



ORIGINAL PAPER

THE INFLUENCE OF DIFFERENT TYPES OF NOISE ON THE VELOCITY UNCERTAINTIES IN GPS TIME SERIES ANALYSIS

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ABSTRACT

Due to the development of GPS technology, nowadays we are able to determine geodynamic plate motion around the world. Using an appropriate noise technique and understanding all the stochastic effects we are able to do a proper GPS time series analysis in which the sources of noise can be classified as: white noise, flicker noise and random walk.

We study the area from the Caribbean Sea, taking the data from two GPS stations for a period of 7.5 years. We use spectral analysis and Maximum Likelihood Estimation. In the first part of the analysis we simultaneously estimate the velocity and amplitudes of the noise with integer spectral index and in the second part we estimate the spectral index. The noise model that presented the higher values of the log likelihood is a combination between power law and white noise which best describes the noise characteristics of all three components. In all the cases the noise amplitudes presented higher values for the vertical component. Also, the rate uncertainties for the power law plus white were higher by a factor of 10, then by using only white noise model.

INTRODUCTION

To be able to use the GPS technology in geodynamics, we need to investigate the velocity uncertainties from GPS position time series. Several research groups demonstrated that GPS time series are affected not only by white noise (no time dependence) but also by colored noise (time-correlated noise). By understanding the error spectrum of the Global Positioning System (GPS) we can obtain more precise results which can be employed in geodynamic applications (Nistor and Buda, 2014).

The GPS techniques, has proven over the years that it can be an outstanding tool for plate tectonics studies such as: crustal motions and deformation. By using the position time series as a result of GPS measurements, the horizontal and vertical velocities can be determined. A proper analysis of the position time series in geodynamic determinations invokes estimates of velocity and their uncertainties to be unbiased. The geodetic velocity and their uncertainties are computed indirectly through repeated position measurements of given points (Hackl et al., 2011). These measurements have to be taken for several years or more to obtain accurate velocity estimates, thus resulting in a large variety of errors that can corrupt the data.

A constant long term signal (interseismic) rate is not the only contribution to the antenna motion, but also, offsets due to antenna changes or coseismic displacements, annual or semiannual seasonal deformations, or postseismic deformation (Hackl et al., 2011).

The noise model have to take into account the seasonal variation which results from different

geophysical sources and systematic modeling errors (Dong et al., 2002). The research done by (Dong et al., 2002; van Dam et al., 2001) presented that the coordinates obtained from continuous GPS measurements are affected by seasonal variations with annual and semiannual periods which are present into global and regional networks. (Blewitt and Lavallée, 2002) came to the conclusion that if it isn't taken into account the annual variation signal the results of the site velocity will be greatly bias.

By using only a white noise model (Zhang et al., 1997) concluded that the rate uncertainties were 3-6 times smaller than by invoking a combination of white and flicker noise. Also (Mao et al., 1999) presented that if the correlated noise was neglected the rate uncertainties were underestimated by as much as an order of magnitude. The reduction of the white noise effect can be done by frequent measurements and averaging, although this is not the case for colored noise which is time correlated. This source can be: mismodeled satellite orbits, mismodeled antenna phase center, mismodeled atmospheric effects (Mao et al., 1999). The presence of random walk was detected in continuous measurements of strainmeters as well as in very short GPS baseline (Langbein and Johnson, 1997).

By taking into account the parameters like spectral index, amplitude of the noise and sampling interval we can estimate the standard error in rate (Williams, 2003). From this we can conclude that the chosen noise model greatly affects the rate uncertainty and a classification and quantification of noise components has to be done before using the data in geodynamic applications.

For characterizing the noise in time series analysis different techniques can be used: the maximum likelihood estimation (MLE) method, power spectral method and Least Squares Variance Component Estimation (LS-VCE). The first one is used to examine the data from covariance matrix in the time /space domain, and the second one is used to examine the data in the frequency domain. To model the noise effectively it is recommended to use the MLE method in contrast to the classical power spectra technique. The MLE method is generally used to compute the amount of white noise, flicker noise and random walk noise in the time series (Zhang et al., 1997; Langbein and Johnson, 1997; Mao et al., 1999; Williams et al., 2004; Langbein and Bock, 2004).

Taking the work done by (Zhang et al., 1997), they analyzed the time series from 10 continuous GPS sites in Southern California, for a period of 19 months, in which the time series presented significant colored noise. Flicker noise plus white noise, or fractional white noise best describes the time series instead of random walk plus white noise. Due to shortness of the time series they could not rule out the existence of random-walk noise as detected by other geodetic instruments. To be able to estimate the amount of random walk plus white noise or flicker plus white noise contained in each time series (Zhang et al., 1997) used maximum-likelihood technique to assess the velocity uncertainties.

MATHEMATICAL MODEL

The form of the power spectrum P_x , that describes many types of geophysical data whose behavior in the time domain denoted by $x(t)$ given by (Mandelbrot and Van Ness, 1968; Agnew, 1992) is:

$$P_x(f) = P_0 \left(\frac{f}{f_0} \right)^k \tag{1}$$

where, f is the spatial or temporal frequency, P_0 and f_0 are normalizing constants and k is the spectral index. Typically, the spectral index, k , lies within the range -3 to 1 (Agnew, 1992). The process within this range is subdivided into “fractional Brownian motion” with $-3 < k < -1$ and “fractional white noise” with $-1 < k < 1$ (Mandelbrot, 1977, 1983). Within this stochastic model occur special cases at the integer values. At $k=0$ we are dealing with classical white noise, at $k=-1$ we are subject to flicker noise and $k=-2$ we have Brownian motion – the so-called “random walk”. To refer to power law processes that differ from classical white noise, we will use the term colored noise.

Taking the work done by (Williams, 2008) to be able to estimate the noise components and the parameters of the linear function, the likelihood l from a set of observation x have to be maximized. If we are assuming that the distribution is Gaussian the likelihood is:

$$l(x, C) = \frac{1}{(2\pi)^{\frac{N}{2}} (\det C)^{\frac{1}{2}}} \exp(-0.5 \hat{v}^T C^{-1} \hat{v}) \tag{2}$$

where:

- \det represents the determinant of the matrix,
- C represents the covariance matrix of the assumed noise in the data
- N is the number of epochs and
- \hat{v} is the postfit residuals of the linear function using weighted least squares with the same covariance matrix C .

In terms of stability the logarithm of the likelihood has to be maximized or minimize the negative:

$$MLE = \ln[l(x, C)] = \frac{1}{2} [\ln(\det C) + \hat{v}^T C^{-1} \hat{v} + N \ln(2\pi)] \tag{3}$$

due to the fact that the maximum is unaffected by the monotonic transformations.

The typical model is composed by an intercept, a linear trend (velocity), sinusoidal terms represented by annual and semiannual signal, terms for offsets and in the case of a large coseismic event, a term to describe the postseismic motion (Nikolaidis, 2002). The covariance matrix C can correspond to different stochastic noise such as white, power law, moving average, autoregressive, first-order Gauss Markov, band pass and also combination between these different types of noise. If it is assumed that we are dealing with a white noise component and power law then the matrix is:

$$C = a_w^2 I + b_k^2 J_k \tag{4}$$

where a_w and b_k represents the white respectively the power law noise amplitudes, I is the $n \times n$ identity matrix and J_k is the power law covariance matrix with spectral index k . Although the equation (4) presents only two types of noise, the time series may contain more than this two types of noise and sometime may not be power law noise (Williams et al., 2004).

By fitting a straight line through a series of n points x_i taken at time t_i which represents the basic liner regression problem the determination of rate uncertainties is given by:

$$x_i = x_0 + r t_i + \epsilon_x(t_i) \tag{5}$$

where $\epsilon_x(t_i)$ represents the error term.

Assuming that $\epsilon_x(t_i)$ is subjected to linear combination of independent random variables and it is identically distributed, $\alpha(t_i)$, and a sequence of temporally correlated random variables, $\beta(t_i)$ such as:

$$\epsilon_x(t_i) = a \alpha(t_i) + b_k \beta(t_i) \tag{6}$$

the amplitude of white noise is represented by the

scalar factor a and $b_{k \neq 0}$ is the scale factor of colored noise of spectral index k .

By assuming that the covariance matrix presents time-dependent positions and the station monuments are subject to a random walk process, then the formal uncertainty of the estimated velocities is approximated by (Zhang et al., 1997; Williams, 2003; Bos et al., 2008):

$$\sigma_r^2 \approx \frac{b^2}{T} = \frac{b^2}{\Delta T(N-1)} \quad (7)$$

where, b represents the random walk noise amplitude, N the number of measurements and T is the total time span. Equation (7) expresses that in the presence of heavily correlated time series, velocity uncertainties are significantly influenced with respect to those from uncorrelated time series. Increasing the observation span by adding more (correlated) position barely reduced the formal rate uncertainty. Also, if the observation span is kept constant then changing the sampling interval, does not affect at all the estimated formal uncertainties.

Another method to assess the presence of the noise in GPS time series analysis is to use the least squares variance component estimation described by (Amiri-Simkooei et al., 2007). Considering the linear model of observation as:

$$E(\underline{y}) = Ax, D(\underline{y}) = Q_y = \sum_{k=1}^p \sigma_k Q_k \quad (8)$$

where the design matrix A has the dimension $m \times n$ is assumed to be a full column rank, the $m \times m$ covariance matrix Q_y of the m - observable vector \underline{y} is assumed to be positive definite, y the n -vector of parameters has to be estimated and E and D are the expectation and dispersion operators where the underscore represents a random variable. The $m \times m$ cofactor matrix Q_k is assumed to be symmetric

such that the sum $\sum_{k=1}^p \sigma_k Q_k$ is positive definite. In this case the cofactor matrix Q_k should be imposed to fulfill the stochastic condition of being linearly independent to have a regular solution. There are several advantages for this method but this is outside the articles scope. For more information refer to (Amiri-Simkooei, 2007; Xu et al., 2006).

PROCESSING AND RESULTS

The experiment was conducted by using the data from the GPS station in Caribbean Sea – Bggy and Greo. The data were downloaded from UNAVCO in *.pos format. For the time series analysis we used the Hector software (Bos et al., 2012).

The data from the GPS station are for a period of 7.5 years from 2007.5 until 2015. For estimating the rate uncertainties and the parameters of the noise model we have used the Maximum Likelihood Estimation (MLE). We have tried different type of noise to see which is more appropriate for our experiment. The authors had chosen different types of

noise based on the extensive studies done by (Santamaria-Gómez et al., 2011; Amiri-Simkooei et al., 2008; Bos et al., 2008; Langbein, 2012; Williams, 2003) and shown that the most present types of noise in GPS time series analysis is: Power - law plus white noise, white noise only, a combination between white noise and random walk and a combination between white noise and flicker noise. To verify if the chosen model behaves “good enough” we have computed the power spectra density from the generated residuals after subtracting the least squares which estimated the linear motion.

The top panel of Figure 1 presents the daily observation for station Greo and the bottom panel presents the daily observation for station Bggy.

First a linear trend using ordinary least-square is applied to the raw data and the resulted residuals are ordered by size and then the values that are less than three times the interquartile range, below or above the median are considered outliers. This approach was recommended by (Langbein and Bock, 2004). All statistical methods contain different types and number of assumptions (Nistor and Ionascu, 2013).

Taking into account the work done by (van Dam et al., 2001), GPS time series analyses have significant annual signals. The effects of these annual and semiannual effects can significantly bias the velocity estimates. If we are talking about global reference frames, then the dominant cause of annual signals, is the surface loading, due to hydrology and atmospheric pressure. Over short data spans the seasonal variations, which is best described by a deterministic model, contributes to velocity error. Due to the fact that the spectral index was $1 < k < 2$ the power spectrum confirms the presence of significant annual harmonic frequencies, from which we can conclude that we are dealing with repeating signals. For the determination of site velocity and initial position it is strongly recommended to estimate them simultaneously with annual and semiannual sinusoidal signal, because they bias the estimation of site velocity for high accuracy purposes. Assuming that there isn't an annual signal in the estimation process, the results tend to be more than optimistic and (Dong et al., 2002) demonstrated that the time series exhibit an annual variation with an amplitude of a few millimeters. So, to remove a seasonal variations we can fit a sinusoid with a period of one year during the detrend operation, which was also the case for our experiment - Figure 2.

In Figure 2 the sinusoidal red lines represent the ± 1 standard deviation of the computed model – estimated trend, and the green lines represent ± 3 time's standard deviation.

We have computed the log likelihood values using the maximum likelihood estimator – MLE - for both sites and for each component: North, East and Up. Each of these components was treated separately obtaining in the first step six values to compare. To evaluate the time series and the amplitude of the noise we used white noise $k = 0$, colored noise with integer spectral index $k = -1$ for flicker noise, $k = -2$ for random walk and colored noise with fractional

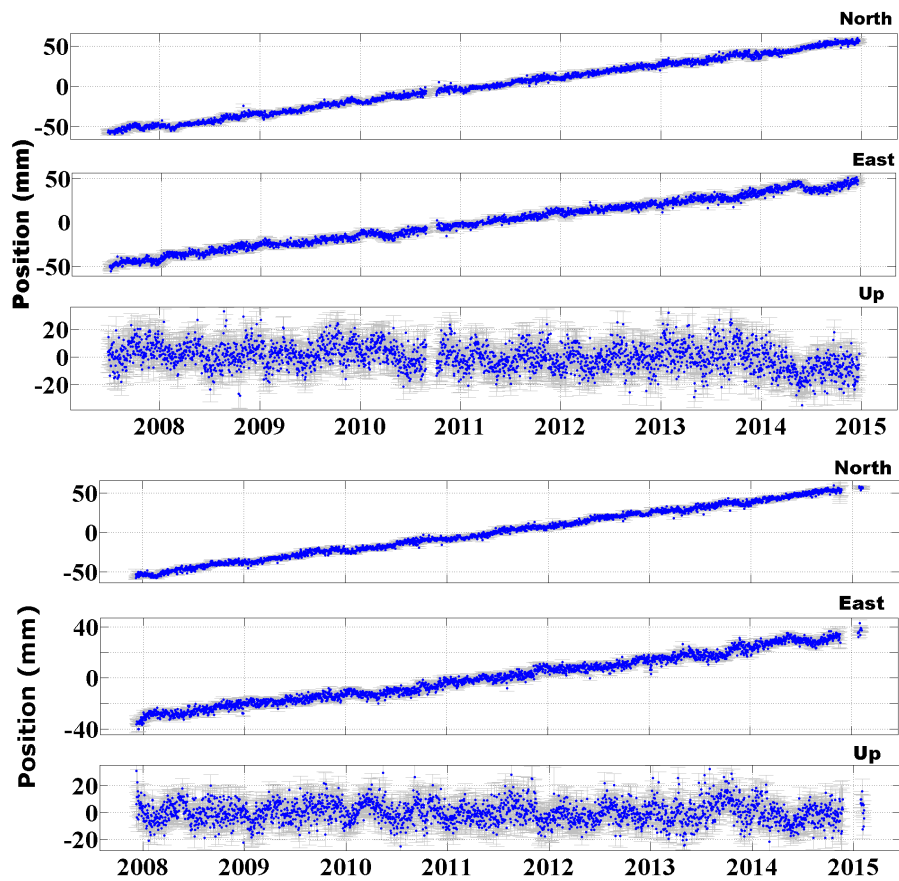


Fig. 1 The top panel presents the daily observation for station Greo and the bottom panel the daily observation for station Bggy.

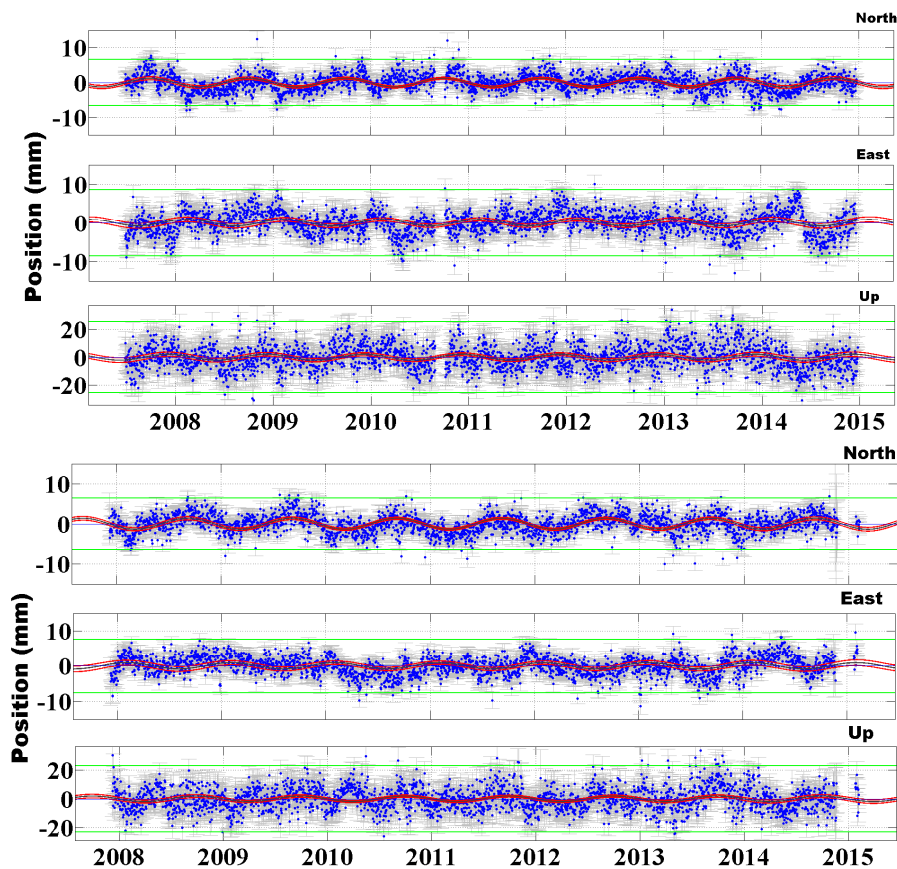


Fig. 2 Detrending daily observation taking into account seasonal variation – top panel represents the data for station Greo and the bottom panel the data for station Bggy.

spectral index. In the first part of the analysis a combination between white noise plus flicker noise was computed, white noise plus random walk and then only white noise. In this case, the spectral index was imposed – for flicker noise $k = -1$ and for random walk $k = -2$. A higher value of the log likelihood indicates the preferred noise model. In our experiment the highest value was when we used a combination of white noise plus flicker noise.

The values for log likelihood values are presented in Table 1.

Because the combination of white noise plus flicker noise is the recommended noise model in terms of log likelihood values, only the noise amplitude and the formal uncertainties of the estimated parameters are presented in Table 2.

From these values we can see that the horizontal component is less noisy for white noise amplitudes in comparison with the vertical component by a factor ~ 4 and the flicker noise amplitudes in comparison with the vertical component by a factor ~ 3 . Also, as expected the East component is noisier than the North component on all noise models except the white noise on Greo station because of the incomplete integer-cycles phase ambiguity resolution on global solutions. Also the uncertainties of the flicker noise are about 4 to 6 times larger than the uncertainties of the white noise. Prior to estimating the noise components the annual signal was removed.

In the second part of the analysis we estimated the spectral index for the power law noise plus white noise. The results of the amplitudes are presented in Table 3. The only notable difference is in the case of Bggy station for the East component for white noise amplitude.

Although we have used different types of noise model, by analyzing the data from Table 4, we can draw the conclusion that the noise model does not noticeably affect the velocity estimates. Taking into account the work done by (Zhang et al., 1997) they observed, that in the case of white plus flicker noise model, the rate uncertainties were 3-6 times larger than in the case of only with noise model. In our case the difference in rate uncertainties is almost 10 times higher. In the case of white noise model, the velocity uncertainty are inversely proportional to the total interval time and the square root of the number of measurements, but we have to take into account that the white noise model fits poorly the data and in time series we are dealing with colored noise.

The estimated spectral index in the case of Bggy station was -0.7998 , -0.7766 , -1.0948 for the North, East and Up component. For Greo station we have obtained -1.1033 , -1.2521 , -1.0335 for the North, East and Up component. Comparing the results of the noise model – white plus flicker noise and power law plus white noise in terms of log likelihood the power law plus white noise presented the higher value in 100 % although the differences between them are very small. A higher value of the log likelihood is more significant and thus giving us the most probably candidate for the best noise model. The problem is that by using the power law plus white noise we

estimate an extra parameter and thus resulting in a higher value of log likelihood. By taking the work from (Langbein, 2004) when we have to compare two noise models with different number of parameters, as in our case, a threshold of a difference of 2.6 points can be imposed. Although we take into account this recommendation, the log likelihood in the case of power law plus white presented the highest value which confirms that this model is the preferred model.

The removed seasonal amplitude of the station Greo in the North component was 1.26 mm, for the East component it was 0.79 mm and for the Up component was 2.19 mm. The phase –lag for Greo station was 1.53 rad for the North component, 0.61 rad for the East component -0.57 rad and for the Up component. In the case of Bggy station the removed seasonal amplitude in the North component was 1.35 mm, for the East component it was 0.93 mm and for the Up component 1.92 mm. The phase –lag for Bggy station was 1.16 rad, for the East component 0.69 rad and 1.55 rad for the Up component.

In the next part of the computation we have used the power spectra analysis – this represents the difference between observations minus the estimated linear trend and additional signals. This is done after fitting and removing the linear trends expected for purely tectonic motion. The results are presented in Figure 3.

In the plot the red points marked with “x” represents the computed spectrum for the observation and the solid green line represents the fitted power-law plus white noise. The frequency is given in cpy – cycles per year. From the plot it can be observed that at high frequencies the noise is flat which represents a property of the white noise and for the lower frequencies the spectrum obeys a power-law. In the case of both stations the slope of the power – law noise is around one, which indicated the presence of the flicker noise. The chosen noise model – power-law plus white noise was chosen due to the fact that by analyzing the quality of the noise model in terms of log likelihood, it presented the highest values. Although the preferred noise model was power law plus white noise, we cannot exclude first-order Gauss Markov noise model (Nistor and Buda, 2016).

DISCUSSION

By using GPS technology in geophysical application - for example in plate tectonics studies, as crustal motion, deformation – we need to introduce in the estimation process of determining the site velocity and initial position, the annual and semiannual sinusoidal signal. Because of the effect created by the presence of colored noise in time series analysis, which has significant effect on the uncertainty of rate estimation, we need to use the proper noise method and combination to estimate the noise.

There are many reasons for understanding the noise that it is contained in GPS time series which is important for geodetic and geophysical applications. One of the most important features is that by understanding the type of noise we can estimate realistic site velocity and uncertainties. In our case the

Table 1 Log likelihood values - The higher value indicates the recommended noise model.

Stations	Models											
	White			White + flicker			White +random walk			Power law +white		
	North	East	Up	North	East	Up	North	East	Up	North	East	Up
Greo	12660.748	11966.703	8988.956	13053.658	12597.207	9328.575	13028.361	12588.149	9306.945	13080.904	12612.380	9336.972
Bggy	11991.796	11664.321	8816.046	12321.876	12159.205	9043.155	12290.772	12106.093	9027.789	12374.236	12190.936	9070.840

Table 2 White noise and flicker noise amplitude estimated for the North, East,Up component and the related uncertainties

Stations	Models											
	White noise – mm						Flicker noise - mm/yr ^{1/4}					
	North	σ_w	East	σ_w	Up	σ_w	North	σ_f	East	σ_f	Up	σ_f
Greo	1.4383	0.0475	1.6234	0.0593	6.0689	0.1703	5.0895	0.2468	6.2987	0.2942	17.7528	0.9617
Bggy	1.4719	0.0493	1.2562	0.0751	5.5564	0.1542	4.8287	0.2682	6.6236	0.3095	15.9907	0.8770

Table 3 White noise and flicker noise amplitude estimated for the North, East,Up component and the related uncertainties with estimated spectral.

Stations	Models											
	White noise – mm						Flicker noise - mm/yr ^{1/4}					
	North	σ_w	East	σ_w	Up	σ_w	North	σ_f	East	σ_f	Up	σ_f
Greo	1.5159	0.0417	1.8447	0.0434	6.0689	0.1703	5.2646	0.2694	6.7051	0.3606	17.9244	0.9879
Bggy	1.2398	0.0726	0.6224	0.1980	5.5564	0.1542	4.5618	0.2255	6.0678	0.2470	16.3994	0.9414

Table 4 Velocity and their uncertainties with integer spectral index.

Models						
Stations	White noise only					
	North mm/year	Uncertainties mm/year	East mm/year	Uncertainties mm/year	Up mm/year	Uncertainties mm/year
Greo	15.5108	0.0233	12.6824	0.0284	-0.9651	0.0851
Bggy	15.4326	0.0216	9.1461	0.0253	-0.2338	0.0764
Stations	White + Flicker noise					
	North mm/year	Uncertainties mm/year	East mm/year	Uncertainties mm/year	Up mm/year	Uncertainties mm/year
Greo	15.3284	0.2290	12.6542	0.2829	-1.1192	0.8030
Bggy	15.4973	0.2177	9.5247	0.2959	-0.7453	0.7242

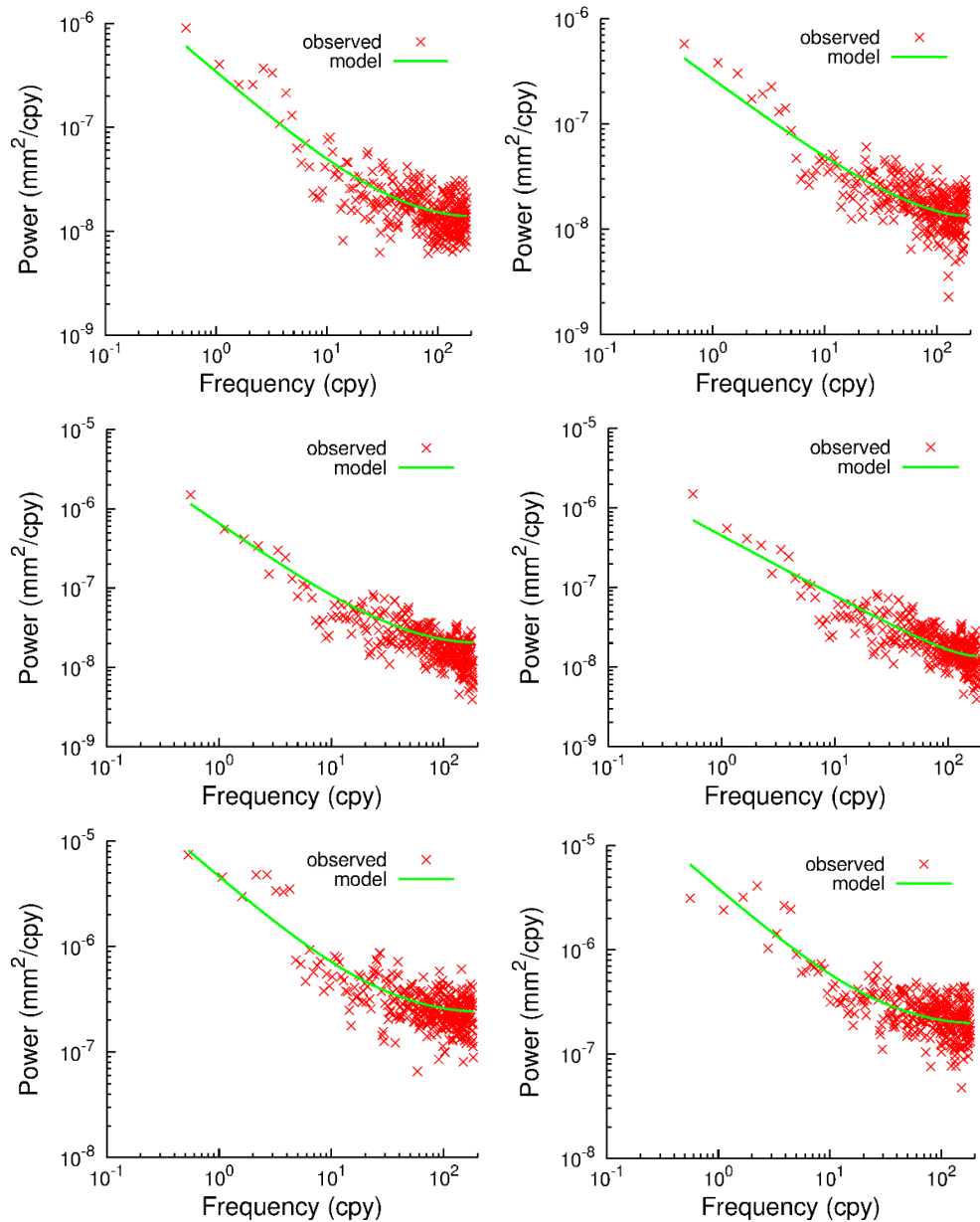


Fig. 3 On the left side it is the PSD – power spectra density - for North, East and Up component for station Greo; on the right side it is the PSD for North, East and Up component for station Bggy.

recommended combination for determining the noise, is power law plus white noise. In the case of a white noise model the total velocity was underestimated by a factor of 10. The identification of what type of noise

is present in the GPS time series helps us to locate or understand the sources of noise and thus indicating how to reduce or to eliminate their effects in the future. Also time series analysis for identifying the

noise can be done on points determined by using very long baseline interferometry - VLBI - (Nistor and Buda, 2015b) or precise point positioning (Nistor and Buda, 2015a). Because the four major space geodetic techniques are combined to obtain an accurate terrestrial reference system, there is the need to understand the noise and their possible biases.

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