satellite positions illustrated by the example of GLONASS broadcast ephemeris. The GLONASS broadcast ephemeris are transmitted as a half-hourly satellite state vector, expressed in PZ-90.02 geocentric Cartesian coordinates and are nominally valid for 30 minutes.

According to GLONASS ICD 2008 the computation of the satelite positron at time t from a satellite state vector with reference time t_e requires the numerical integration where the longest integration period recommended should be 15 min forward and backward from the current state vector.

The proposed analytical algorithm provides good results when calculating positions of the GLONASS satellites in the time-range of 30 minutes forward and 30 minutes backward from the given current state vector. Results presented in Figure 2, are more accurate then the mean square error of broadcast positions of GLONASS-M satellites, which is about 10 m (GLONASS ICD, 2008, Table 4.2). The analytical solution, obtained from the generalized problem of two fixed centers, demands less time for calculation than the method of numerical integration.

ACKNOWLEDGEMENT

This work has been partially supported by the project No. 11.11.150. 006, AGH – University of Science and Technology in Krakow.

The authors are grateful to anonymous reviewers for their constructive comments and suggestions.

REFERENCES

Aksenov, P., Grebenikov, A. and Demin, V.G.: 1963, The generalized problem of motion about two fixed centers and its application to the theory of artificial Earth satellites, translated in Soviet Astron. – AJ. 7, No. 2, American Institute of Physics, 276–283.

Aksenov, E.P.: 1969, Mechanical interpretation of the force function in the generalized problem of two fixed centers, Soviet Astron. – AJ. 12, No 4, American Institute of Physics, 681–685.

Aksenov, E.P.: 1977, Theory of motion artificial Earth's satellites (in Russian), Nauka Press, Moscow, pp.360.

Demin,V.G.: 1970, Motion of an artificial satellite in an eccentric gravitation field, translated and published as NASA Technical translation, TT F-579, Wshington D.C. (Translation of Dvizheniye Iskusstvennogo Sputnika v Netsentral'nom Pole Tyagoteniya, Nauka Press, Moscow, 1968).

Emelyanov, N.V.: 1992, The acting analytical theory of artificial Earth satellites, Astronomical & Astrophysical Transactions, 1, 119 - 127, Gordon and Breach Science Publishers S. A.

Escobal, P.R.: 1965, Methods of orbit determination, J. Willey & Sons, Inc.

GLONASS Interface Control Document (ICD), Edition 5.1, Moscow, 2008.

Lukyanov, L.G., Emelyanov, N.V. and Shirmin, G.I.: 2005, Generalized problem of two fixed centers or the Darboux-Gredeaks problem, Cosmic Research, 43, No. 3, Springer Verlag, 186–191.

NIMA Technical Report, TR8350.2, 2000.