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ORIGINAL PAPER

GEOID HEIGHT COMPUTATION IN STRIP-AREA PROJECTS BY USING LEAST-SQUARES COLLOCATION

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ABSTRACT

GPS/leveling technique is the most effective engineering method for the determination of heights and height differences. This method is based on the principal of conversion of ellipsoid heights to orthometric heights. For transformation, polynomial surface models are generally used for study region or area. The accuracy of results depends on the location and distribution of selected reference stations with known ellipsoidal and orthometric heights. Especially, in the strip area projects (highway, railway, channel etc.) polynomial curve model is used instead of polynomial surface model due to reference stations disturbing along a route.
In this research, Least Squares Collocation (LSC) methods used in determining the geoid heights of a strip area were examined. For this purpose, GPS/leveling data of Bozkurt-Dinar (Afyonkarahisar) train project which is approximately 75 km was used in order to examine LSC methods. The ground control stations of the project were classified as reference and test for the purpose of this research. The geoid heights of test stations were calculated by curve polynomial with different degrees. Additionally geoid heights at the same points were calculated using polynomial curve fitting based on LSC which is suggested by this research. The geoid heights by computed using LSC approach when compared to polynomial curve method were observed to yield better results.

1. INTRODUCTION

Vertical positioning for engineering surveys is a harder task relative tool that of horizontal positioning. It requires the true position of the plumb line in the Earth gravity's field and its reference surface for heights above mean sea level, i.e. namely geoid. Three types of leveling, i) geometric, ii) trigonometric and iii) GPS are used for the differential measurements to produce the heights with respect to the plumb line and geoid model (Deng et al., 2013).

In GPS technique, point positions are defined as geocentric (x,y,z) or as geodetic latitude (φ), longitude (λ) and ellipsoidal heights (h) (Seeber, 2003; Daho et al., 2006; Doganalp and Selvi, 2015). While GNSS based new methods may be used in the near future (Alcay et al., 2012; Yigit et al., 2014), nowadays GPS/leveling method is demonstrated as one of the most effective methods for determining point heights. This method is based on the transformation of ellipsoidal heights to orthometric heights (Fotopoulos, 2003, 2005; Erol and Erol, 2013). The users need the conversion between the ellipsoidal height and orthometric height in many GPS applications (Seeber, 2003; Fotopoulos, 2003; Erol et al., 2005; El-Hallaq, 2012; Erol and Erol, 2013). In the literature, some successive results can be seen on the improvement of local geoid models taking as reference the standards (e.g., Erol et al., 2005, 2008; El-Hallaq, 2012). However, in the strip-area engineering projects, the number of reference stations or the distribution of available stations are likely to be problematic. Mostly, additional leveling measurements can required to form a good surface model in the study area. This situation leads to an extra burden in terms of time and cost. The optimum solution for these type applications can obtain using geoid models with high accuracy. To provide this, the precise estimation of geoid height values produced with the basis of GPS/leveling measurements is necessary. In such strip-area engineering projects, geoid heights (undulation) can be calculated by according to Least Square Collocation (LSC) method instead of polynomial surface models.

The aim of this research is to solve problems of polynomial curve fitting which is applied in the transformation of ellipsoidal heights to orthometric heights. For this purpose, geoid undulation values were calculated with the help of data of Bozkurt-Dinar (Afyonkarahisar) train project using polynomial curve model and LSC method. When the results were examined for the mentioned problem, LSC method according to polynomial curve model has been shown to give better results.

2. INTERPOLATION BY POLYNOMIAL CURVE FITTING

While forming local GPS/leveling geoid model, in the study area one makes use of the reference stations whose ellipsoidal *(h)* and Helmert orthometric *(H)* heights are known. Using a data set which consists of good positional estimates within a few cm, one can form a local geoid model by using an analytic interpolation polynomial (Sahrum et al., 2009; Deng et al., 2013). Polynomial interpolation methods are widely used especially in determining local geoid heights with the GPS/leveling method. The main

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purpose of this method is based on the expression of the studied area with only one function. Considering the distribution of the reference stations and the dimensions of the study area, Equation (1) can be defined for the curve polynomial function.

$$N(x)=\sum_{i=0}^{n} a_i x^i = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n \tag{1}$$

Here, x denotes the distance from origin for reference points, a_i indicates polynomial coefficients and n represents the degree of the polynomial. In order to generate the geoid model at reference stations, geoid heights are calculated by the well-known formula (Heiskanen and Moritz, 1967; Erol and Erol, 2013; Deng et al., 2013, Doganalp and Selvi, 2015).

$$N = h - H \tag{2}$$

where h is the ellipsoid height is measured from GPS surveying, H is the orthometric height measured from spirit leveling. The curvature of the plumb line and vertical deflection whose total effect occurs in sub-mm level are neglected in Equation (2). Observations, namely geoid heights can be described as a function of unknown coefficients for the preferred model by (1). Using matrices, the system of observation equations, is defined by $\mathbf{l} = \mathbf{A}\hat{\mathbf{x}}$ where $\mathbf{A}$ denotes the design matrix, $\hat{\mathbf{x}}$ is a vector of system parameters and $\mathbf{l}$ is the observation vector;

$$\mathbf{A}=\begin{pmatrix} 1 & x_1 & x_1^2 & \dots & x_1^n \\ 1 & x_2 & x_2^2 & \dots & x_2^n \\ 1 & x_3 & x_3^2 & \dots & x_3^n \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & x_k & x_k^2 & \dots & x_k^n \end{pmatrix},\ \hat{\mathbf{x}}=\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_k \end{pmatrix},\mathbf{l}=\begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ \vdots \\ N_k \end{pmatrix} \tag{3}$$

where k denotes the number of reference points and n represents the degree of the polynomial. In the cases where the number of reference stations is equal to that of parameters (a_i), coefficient matrix $\mathbf{A}$ is a square matrix and the problem has a unique solution (Mikhail and Ackermann, 1976). Whereas in the case where the system is over determined, the solution is more than the unique solution and one can get an approximate solution by

$$\hat{\mathbf{x}} = (\mathbf{A}^{\mathbf{T}}\mathbf{P}\mathbf{A})^{-1}(\mathbf{A}^{\mathbf{T}}\mathbf{P}\mathbf{l}) \tag{4}$$

Equation (4) which is a well-known technique, the Least-Square Adjustment (LSA) of observations, with equal weighted. Once a solution has been found for the model, it can be used for predicting the geoid heights at points where leveling measurements are not performed. In the above solution, it is assumed that the weights of the GPS/leveling data are equal to each other. An alternative procedure which uses a priori stochastic model given by $\mathbf{P}$ is called Gauss-Markov model,

$$\mathbf{v} = \mathbf{A}\hat{\mathbf{x}} - \mathbf{l} \quad ; \quad \mathbf{P} = diag\,(P_1, P_2, \dots, P_n) \tag{5}$$

where $\mathbf{v}$ represents the residual vector of observations. In Equation (5), $P_1, P_2, \dots, P_n$ are weights of observations. The standard deviation of a unit weighted observation is estimated from the residuals by Equation (6).

$$m_0 = \sqrt{\frac{\mathbf{v}^{\mathbf{T}}\mathbf{P}\mathbf{v}}{n-u}} \tag{6}$$

where n: number of observations and u: number of unknown parameters. After forming the curve model, it can be used for interpolation of some points at which geoid heights are not known. For such points, according to the linear model the coefficient matrix is as illustrated in Equation (7).

$$\mathbf{A}_p = \begin{pmatrix} 1 & x_1 & x_1^2 & \dots & x_1^n \\ 1 & x_2 & x_2^2 & \dots & x_2^n \\ 1 & x_3 & x_3^2 & \dots & x_3^n \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & x_p & x_p^2 & \dots & x_p^n \end{pmatrix} \tag{7}$$

where p denotes the number of points to be predicted and n represents the degree of the polynomial. The interpolation is obtained by the matrix multiplication

$$\mathbf{l}_p = \mathbf{A}_p \hat{\mathbf{x}} \tag{8}$$

Using the equations (3) – (8), the least square solution for the curve polynomial can be easily re-modified regarding to the surface polynomial model. In the applications of local geoid determination, it is hard to decide the degree of surface or curve polynomial at first glance. Although the degree of the polynomial is determined by upper limit, i.e., number of observations and unknown parameters, the best suitable polynomial degree for the area is generally determined according to the trial and error method. Starting from the first degree, the a posteriori variance of unit weight of adjustment results is convenient for this analysis. Theoretically, it is expected that model and observations approximate to each other as the degree of the polynomial increases. As a result, the a posteriori variance of the model decreases (see e.g., Gikas et al., 1995; Zinn, 1998; Ustun, 2001, Ceylan et al., 2011). However, due to ill-conditioned problems of the normal equation system, precision losses in the estimated parameters might result in bigger model errors depending on the increase of model degree. So, it can be said that the proper value is the previous polynomial degree before the model error starts increasing (Ceylan et al., 2011; Doganalp and Selvi, 2015).

3. LEAST SQUARES COLLOCATION (LSC)

The functional model used for interpolation does not often represent the measurements. In such situations, a simple functional model together with a stochastic model should be assumed. Collocation is the most general form of the adjustment process

Table 1 Typical covariance functions.

Gaussian function	$C(q) = C_0 e^{-q^2 q_0^2}$	Markov's function	$C(q) = C_0 e^{-q/q_0}$
Hirvonen function	$C(q) = C_0 \Big/ 1 + \left(\dfrac{q}{q_0}\right)^2$	Markov's second model function	$C(q) = C_0 (1 + q/q_0) e^{-q/q_0}$
Lauer function	$C(q) = C_0 \big/ q^{q_0}$	Exponential function	$C(q) = C_0 e^{-q/q_0}$

where C_0 is variance, q_0 is a coefficient and q is the distance between points. Because of Hirvonen function is commonly used in geodetic studies, this function was chosen for computation of experimental covariances in this study. In order to use this function, the values of C_0, q_0 should be estimated. For this purpose, first of all, LSA method is performed using measurements and one model is determined and then the unknown parameters are computed with Equation (4).

which includes LSA, filtering and prediction steps with in a combined algorithm. LSC method has been used and applied successfully in the geodetic problems such as physical geodesy, geoid determination and satellite mission applications (Tscherning, 1986; Lee et al., 2008, 2013). The most important difference between the classical LSA and LSC model is the signal and noise terms are added on the functional model. Thus, the general collocation model is:

$$\mathbf{l} + \mathbf{v} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{s} \tag{9}$$

where $\mathbf{l}$ is a measurement vector, $\mathbf{v}$ is randomly distributed uncorrelated residual vectors related to measurements and due to non-systematic errors is called *noise*, $\mathbf{s}$ is a vector of all signal quantities to be estimated and due to systematic errors is called *signal*, $\mathbf{A}\hat{\mathbf{x}}$ is the trend or deterministic part. The statistical behavior of $\mathbf{s}$ is described by a cofactor matrix whose elements are determined by an appropriate correlation function (Kutoglu et al., 2006). LSC method is generally performed in three phases ***i)*** determination of the trend, ***ii)*** computation of experimental covariances, ***iii)*** prediction of the signals at estimation points.

i. Determination of the trend:

Before starting LSC process, an analytic function fitting the measurements is determined and the measurements are examined using this model. The residual values which are obtained via a pre-adjustment indicate whether is proper of the selected function for the measurements. In general, the functions with the smallest standard deviation are chosen. The type of function could be linear, polynomial, spline, trigonometric or a function of higher degree. However, there is an important point to be considered (Wolf, 1955; Demirel, 1977; Ceylan et al. 2011; Doganalp and Selvi, 2015). The increase in the function degree will cause an increase in the number of unknowns. Therefore, function degree should be selected very carefully. In this study, curve polynomial function has been selected as the analytical function.

ii. Computation of experimental covariances:

The most important difference between collocation and LSA method lies in the inclusion of a signal term within functional model. Therefore, the correlated signals and the covariance matrix which is calculated from experimental covariances should be included in the adjustment. The statistical behavior of the signals (s) is described by a cofactor matrix whose elements are determined by an appropriate correlation function (Kutoglu et al., 2006). The elements of this matrix are determined with a suitable covariance function. Thus, the covariance functions of the signals are determined empirically with the help of the covariance functions. Hence, before adjustment, one should determine the covariance matrix which is calculated using the distances (q) between measurement and estimation points. Also, the covariance matrices are unknown in the general form LSC solution. The covariance function of a non-stochastic field in its most general form is inhomogeneous and anisotropic. Hence the covariance function depends by definition only upon the distance (q) between two points (Egli et al., 2007). In calculating covariance matrices, one makes use of experimental covariance functions fitting the measurements and the model. Some covariance functions depending on the q parameter are given Table 1 (Tscherning and Rapp, 1974; Moritz, 1976; Musyoka, 1999; Krynski and Lyszkowicz, 2006).

iii. Prediction of the signals at estimation points:

In the collocation model, the estimated values are signals. For the estimation of these values, signals of known points and correlation between them are used. The stochastic quantity (*z*), which remains after subtracting the deterministic part from measurements by pre-adjustment, consists of noise (*n*) and signal (*s*). The purpose is to separate signals from the stochastic quantity. To this end, covariance matrices which show correlation between signals and covariance function formed with the help of experimental covariances are calculated. Measurement (reference) point (**s**) and estimation point signals ($\mathbf{s_p}$) are called internal and external signals, respectively. For solve collocation problem, correlations related to signals or weight coefficients should be given. The weight coefficient matrix is determined by using a covariance function. If the number of the estimation points is *m* and number of the measurement points is *n*, then the covariance matrices of internal signals $(\mathbf{C}_{ss})$, external signals $(\mathbf{C}_{s_p s_p})$ and the cross covariance matrix $(\mathbf{C}_{s_p s})$ between internal and external signals are given by Equation (10).

$$\mathbf{C}_{ss}=\begin{bmatrix} C_s(0) & C_{s_1s_2} & \dots & C_{s_1s_n} \\ C_{s_2s_1} & C_s(0) & \dots & C_{s_2s_n} \\ \dots & \dots & \dots & \dots \\ C_{s_ns_1} & C_{s_ns_1} & \dots & C_s(0) \end{bmatrix},\ \mathbf{C}_{s_ps_p}=\begin{bmatrix} C_s(0) & C_{s_1s_2} & \dots & C_{s_1s_m} \\ C_{s_2s_1} & C_s(0) & \dots & C_{s_2s_m} \\ \dots & \dots & \dots & \dots \\ C_{s_ms_1} & C_{s_ms_1} & \dots & C_s(0) \end{bmatrix},\ \mathbf{C}_{s_ps}=\begin{bmatrix} C_s(0) & C_{s_1s_2} & \dots & C_{s_1s_n} \\ C_{s_2s_1} & C_s(0) & \dots & C_{s_2s_n} \\ \dots & \dots & \dots & \dots \\ C_{s_ms_1} & C_{s_ms_1} & \dots & C_s(0) \end{bmatrix} \quad (10)$$

When the covariance matrices are divided by selected suitable μ_0^2 value (for example, variance value of signals), the weight coefficient matrices are obtained by Equation (11).

$$\mathbf{Q}_{ss}=\frac{\mathbf{C}_{ss}}{\mu_0^2}=\begin{bmatrix} Q_{s_1s_1} & Q_{s_ss_2} & . & . & . & Q_{s_1s_n} \\ Q_{s_2s_1} & Q_{s_2s_2} & . & . & . & Q_{s_2s_n} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ Q_{s_ns_1} & Q_{s_ns_2} & . & . & . & Q_{s_ns_n} \end{bmatrix},\ \mathbf{Q}_{s_ps_p}=\frac{\mathbf{C}_{s_ps_p}}{\mu_0^2}=\begin{bmatrix} Q_{s_Is_I} & Q_{s_Is_{II}} & . & . & . & Q_{s_Is_m} \\ Q_{s_{II}s_I} & Q_{s_{II}s_{II}} & . & . & . & Q_{s_{II}s_m} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ Q_{s_Is_m} & Q_{s_{II}s_m} & . & . & . & Q_{s_ms_m} \end{bmatrix}$$

$$\mathbf{Q}_{s_ps}=\frac{\mathbf{C}_{s_ps}}{\mu_0^2}=\begin{bmatrix} Q_{s_1s_1} & Q_{s_1s_2} & . & . & . & Q_{s_1s_n} \\ Q_{s_{II}s_1} & Q_{s_{II}s_2} & . & . & . & Q_{s_{II}s_n} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ Q_{s_ms_1} & Q_{s_ms_2} & . & . & . & Q_{s_ms_n} \end{bmatrix},\ \mathbf{Q}_{ll}=\frac{\mathbf{C}_{ll}}{\mu_0^2}=\begin{bmatrix} Q_{L_1L_1} & 0 & . & . & . & 0 \\ 0 & Q_{L_2L_2} & 0 & . & . & . \\ . & 0 & . & & & . \\ . & . & & . & & . \\ . & . & & & . & 0 \\ 0 & 0 & . & . & 0 & Q_{L_nL_n} \end{bmatrix} \quad (11)$$

Here, $\mathbf{Q}_{ss}$ denotes the weight coefficient matrix between internal signals, $\mathbf{Q}_{s_ps_p}$ indicates the weight coefficient matrix between external signals, $\mathbf{Q}_{s_ps}$ represents the cross weight coefficient matrix between internal and external signals and $\mathbf{Q}_{ll}$ indicates the weight coefficient matrix of measurements. Since there is no correlation between measurement errors and signals, the total covariance matrix of the observations in the LSC model is obtained by the sum of the covariance matrices of the signal and the noise.

$$\overline{\mathbf{Q}}=\mathbf{Q}_{ss}+\mathbf{Q}_{ll} \quad (12)$$

The equations expressing the solution for the systematic parameters, the signal quantities (**s**) at measurement points and the observational noise (**n**) are as follows (Collier and Leahy, 1992):

$$\mathbf{x}=\left(\mathbf{A}^T\overline{\mathbf{Q}}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^T\overline{\mathbf{Q}}^{-1}\mathbf{l} \quad (13)$$

$$\mathbf{s}=\mathbf{Q}_{ss}\overline{\mathbf{Q}}^{-1}(\mathbf{l}-\mathbf{Ax}) \quad (14)$$

$$\mathbf{n}=\mathbf{Q}_{ll}\overline{\mathbf{Q}}^{-1}(\mathbf{l}-\mathbf{Ax}) \quad (15)$$

The signals ($\mathbf{s}_p$) and the calculated geoid values ($\mathbf{L}_p$) in estimation points are obtained using the equations.

$$\mathbf{s}_p=\mathbf{Q}_{s_ps}\overline{\mathbf{Q}}^{-1}(\mathbf{l}-\mathbf{Ax}) \quad (16)$$

$$\mathbf{L}_p=\mathbf{A}_p\mathbf{x}+\mathbf{s}_p \quad (17)$$

$$rms=\pm\sqrt{\frac{1}{m}\sum_{i=0}^{m}(\mathbf{L}_p-\mathbf{l})^2} \quad (18)$$

where $\mathbf{s}_p$ and $\mathbf{L}_p$ respectively indicate the signals and geoid undulation values in estimated points (*p*), *m* is the number of estimation point and *rms* represents the root mean square error of the geoid heights differences. For details about collocation method, see (Tscherning and Rapp, 1974; Moritz, 1976, 1978; Balmino, 1978; Koch, 1999).

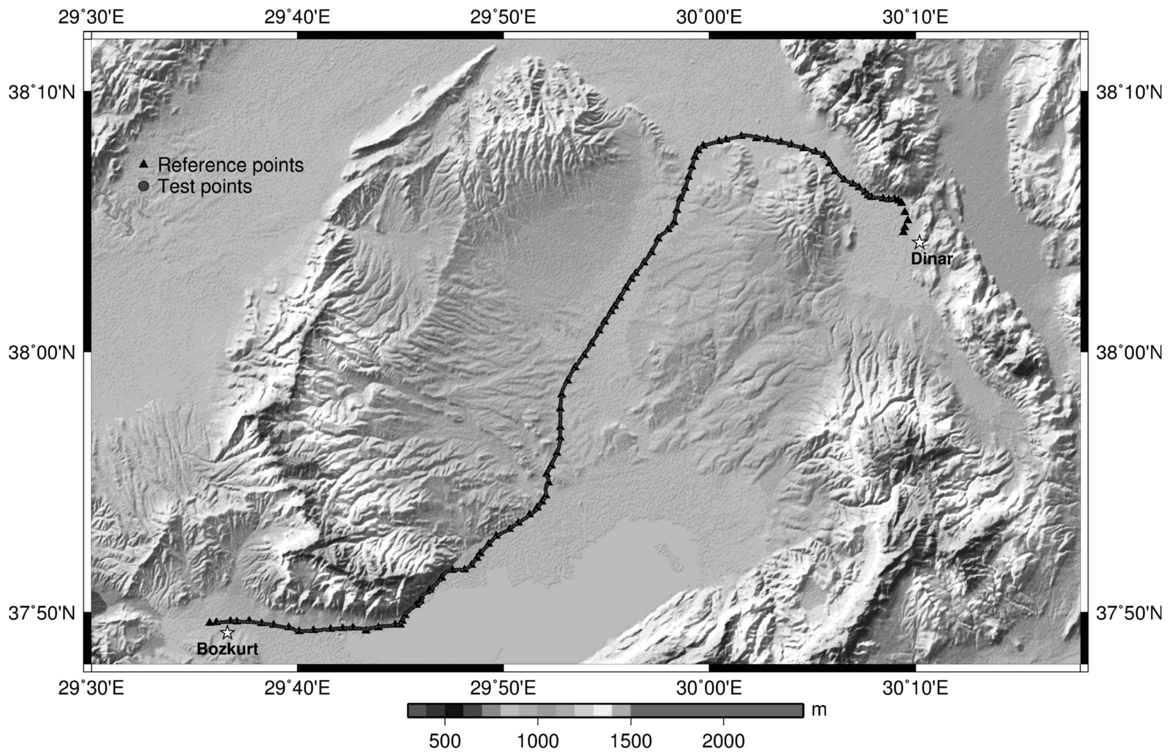

Fig. 1 The study area, Bozkurt-Dinar (Afyonkarahisar) Train Project.

4. APPLICATION

The study area is Bozkurt-Dinar (Afyonkarahisar) train project. The length of the project is approximately 75 km. It consists of total 449 points, 99 of which are benchmarks and 350 of which are traverse stations. Orthometric heights (*H*) of the benchmarks and traverse stations were carried out with geometric levelling method in the datum of Turkey National Vertical Network (TUDKA). The geographic coordinates including ellipsoidal heights (*h*) were determined in static positioning mode and referred to Turkish National Fundamental GPS Network (TUTGA). Points in the study area were classified as reference and test (ground control) points. Within the scope of application, benchmarks were selected as reference points and traverse stations were selected as test (ground control) points (Fig. 1). While the orthometric height (*H*) values of the reference points (99 pts) ranging between 833 and 900 m, ellipsoidal height (*h*) values ranging from 869 to 936 m. Similarly, according to the test points (350 pts) heights are ranging between 833-898 m and 869-934 m for orthometric and ellipsoidal heights, respectively (Fig. 2).

In this section, the computation process of the geoid heights (undulations) through the LSC method in the strip area was presented. A software package developed by the author according to the mathematical theory was used for the geoid height determination evaluations. Firstly, the geoid models for curve fitting methods were established in least-squares sense and then this model with LSC method was implied in the study area. SRTM-3 data and the Generic Mapping Tools (GMT) software were used for Figure 1 (Wessel and Smith, 1998). The Shuttle Radar Topography Mission (SRTM) contains elevation data with 3 arc-second resolution and 16 m absolute height error (90 percent confidence level). These data are freely available via the Internet for approximately 80 percent of the Earth's land mass (Bildirici et al., 2009).

4.1. DETERMINING OF GEOID HEIGHTS BY CURVE MODEL

In this study area, the surface polynomial can be applied in different degrees. However, since the reference stations are located along the route and point distribution is irregular, surface fitting method is affected negatively. Besides, for higher degree solutions unknown parameters give insignificant results. Therefore, the curve fitting method is appropriate for strip area projects such as highway, railway etc. The polynomial curve model is applied as a distance variation of test points along, geoid heights and coordinate values of 99 reference stations in the study area. In this approach, the polynomial curve was crossed by means of assuming the kilometers of reference points as abscissa and geoid heights as ordinate values. Because the curve model according to the surface model is less than the number of unknown parameters, degree of the model could be increased up to 13. The significance of the parameters for the higher degree model decreases. After curve fitting method, the best of root mean square error for 350 test points has been computed approximately ±1.9 cm for 10^{th} degree. Model statistics and the results of geoid height comparisons for curve fitting are given in Table 2. At the test points, the concordance of

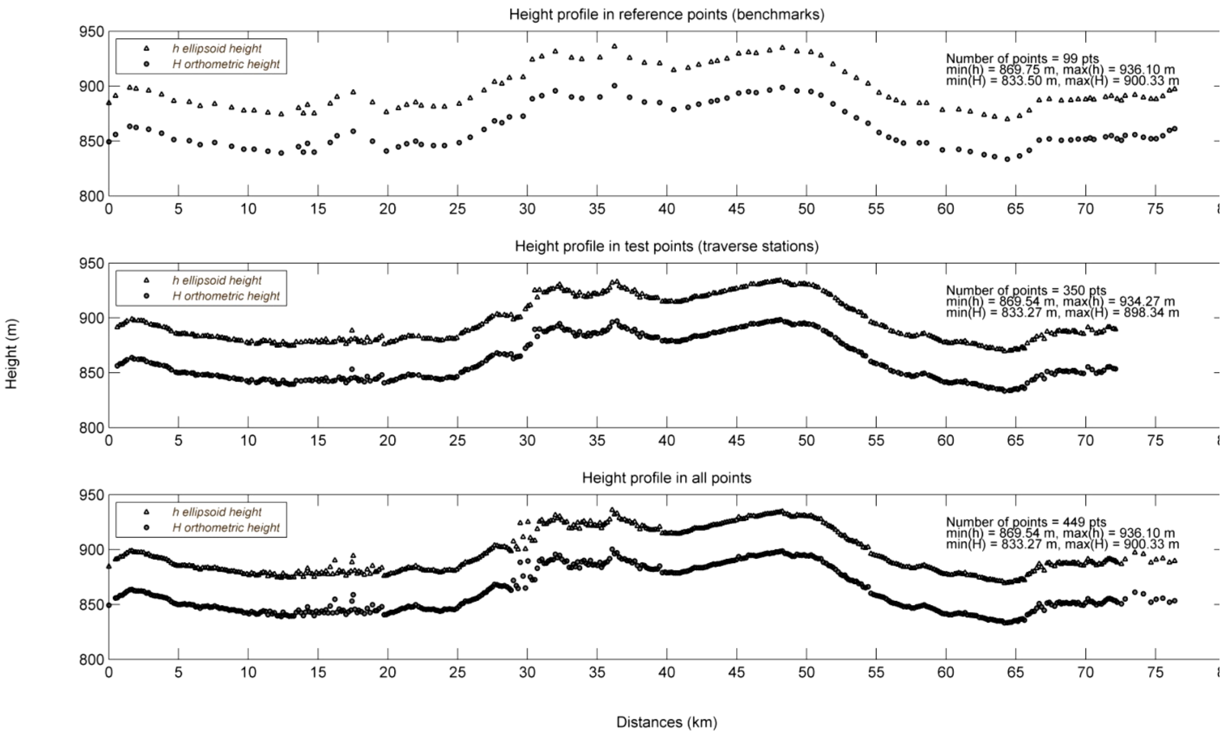

Fig. 2 Height profile in the study area.

Table 2 The results of model and comparison statistics at control points for curve model.

Degree of polynomial	Model statistics			Geoid height differences			
	min (cm)	max (cm)	m_o (cm)	min (cm)	max (cm)	mean (cm)	rms (cm)
1st	-16.4	27.9	±10.7	-19.8	13.5	-2.8	±8.9
2nd	-17.2	17.8	±8.9	-18.5	12.9	-1.4	±8.2
3rd	-5.7	8.4	±3.0	-9.2	9.1	-0.5	±3.4
4th	-6.5	8.1	±2.9	-8.3	8.8	-0.4	±3.3
5th	-6.2	7.1	±2.8	-8.1	7.8	-0.4	±3.1
6th	-5.8	7.1	±2.8	-8.5	7.7	-0.4	±3.0
7th	-4.9	5.5	±2.3	-6.6	6.1	-0.2	±2.4
8th	-5.0	4.9	±2.2	-6.5	5.6	-0.2	±2.2
9th	-5.1	4.8	±2.2	-6.9	6.0	-0.1	±2.3
10th	**-4.1**	**5.5**	**±2.1**	**-5.7**	**5.7**	**0.1**	**±1.9**
11th	-4.1	279.0	±89.3	-5.7	165.8	16.4	±38.4
12th	-4.0	868.7	±289.8	-5.6	545.9	62.2	±134.0

between curve model and true geoid undulation values (GPS/Lev) are shown in Figure 3. Also, the geoid height differences in detail between true values and 10th degree curve model are shown in Figure 4.

4.2. DETERMINING OF GEOID HEIGHTS BY LSC

Firstly, in order to calculate signals at measurement (**s**) and estimation points ($\mathbf{s_p}$), a covariance function which to express correlation between them is defined. Because of that it is the commonly used function in geodetic studies in this study Hirvonen function was chosen. Variance and covariance values obtained by experimental methods are used to define covariance function. Covariance values between signals at reference stations were calculated for 1, 2, 3, 4, 6, 10 and 20 km intervals. Similarly, geoid heights differences for curve model at test stations were calculated for these intervals (Table 3). When the collocation results were examined, the best results in terms of geoid height differences were obtained in the 2nd degree and 2 km range for the curve collocation. The rms value of ±1.5 cm for the curve collocation was obtained. Also, the geoid height differences in detail between true values and 2nd degree LSC curve model are shown in Figure 5.

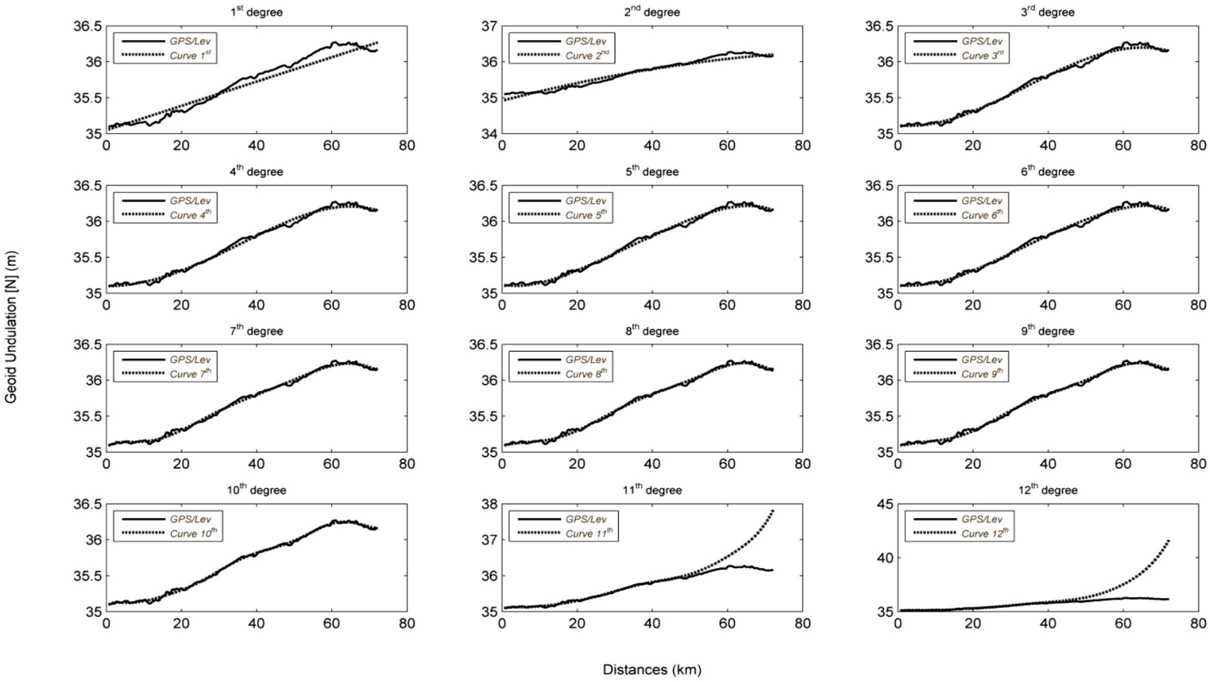

Fig. 3 The change of geoid heights for curve model.

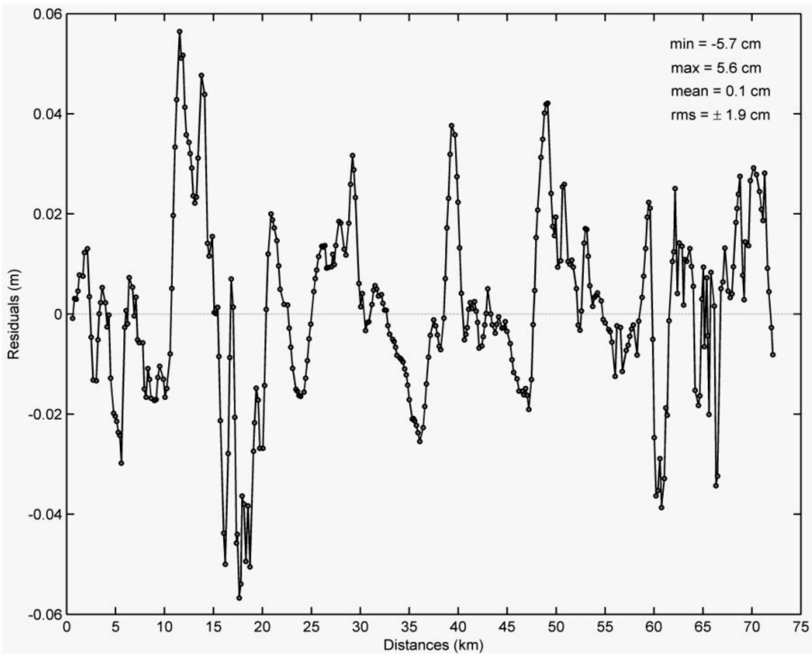

Fig. 4 The geoid height differences between true values and 10^{th} degree curve model.

When the Table 3 was examined in more detail, it can be seen that the unknown parameters corresponding to higher degree solutions give insignificant results for LSC approach. In addition, the best results in terms of geoid height differences were obtained in the 2^{nd} degree and 2 km range for the curve collocation. The concordance between true geoid undulations (GPS/Lev) and the values obtained by LSC at test stations is depicted in Figure 6. As the Figure 6 illustrates, there is good concordance between collocation model and true geoid undulations.

5. CONCLUSIONS AND SUGGESTIONS

Determination of the orthometric heights with leveling technique is a simple but extra effort, cost and time consuming procedure compared to GPS. Therefore, nowadays GPS/leveling method is demonstrated as one of the most effective methods for determining point heights. The basic principle of the method is based on the transformation of ellipsoidal heights to orthometric heights. This transformation procedure can be performed by using geoid heights (undulations) values. Unknown geoid heights at any

Table 3 The results of LSC for curve models.

Degree	km	min (cm)	max (cm)	mean (cm)	rms (cm)	Degree	km	min (cm)	max (cm)	mean (cm)	rms (cm)
1st	1	-36383.1	1835.0	-127.7	±1988.8	4th	1	-1119.4	1224.1	17.2	±408.2
	2	-21921.8	15146.8	-35.6	±1660.5		2	-57.8	55.9	-1.0	±30.3
	3	-10.8	8.5	-0.7	±3.8		3	-33.7	29.8	-0.2	±16.6
	4	-5.8	6.0	-0.3	±2.1		4	-36.9	45.4	3.8	±22.8
	6	-5.5	5.7	-0.2	±1.9		6	-29.9	26.6	1.8	±15.5
	10	-4.8	5.0	-0.2	±1.7		10	-28.7	26.6	1.8	±15.2
	20	-4.8	4.8	-0.2	±1.6		20	---	---	---	---
2nd	1	-52421.2	10128.5	-248.7	±3075.6	5th	1	-183.5	175.5	8.9	±60.1
	2	**-5.3**	**4.5**	**-0.2**	**±1.5**		2	-367.0	450.1	15.3	±216.8
	3	-5.1	4.6	-0.2	±1.5		3	-86.7	99.1	2.9	±49.3
	4	-5.1	4.6	-0.2	±1.5		4	-201.5	222.9	6.3	±116.6
	6	-5.0	5.1	-0.2	±1.7		6	-41.4	49.7	-0.5	±24.8
	10	-5.0	4.9	-0.2	±1.6		10	-92.6	97.1	-0.6	±49.4
	20	-5.1	4.6	-0.2	±1.5		20	---	---	---	---
3rd	1	-249.8	222.2	12.2	±103.1	6th	1	-153.8	121.9	4.3	±60.6
	2	-59.2	62.8	4.2	±30.7		2	-195.4	245.5	8.6	±112.2
	3	-51.3	60.8	3.3	±33.4		3	-134.8	151.3	4.3	±77.9
	4	-32.5	47.5	2.9	±22.1		4	-136.6	153.1	4.3	±78.9
	6	-128.4	137.8	-1.9	±76.3		6	-87.2	97.5	-1.8	±51.1
	10	-103.8	115.7	-1.2	±62.5		10	-63.0	86.1	3.9	±33.6
	20	---	---	---	---		20	---	---	---	---

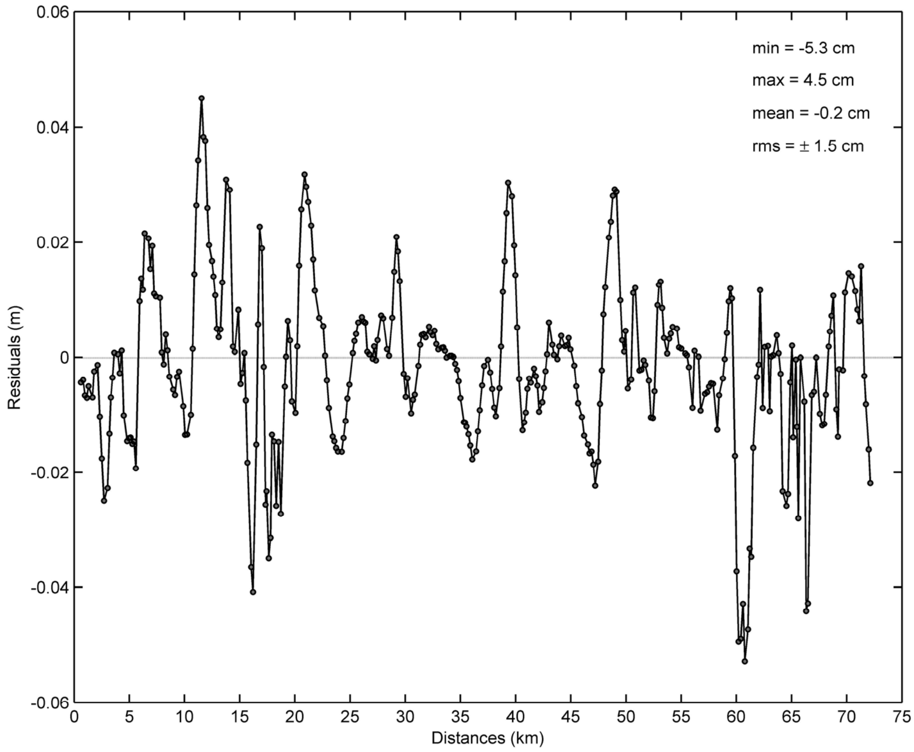

Fig. 5 The geoid height differences between true values and 2nd degree LSC curve model.

points can be estimated by using the known GPS/leveling geoid heights. The accuracy of the procedure depends on several factors. These factors can be generally classified as the number of reference stations or the distribution of available stations and interpolation method.

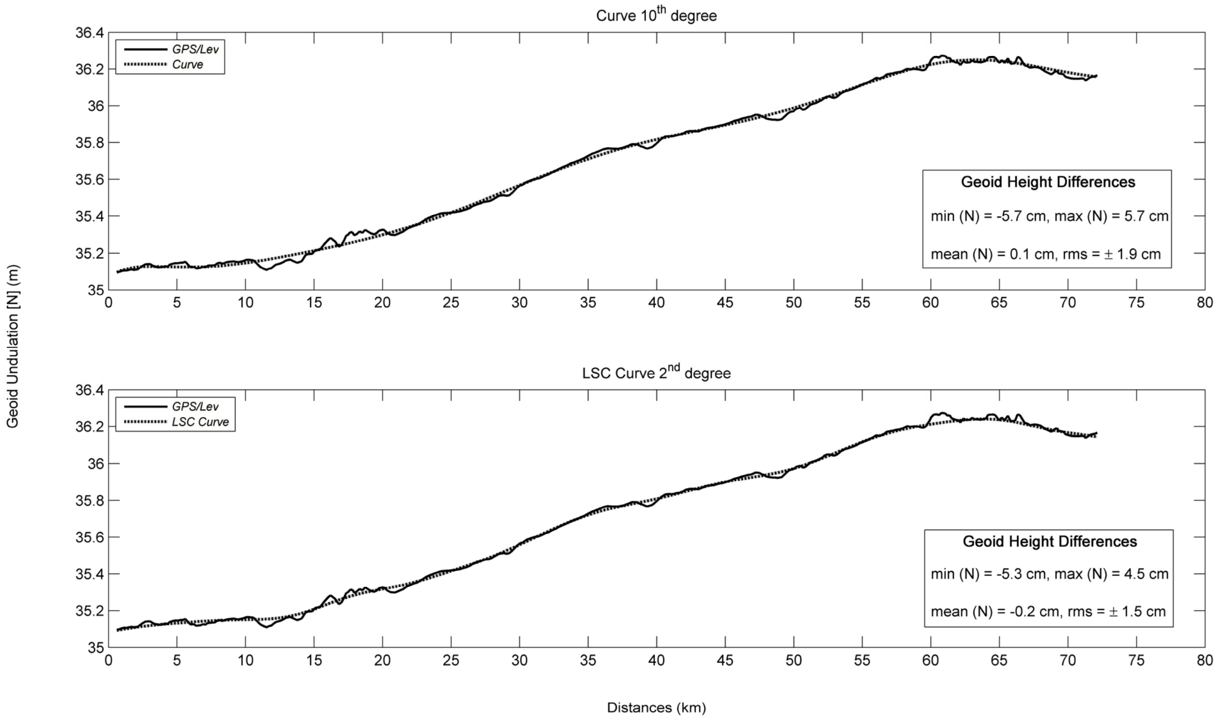

Fig. 6 The change of geoid height differences.

Generally, polynomial surface models are preferred on determining the heights based on GPS/leveling. But, the accuracy of these models should require the homogenously dispersed points within the study area. However, in the strip-area projects, the number of reference stations or the distribution of available stations are likely to be problematic. Therefore, more precise determination of heights can be obtained by using the different interpolation methods for these type areas. Although the surface polynomials can be applied in different degrees in this type area, surface models are affected negatively because of irregular distribution of points. Therefore, the polynomial curve fitting methods could be applied instead of surface models in these cases. On the other hand, as it can also be seen from application results, the use of polynomial methods with LSC creates a positive effect and increases the accuracy of results.

The structure of study area is nearly flat terrain and consists of 99 reference and 350 test (ground control) points with a length of route 75 km. In the application, the best rms value was obtained as ±1.9 cm for the curve polynomial. Moreover, as a result of the use of curve polynomial with the collocation method, the rms value was calculated as ±1.5 cm. When the results were examined, the geoid height values obtained from the curve model together with LSC has been shown to give better results than the curve model. On the other hand, the calculated geoid heights using curve models can be seen adequate for engineering applications. But it takes too much computing time to solve of results in different degrees for curve model. This process time in the geoid heights computations can be reduced using by curve model together LSC method. In this study, the best results for the curve model were obtained the 10^{th} degree. On the other hand, as a result of the curve model cooperation with LSC was found to be adequate results for the 2^{nd} degree model. For obtaining more accurate results in the strip area projects geoid heights curve model can be used with LSC method. Also, the advantage of LSC method is that it rules out the necessity of choosing polynomials with higher degrees. Therefore, the lower degree polynomial curve model with LSC method was found to be sufficient for geoid undulations determination in strip area projects.

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