Introduction

Multipath in GPS observations is generally considered as a disturbing factor systematically influencing static and kinematic positioning. The broadcasted satellite microwave signal directly reaching the GPS antenna is interfered with the satellite signal reflected from the buildings and terrain around the antenna – systematical effect which is usually described in publications dealing with precise geodetic satellite surveying e.g. in (Hoffmann-Wellenhof et al., 2001). Motion of the GPS satellite on the sky as well as the features of the reflection surface affects the multipath characteristics, namely its spatial and temporal variability. Recently, several studies have shown that the analysis of multipath caused by reflections from horizontal surface at permanently observing GPS sites allows to infer the properties of the environment of the antenna, namely the variability of depth of the reflecting surface and consecutively to deduce the variations of soil moisture or the changes of snow cover. Two types of GPS observables are efficient for such analyses of multipath: the signal-to-noise ratio (SNR) used by Larson (Larson et al., 2008, Larson et al., 2009) or the geometry-free linear combination of carrier phases (L4) applied by (Ozeki and Heki, 2011). The pseudorange/carrier phase multipath combinations MP1 and MP2 (Estey and Meertens, 1999) conventionally applied for multipath assessment are not suitable for monitoring the multipath time variability as long as the centimeter accuracy of the height of reflecting surface is expected.

We examined and compared both approaches on several permanent GPS stations in Slovakia, situated in various environments. The selection of GPS sites suitable for testing the potential of snow depth sensing was limited by various factors, namely: GPS antenna monumentation on stable pillar surrounded by flat ground, no vertical obstacles around the pillar (at lest in some sectors), continuous GPS observations in the winter period, snow cover during winter season reaches up to 40 cm and the snow depth manual measurements are regularly performed. Due to terrain topography, buildings, structures and other obstacles in the vicinity of pillar where the GPS antenna is mounted only narrow azimuthal sectors in these sites are suitable for analyzing reflected signals from ground. Comparisons of series of manual snow depth sensing with data resulting from GPS multipath analyses generally shows consistency better than 5 cm, however some periods where the biases are at 10 cm level were recognized.
Interference of the direct satellite signal with amplitude $A_d$ and the reflected indirect signal with amplitude $A_r$ cause the change of signal strength $S$ and multipath induced carrier wave phase shift $\delta \varphi$ of the complete received signal (direct and indirect). According to (Billich et al., 2008; Ozeki and Heki, 2011) they are expressed as functions of multipath parameters $\psi, A_d$ and $A_r$ (or $\alpha = A_r/A_d$) as

$$S^2 = A_d^2 + A_r^2 + 2A_dA_r \cos \psi$$

(4)

$$\tan(\delta \varphi) = \frac{A_r \sin \psi}{A_d + A_r \cos \psi} = \frac{\alpha \sin \psi}{1 + \alpha \cos \psi}$$

(5)

The time variability of $\psi$ due to slow change of elevation angle $\theta$ is manifested as periodic oscillations of signal strength $S$ and phase shift $\delta \varphi$. As coefficient of the microwave reflectivity at the ground is small ($\alpha \approx 0.15$) the periodic part $S_p$ of signal strength can be approximated as

$$S_p(A_d, \alpha, h, \lambda, \theta) \approx \alpha A_d \cos \left(\frac{4\pi h \sin \theta}{\lambda}\right)$$

(6)

and the phase shift is

$$\delta \varphi(h, \alpha, \lambda, \theta) = \tan^{-1} \left[\frac{\alpha \sin \left(\frac{4\pi h \sin \theta}{\lambda}\right)}{1 + \alpha \cos \left(\frac{4\pi h \sin \theta}{\lambda}\right)}\right]$$

(7)

Analyses of series of satellite signal strength and carrier phase as functions of elevation angle $\theta$ enable to estimate the value of actual antenna height $h$ for the GPS signal with wavelength $\lambda$. The period of $S$ and $\delta \varphi$ decreases with increasing $h$ according to (3), (6) and (7).

The possibility of obtaining the actual height $h$ of antenna phase centre above the surrounding reflecting surface from multipath analysis enables to detect the potential variability of the height of the reflector provided the stable monumentation of GPS antenna. Application for snow depth sensing is demonstrated in Figure 2, where $h_G$ is the antenna height above ground without snow cover. Then the $h_{SNOW}$ is simply deduced using actual $h$ estimated from multipath as

$$h_{SNOW} = h_G - h$$

(8)

According to (6) and (7) two types of GPS observables are suitable for estimation of $h$, namely signal-to-noise ratio (SNR) and carrier phase, both generally included in GPS RINEX observation files (Gurtner, 1994).

The SNR data used for multipath studies e.g. in (Larson et al., 2009; Larson and Nievisnki, 2012; Billich et al., 2008) have the advantage in their smoothness and independence of ionosphere fluctuations. The drawback of SNR observables is that they are reported in logarithmic scale (in dB), their
Fig. 2. Effect of snow coverage with $h_{\text{SNOW}}$ on the effective antenna height $h$ above the reflecting surface. If the value $h_0$ represents the effective antenna height above ground without snow coverage then $h_{\text{SNOW}} = h_0 - h$.

resolution is in some cases limited as they are rounded to 1 dB, and the value of SNR is receiver dependent, according to the applied code correlation technique (Hoffmenn-Wellenhof et al., 2001). Moreover the SNR is not always included in all RINEX data files which are archived in GNSS data centres.

The geometry-free linear combination $L4$ of carrier phases used by (Ozeki and Heki, 2011) is simple difference between phase observations $L1 - L2$, where the carrier phases are multiplied by wavelengths $\lambda_1 \approx 19.0$ cm and $\lambda_2 \approx 24.4$ cm. The geometrical content of $L4$ is removed by differentiation and reduction for initial ambiguities. The time variability of $L4$ is then dependent on ionosphere disturbances and multipath issues only. The $L1$ and $L2$ phase observables are always included RINEX files, non-rounded and with sufficient temporal resolution.

3. STRATEGY FOR ESTIMATION OF ACTUAL ANTENNA HEIGHT FROM SIGNAL-TO-NOISE RATIO AND GEOMETRY-FREE LINEAR COMBINATION OF CARRIER PHASES

Multipath periodic fluctuations presented in signal-to-noise ratio series as well as in series of $L4$ linear combination of carrier phases can be applied for estimate of the effective antenna phase centre height $h$ above the reflecting horizontal surface. If the observation RINEX files include carrier phase data $L1$ and $L2$, as well as SNR data for $L1$ and $L2$ frequencies (denoted here as $\text{SNR}_{L1}$ and $\text{SNR}_{L2}$) it will be possible to estimate $h$ independently from three independent observables.

The GPS observations are recorded and stored with sampling rate 15 s or 30 s. In order to apply eq. (3) for evaluation of effective antenna height $h$ from frequency $f_\delta$ of multipath induced periodic variations the original time argument $t$ of the series $\text{SNR}_{L1}$, $\text{SNR}_{L2}$ and $L4$ in RINEX files is transformed to new argument $s$ – the independent variable $s = \sin \theta$ for each analyzed satellite. The actual elevation angle $\theta$ of the observed satellite at epoch $t$ is computed from satellite ephemeris and site coordinates.

The SNR series during the satellite observations are sum of the direct signal component which generally increases with satellite elevation and the multipath component including the ground reflections. A multipath effect is diminishing with increase of satellite elevation angle and for study of multipath effect is sufficient to analyse the SNR below 30° elevation. The SNR values given in RINEX files expressing the ratio of signal power to power of noise are in logarithmic scale in dB (Gurtner, 1994, Misra and Enge, 2001). For further analyses related to study of the multipath periodic variability the $\text{SNR}_{\text{lin}}$ data are converted to the ratio of signal amplitude to noise amplitude expressed in linear scale $\text{SNR}_{\text{lin}}$ according to the formula $\text{SNR}_{\text{lin}} = 10^{\text{SNR}_{\text{db}}} / 20$ (Billich et al., 2008).

The long-term part of the $\text{SNR}_{\text{lin}}$ due to satellite elevation change is removed by low-order ($n=5$) polynomial. The frequency of periodic part is estimated from the residual series $\delta \text{SNR}$ by means of spectral analysis. The residual $\delta \text{SNR}_{L1}$ and $\delta \text{SNR}_{L2}$ from $\text{SNR}_{L1}$ and $\text{SNR}_{L2}$ (transformed to linear scale as $\text{SNR}_{\text{lin,}L1}$ and $\text{SNR}_{\text{lin,}L2}$) are formed and analysed separately. If the reflecting surface is horizontal, smooth and no other obstacles are contributing to multipath pattern only one peak should be dominant in the spectra of each series.

Variations of $L4$ series besides the multipath from ground reflections are strongly influenced also by the time varying ionosphere delays. Similarly to SNR, only the observations below 30° elevation angle are analysed. The disturbing unpredictable ionosphere influences are removed by polynomial of higher order ($n=14$) and the residual series $\delta L4$ is used for estimation of dominant frequencies corresponding to multipath from reflections from horizontal surface. Unlike to $\delta \text{SNR}_{L1}$ or $\delta \text{SNR}_{L2}$ in $\delta L4$ two dominant peaks in spectra are expected, as the $L4$ is composed from two carriers $L1$ and $L2$, each separately influencing the multipath pattern of the geometry-free linear combination.
Fig. 3 Observed series of SNR1, SNR2 (transformed to linear scale) and L4 for rising satellite PRN23 (elevation angle $\theta$ from $5^\circ$ to $30^\circ$) for two successive days. The original time argument of all the series was transformed to argument $s = \sin \theta$.

The purpose of spectral analyses of the observed $\delta$SNR1, $\delta$SNR2 and $\delta$L4 series is to obtain multipath frequencies $\omega_3 = d\psi / ds$. In the analyzed series the regular sampling time interval $t$ was transformed to the irregularly spaced sampling interval $s = \sin \theta$. In such case the spectral analysis method handling unequally spaced data has to be applied. We used here the least-squares spectral analysis approach (Wells et al., 1985) which besides the dominant frequencies estimates also their amplitudes and level of noise. The maximum spectral peak of $\delta$SNR1 series corresponds to frequency $\omega_{S1}$, the maximum spectral peak of $\delta$SNR2 series to $\omega_{S2}$, and two local maxima in the spectrum of $\delta$L4 series correspond to frequencies $\omega_{L1}$ and $\omega_{L2}$.

Then the from equation (3) four estimates of effective antenna height $h$ are obtained as

\[
    h_{S1} = \frac{\omega_{S1} \lambda_4}{4\pi}, \quad h_{S2} = \frac{\omega_{S2} \lambda_4}{4\pi},
\]

\[
    h_{L1} = \frac{\omega_{L1} \lambda_4}{4\pi}, \quad h_{L2} = \frac{\omega_{L2} \lambda_4}{4\pi}
\]

(9)

Examples of observation series of SNR1, SNR2 (transformed to linear scale) and L4 for rising satellite (elevation angle $\theta$ from $5^\circ$ to $30^\circ$) for two successive days are shown in Figure 3. The original argument $t$ of all the series was transformed to $s = \sin \theta$. The residual series $\delta$SNR1, $\delta$SNR2 and $\delta$L4 which are obtained after elimination if long-term variation modeled by polynomial ($5^{th}$ order for SNR1, SNR2 and $14^{th}$ order for L4) are shown in Figure 4. Such residual series are later on submitted to least-squares spectral analysis to find the dominant frequencies relating to the multipath due to reflections.
from the horizontal surface around the GPS antenna. Figure 5 shows example from permanent station GANP for two days: DOY 344 of 2011 without snow cover and for DOY 43 of 2012 with snow depth ~25 cm. On the horizontal axes of the amplitude spectra are periods \( P \) of argument \( \sin \theta \) which are related to frequencies \( \omega \) used in (9) as \( \omega = 1/P \). The comparison of dominant periods for each of the data series documents the shift of the spectra peak related to change of horizontal reflector height.

4. RESULTS FROM GPS SNOW DEPTH SENSING AT STATION GANP (WINTER 2011/2012) AND LIE1 (WINTER 2012/2013)

The applicability of multipath analyses for snow depth sensing at Slovak permanent GNSS stations will be demonstrated on IGS and EPN permanent station Gánovce (GANP) situated close to town Poprad and permanent GNSS station Liesek (LIE1) which is a part of the Slovak Positioning Service SKPOS situated in North Slovakia, close to the Orava Reservoir. Both stations have GNSS antenna mounted on ~2.5 m concrete pillar situated in slightly undulating terrain. The basic information about the station location and station equipment are summarized in Table 1.

In the winter season the snow cover in GANP and LIE1 reaches up to 40 cm. As it is visible from Figures 6 and 7, there are several buildings and other constructions surrounding the pillars, however free sectors suitable for analysis of multipath reflections from ground are available. The reflection areas for some satellites which we found usable for snow depth sensing are demonstrated by First Fresnel Zones (FFZ) in Figures 6 and 7. They were computed for the terrain without snow according to formulae given in (Larson and Nievinski, 2012) for satellites observed at elevation angles \( \theta = 7^\circ, 15^\circ \) and \( 25^\circ \) on carrier wave L2 with wavelength \( \lambda_2 \). Reflections for satellites observed at low elevation angles cover the area up to 50 m from antenna pillar. Snow cover decreases the effective height \( h \) and diminishes the reflecting area. The 0.5 m snow layer reduces about 15% the maximum distance from pillar applied for estimation of dominant multipath frequencies.

The fundamental requirement for successful estimation of effective antenna height \( h \) from (9) is identification of clear spectral peak corresponding to
Fig. 5  Examples of amplitude spectra of $\delta SNR_1$, $\delta SNR_2$ and $\delta L_4$ from permanent station GANP for two days: DOY 344 of 2011 without snow cover and for DOY 43 of 2012 with snow depth ~ 25 cm. Comparison of dominant periods for each of the data series documents the shift of the spectra peak related to change of horizontal reflector height.

Table 1  Analyzed stations

<table>
<thead>
<tr>
<th>GNSS permanent station</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Receiver</th>
<th>Antenna height (pillar + centering rod + phase center)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gánovce GANP</td>
<td>49° 02’ 05”</td>
<td>20° 19’ 23”</td>
<td>TRIMBLE NETR8</td>
<td>2.76 m</td>
</tr>
<tr>
<td></td>
<td>704 m</td>
<td></td>
<td>TRM55971.NONE</td>
<td></td>
</tr>
<tr>
<td>Liesek LIE1</td>
<td>49° 22’ 09”</td>
<td>19° 40’ 42”</td>
<td>TRIMBLE NETR9</td>
<td>3.10 m</td>
</tr>
<tr>
<td></td>
<td>692 m</td>
<td></td>
<td>TRM59800.NONE</td>
<td></td>
</tr>
</tbody>
</table>
dominant frequency $\omega$ in the $\delta SNR$ series and two dominant spectral peaks in the $\delta L4$ series. For snow depth sensing is essential that the clearly distinguished spectral maxima are available for the processed rising or descending satellite during the whole winter season. If the GPS antenna pillar is situated in environment with various obstacles and variable terrain topography the choice of relevant satellites continuously observable above horizon up to $30^\circ$ is limited. The restrictions mentioned significantly reduce the number of potential satellites and sectors suitable for continuous snow sensing during the winter season. We found for GANP and LIE1 sites only several (3 - 4) azimuthal sectors with rising or winter season. We found for GANP and LIE1 sites suitable for continuous snow sensing during the limited. The restrictions mentioned significantly reduce the number of potential satellites and sectors for snow depth variability. In this paper we present the results obtained from descending satellite PRN 23 at site GANP (Fig. 6) and from the rising satellite PRN15 at site LIE1 (Fig. 7). It is obvious that such results will be representative only for narrow azimuth sector. The reliability of the estimated effective antenna height will be based on combination of separate estimates $h_{S1}$, $h_{S2}$ and $h_{L4}$ from $\delta SNR1$, $\delta SNR2$, and $\delta L4$ series. As the representative value from analysis of $\delta L4$ series will be used the mean of $h_{L1}$ and $h_{L2}$ (9) obtained from two dominant spectral peaks in $\delta L4$ spectra. The reference height $h_{REF}$ applied for snow depth sensing in (8) was estimated as mean value from of effective antenna height determined in time span without snow cover. It is worth to mention that due to terrain topography the estimated $h_{REF}$ from observations in selected azimuth sector could be significantly different from the value evaluated from heights of pillar, antenna reference point and antenna phase centre given in Table 1.

The analysed period for GANP permanent station covers the interval from Dec. 1, 2011 till March 3, 2012. Snow depth manual measurements are adopted from meteo-service at Poprad-Tatry international airport (www.weatheronline.co.uk), ~ 6.5 km away from GANP site. Since the elevation of Poprad-Tatry airport (696 m) is similar to Gánovce elevation and both sites are situated in the same climatic zone, we expect that the snow depth data are representative also for GANP GNSS permanent station. Descending satellite PRN 23 (elevation angle $\theta < 30^\circ$) was continuously observed each day in azimuth sector from $80^\circ$ to $110^\circ$ during the whole analysed interval. Reference height $h_0 = 2.90$ m for this azimuth sector was estimated from 15 days without snow cover.

The effective heights $h_{S1}$, $h_{S2}$ and $h_{L4}$ above the reflecting surface were estimated independently for each day separately from $\delta SNR1$, $\delta SNR2$, and $\delta L4$ series. Amplitudes of dominant frequencies for $\delta SNR1$ series are in range 10 – 20 (in linear scale), amplitudes of dominant frequencies for $\delta SNR2$ are in range 2 – 4 (in linear scale). Amplitudes for two dominant peaks of $\delta L4$ are from 3 to 8 mm.

In majority of $\delta SNR1$ and $\delta SNR2$ series only one peak was dominating. Amplitudes related to other non-dominating peaks are less than 70 % of the amplitude of the main peak. In such case the $h_{S1}$, $h_{S2}$ were computed from the frequencies related to major peak in each series. For several $\delta SNR1$ or $\delta SNR2$ series the dominant peak leads to unrealistic estimate of $h$ or more peaks with similar amplitudes are visible in the spectra (about 5 such non-standard situations occurred in the analyzed interval of PRN23 series). Such cases were solved individually (setting the polynomial of higher order up to $n=10$ for removing the long-term effects or by shortening the interval up to 50 % of the original length to remove the anomalous part of the series).

Analysis of the $\delta L4$ series is more complex. Firstly, the amplitudes of two dominant peaks are approximately equal and affiliation to $\delta L4$ will take into consideration the expected effective antenna height above the reflecting surface. Secondly, the occurrence of more than two dominant peaks with approximately equal amplitudes was observed in about 5 % of the analyzed $\delta L4$ series. We assume that it is consequence of not completely removing ionosphere effect by polynomial approximation. Such situations were solved individually, usually by shortening the analyzed $\delta L4$ series or by selection of second and third dominant peaks for evaluation of $h_{L1}$ and $h_{L2}$.

Heights above the reflecting surface $h_{S1}$, $h_{S2}$ and $h_{L4}$ estimated from analyses of PRN23 observations at GANP permanent station are plotted in Figure 8. The progressive reduction of effective antenna heights $h$ after January 10, 2012 is clearly visible as well as the progressive increase of $h$ after February 20, 2012. The biases among antenna heights derived from three different observables - SNR1, SNR2 and L4 are worth noticing. The differences between the pairs of series are not stable and also some outliers can be identified.

The snow depth inferred from GPS multipath analysis and the manual snow depth measurements are shown in Figure 9. The GPS data for every day are evaluated as mean values and their standard errors from combination of $h_{SNOW}$ obtained from SNR1, SNR2 and L4 analyses. The multipath analysis yields series consistent with manual snow depth measurements, the ~ 10 cm systematic difference between the two types of data is observed from February 1 till 20, 2012. Such difference can be caused by the fact that the localities of manual and GPS-inferred snow depths are separated about 6.5 km.

The analyzed GPS data for LIE1 permanent station are from interval Nov. 28, 2012 till Feb. 27, 2013. However, due to technical problems gaps for 1 or 2 days occurred and the snow dept from GPS was not evaluated. Snow depth manual measurements
performed daily, 30 m away from GPS antenna monumentation are provided by Slovak Hydrometeorological Institute.

Rising satellite PRN 15 (elevation angle $\theta < 30^\circ$) was continuously observed in azimuth sector from 295° to 305°. Reference height $h_G = 3.55$ m for this azimuth sector was estimated from 20 days without snow cover (from Dec. 18, 2012 to Jan. 6, 2013). The estimated effective antenna heights $h_{S1}$, $h_{S2}$ and $h_{L4}$ representing the outputs of spectral analyses of series SNR1, SNR2 and L4 at the GPS site LIE1 are plotted in Figure 10. Consistency between $h_{S1}$ and $h_{S2}$ in the period without snow is better (RMS difference is 0.02 m) than in periods with snow cover (RMS difference is 0.05 m). For periods with snow cover (characterized by $h_{S1} < h_G$ and $h_{S2} < h_G$) the non constant biases between $h_{S1}$ and $h_{S2}$ reaching ±0.10 m are observed. The $h_{L4}$ values are more scattered, however generally they follow the pattern represented by $h_{S1}$ and $h_{S2}$. Figure 11 shows the comparison of manual snow measurements and the values $h_{SNOW}$ from the multipath analysis obtained as mean of SNR1, SNR2 and L4 inferred estimates. The GPS data mimic the manual measured snow heights and they are sensitive to sudden snow depth increase and snow melting. Biases between the two series are remarkable after Feb. 8, when the GPS multipath indicates faster snow melting than the manual measurements. However, the possible effect of local topography and larger area representing the multipath data when compared to the manual snow depth sensing may be responsible for such discrepancies.

5. CONCLUSIONS

Information about the actual snow depths is valuable for studying hydrological and climate cycles and for completing the meteorological data related to the long-term monitored GNSS site. The utilization of GPS multipath for snow depth sensing in the vicinity of GNSS permanent station gives a possibility to obtain new type of information without necessity to perform additional observations and do not require additional costs for equipment. The information about snow depth is important also for geodynamic investigations for reduction or interpretations of observations using absolute or relative gravimeters. We have shown that in areas with non-ideal environment for multipath analyses (sites surrounded by buildings, structures and un-even topography) can be obtained relevant snow depth series representing with sufficient reliability the actual situation. Our approach is based on analysis of three independent observation series of setting or ascending satellite observed in narrow azimuthal sector with no significant obstacles, namely the signal-to-noise ratio related to L1 and L2 frequencies (SNR1 and SNR2) and the geometry-free carrier phase linear combination L4. Each series is analyzed separately and the representative value for snow depth for each day is evaluated as mean of three independently estimated effective antenna heights. Mean errors of snow depth estimated from GPS multipath analyses are from 0.02 to 0.12 m. The consistency of GPS estimated snow depth and manual snow cover measurements we found in the range of mean errors of GPS $h_{SNOW}$. However we noticed several biases between the GPS and manually measured snow depths reaching 0.10 m and repeating for several succeeding days. We believe that analyses of multipath performance at GNSS permanent stations and their application for study of environment in the vicinity of antenna monumentation will contribute to better interpretation of observed changes related to geodynamics of monitored sites.

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Fig. 6 Permanent GNSS station GANP and its environment. First Fresnel zones for $\theta = 7^\circ$ (red), $\theta = 10^\circ$ (green) and $\theta = 25^\circ$ (blue) are shown for observed satellites and azimuth sectors with reliable multipath reflections suitable for further analyses.

Fig. 7 Permanent GNSS station LIE1 and its environment. First Fresnel zones for $\theta = 7^\circ$ (red), $\theta = 10^\circ$ (green) and $\theta = 25^\circ$ (blue) are shown for observed satellites and azimuth sectors with reliable multipath reflections suitable for further analyses.
Fig. 8 Effective antenna heights above the horizontal reflection surface at permanent station GANP estimated from multipath analyses of \( \delta \text{SNR1} \), \( \delta \text{SNR2} \) and \( \delta \text{L4} \) series of PRN23 during the winter season 2011/2012.

Fig. 9 Snow depths and their mean errors obtained from multipath GPS analyses of PRN23 at permanent station GANP and manual snow depth measurements at Poprad-Tatry airport.

Fig. 10 Effective antenna heights above the horizontal reflection surface at permanent station LIE1 estimated from multipath analyses of \( \delta \text{SNR1} \), \( \delta \text{SNR2} \) and \( \delta \text{L4} \) series of PRN15 during the winter season 2012/2013.

Fig. 11 Snow depths and their mean errors obtained from multipath GPS analyses of PRN15 at permanent station LIE1 and manual snow depth measurements.