DETERMINATION OF THE HORIZONTAL DEFORMATION FACTOR FOR MINERAL AND FLUIDIZED DEPOSITS EXPLOITATION

Anton SROKA*, Rafał MISA and Krzysztof TAJDUŚ

Strata Mechanics Research Institute of the Polish Academy of Sciences, ul. Reymonta 27, 30-059, Kraków, Poland

*Corresponding author’s e-mail: sroka@img-pan.krakow.pl

ABSTRACT
In this article, the authors use the existing theoretical foundations and partial solutions to provide a general formula, which can be used to determine the value of horizontal deformation factor B for various applications of the Knothe theory. This applies to the calculation of surface deformation not only in the case of hard coal mining, but also in the case of salt mining by borehole. Convergence resulted from mining may cause deformations of caverns used to store liquid and gaseous fuels or the deformations related to the fluidized bed extraction. The calculations for the new analytical model are presented in the examples of the Etzel cavern (Germany) and the Upper Silesian Coal Basin (Poland).

INTRODUCTION
Avershin (1948) published the relationship between horizontal displacement and tilt, known in the literature as the Avershin’s hypothesis. It assumes a linear proportionality between the horizontal displacement vector and the subsidence trough tilt vector. The analysis of the results of horizontal deformations and movement of points due to the convergence of salt caverns used for storing gaseous and liquid fuels together with the analysis of the results of measurements made during the extraction of fluid deposits show clearly that the value of the Avershin’s proportionality index, known in the literature as the horizontal deformation factor B, for the deformations over the cavern fields in a salt rock mass or surface deformations resulting from the extraction of fluid deposits are vastly different from the standard values of this index as provided by Budryk (1953). It was then determined based on measurements of deformations caused solely by the hard coal mining operations.

The solution of the proper determination of the horizontal deformation factor B is an extremely important aspect in the realistic estimation of the expected deformation, in particular of the horizontal deformation, as the most accurately describing the expected damage (Knothe, 1953b; Budryk and Knothe, 1956). This is a necessary action, especially in the increasingly difficult general situation of mining in Europe, where this industry has to deal with increasingly frequent restrictions and formal and political demands.

THEORETICAL FOUNDATIONS
Based on the solutions provided by Sałustowicz (1953) and Litwiniszyn (1953), Budryk (1953) proved that the maximum value for horizontal deformation \( \varepsilon_{\text{max}} \) for the flat issue (so-called: infinite semi-plane) can be described with the following formula:

\[
\varepsilon_{\text{max}} = c \cdot T_{\text{max}},
\]

where:

- \( T_{\text{max}} \) - maximum tilt of the subsidence trough
- \( c \) - constant.

Based on the analysis of measurement data for coal seams with the tilt up to 35°, Budryk calculated the \( c \) parameter value within the limit of 0.52 ÷ 0.60.

In his magnum opus describing the concepts for calculating horizontal deformations and movements, Budryk (1953) writes that: ‘in actuality, the largest horizontal deformations usually do not exceed:

\[
\varepsilon_{\text{max}} = 0.60 \cdot T_{\text{max}}.
\]

This shows that the solution provided by Budryk is applicable to the ultimate values of \( \varepsilon_{\text{max}} \) and therefore should be treated as the worst case scenario.

After accepting the Averhin’s assumption and Knothe’s theory formulas, Budryk uses the determination of (2) to specify the formula for the standard value of the B factor as follows:

\[
B_r = \frac{R}{2\pi} = 0.40 \cdot R
\]

where:

- \( R \) - main influences range radius acc. to Knothe’s definition (1953a).

The value of the influence range radius \( R \) is determined by following formula (4):

\[
R = H \cdot \cot \beta
\]

where:
$H$ - depth of mining operation, 
$\beta$ - angle of main influences acc. to Knothe's definition.

According to Budryk, the standard value of the $B$ factor corresponds with the range of maximum influence on the surface.

Both Knothe and Litwiniszyn in their theories assumed (following Avershin) that between the $U$ horizontal displacement vector and the $T$ tilt vector, there is proportionality:

$$U = -B \cdot T.$$  

Formula (5) shows that the relation between the horizontal deformation tensor $\varepsilon$ and the vertical curvature tensor $K$ is as follows:

$$\varepsilon = -B \cdot K.$$  \hspace{1cm} (6)

The most important indicator of deformation, from the point of view of threat to civil structures, is horizontal deformation (i.a. Knothe, 1953b; Tajdus and Tajdus, 2015; Sroka et al., 2015; Malinowska and Hejmanowski, 2016).

For today’s conditions of mining operation and various application of the Knothe’s theory, formula (6) is more general than (1).

Mining operations at very significant depths cause the mined out fields to be vastly geometrically different from the infinite semi-plane (i.a. Blachowski and Ellefmo, 2012). This prevents the application of formula (1) to forecasting calculations.

The results of the analyses of subsidence and horizontal displacement measurements for the points on the surface above the fields of cavern in a salt rock-mass used to store liquid and gaseous fuels and for the fluidized bed extraction demonstrate that the value of the horizontal deformation factor $B$ in these cases is significantly higher than the standard value provided by Budryk (1953). For instance, the analysis of vertical and horizontal displacements over the Etzel caverns made by Quasnitza (1988) in his doctoral dissertation shows that the value of this factor resulting from identification is:

$$B = 1.0 \cdot R \pm 0.1 \cdot R.$$  \hspace{1cm} (7)

This proves that the most probable value of the $B$ factor in this case is equal to the main effects range radius acc. to the Knothe’s definition – i.e. it is some 2.5 times higher than the standard value provided by Budryk.

A comprehensive description of the formation of the $B$ factor value studies to date can be found in the works of (Knothe and Sroka, 2010) as well as (Tajdus and Tajdus, 2015) and (Sroka et al., 2015). They are mostly applicable to the dependencies described by the following formulas:

$$B = k \cdot H,$$  \hspace{1cm} (8)

$$B = \lambda \cdot R.$$  \hspace{1cm} (9)

It should be noted that formula (9) is preferred in the literature. For hard coal and copper ore mining, the forecast values of $\lambda$ are lower than the standard value provided by Budryk ($\lambda = 0.4$) with an average of 0.32 (Popiolek and Ostrowski, 1978).

**SOLVED ISSUES**

The studies of Dżegniuk (1970) and Sroka (1973) show that the sum of the values of horizontal deformations of the main components of the deformation tensor is 5-10 times larger than the value of vertical deformation.

$$\varepsilon_{xx} + \varepsilon_{yy} + \mu \cdot \varepsilon_{zz} = 0$$  \hspace{1cm} (10)

where:

$\varepsilon_{xx}$, $\varepsilon_{yy}$ - deformation in the horizontal plane,

$\varepsilon_{zz}$ - deformation in the vertical plane,

$5.0 < \mu < 10.0$.

According to Sroka (1973), the value of $\mu$ for the surface horizon, when assuming the soil medium defined as a linear-elastic medium, amounts to:

$$\mu = \frac{1 - \nu}{\nu}$$  \hspace{1cm} (11)

where:

$\nu$ - Poisson’s ratio.

According to Förster (1996) for the small and slowly increasing deformations of the soil medium, the value of the Poisson’s ratio is within the range of $0.10 \pm 0.20$ while in the case of secondary deformations with high values and large increase rate – from 0.20 to 0.40.

The comparison of the aforementioned deformation values shows that the deformations caused by mining operations should be categorised as small. The observations of Dżegniuk (1970) and Sroka (1973) regarding the $\mu$ factor value we can infer that the corresponding range of values of the Poisson’s ratio $\nu$ is:

$$0.09 < \nu < 0.17$$

and corresponds rather well with the scope provided by Förster.

According to Drężała (1989), for the rock mass with the $\tan \beta$ parameter value from 1.5 to 2.5, the value of the Poisson’s ratio is within the range from 0.096 to 0.150. This range is fully compliant with the findings of Dżegniuk (1970) and Sroka (1973).

By analysing the current condition of the Budryk-Knothe theory, Pielok and Sroka (1980/81), under the assumption of a standard horizontal deformation factor value, obtained the following dependency for the surface horizon:

$$\mu = \frac{\sqrt{2 \pi}}{n} \cdot \tan \beta$$  \hspace{1cm} (12)

where:

$n$ - so-called influence area factor in the rock mass.

The $n$ factor is a parameter of an equation describing the changes in the value of the main mining operation influence range radius in the rock mass (formula 13).

$$R(z) = R \left( \frac{z}{H} \right)^{n}$$  \hspace{1cm} (13)

where:
The values of the \( n \) parameter published by various authors are provided in Table 1.

It should be noted that the values of this parameter determined by in situ measurements are practically within the limits:

\[ 0.47 < n < 0.55. \]

This is applicable to the studies of: Sroka and Bartosik-Sroka (1974), Drzęgli (1979, 1989) as well as Preusse (1990), who made the analysis of vertical movements in the rock mass for the mines in the Ruhr Valley.

By transforming formula (12), for any value of the \( B \) factor, we receive:

\[
\mu = \frac{\sqrt{2\pi}}{n} \cdot \tan \beta \cdot \frac{B}{B_n}. \tag{14}
\]

For the assumption \( B = \lambda \cdot R \) and by assuming an equality between formulas (11) and (14), we receive:

\[
\frac{1 - \nu}{\nu} = \frac{2\pi}{n} \cdot \frac{\tan \beta \cdot \lambda}{\lambda} \tag{15}
\]

and, ultimately:

\[
\lambda = \frac{n}{2\pi} \cdot \frac{1 - \nu}{\nu} \cdot \cot \beta. \tag{16}
\]

The quality of formula (16) is also confirmed by previous partial results included in the studies by Drzęgli (e.g. 1975, 1978, 1979, 1989). Drzęgli (1975) provided approximate solutions of the elasticity theory as applied to the calculations of the area surface and rock mass deformation factors. In this study, he assumed that the vertical movement formula is in line with the solution provided by Knothe.

The result shows that the value of the \( n \) factor describing the changes in the range of main influences in the rocks mass is constant and equal to \( n = 0.665 \).

Drzęgli (1989) asserts that the ratio \( S \) between the value of horizontal deformations/movement calculated in accordance with his solution and the values calculated according to the Budryk-Knothe theory is:

\[
S = \frac{0.265}{\tan \beta} \cdot \frac{1 - \nu}{\nu}, \tag{17}
\]

which leads to the value of \( \lambda \) described with formula (18).

\[
\lambda = \frac{0.665}{2\pi} \cdot \frac{1 - \nu}{\nu} \cdot \cot \beta. \tag{18}
\]

Comparing formula (16) to formula (18), it should be noted that they are identical in terms of quality. The difference between them is only that the formula based on Drzęgli’s assertions is based on a constant value of \( n = 0.665 \).

**Table 1** Value of the \( n \) parameter according to different authors (Dzegniuk et al., 2003; Sroka et al., 2005).

<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budryk</td>
<td>1953</td>
<td>( n = \sqrt{2\pi} \cdot \tan \beta )</td>
</tr>
<tr>
<td>Mohr</td>
<td>1958</td>
<td>( n = 0.65 )</td>
</tr>
<tr>
<td>Krzysztoń</td>
<td>1965</td>
<td>( n = 1.0 )</td>
</tr>
<tr>
<td>Drzęgli</td>
<td>1972</td>
<td>( n = 0.525 )</td>
</tr>
<tr>
<td>Sroka, Bartosik-Sroka</td>
<td>1974</td>
<td>( n = 0.50 )</td>
</tr>
<tr>
<td>Drzęgli</td>
<td>1975</td>
<td>( n = 0.665 )</td>
</tr>
<tr>
<td>Gromysz</td>
<td>1977</td>
<td>( n = 0.61 )</td>
</tr>
<tr>
<td>Drzęgli</td>
<td>1979</td>
<td>( 0.47 \leq n \leq 0.49 )</td>
</tr>
<tr>
<td>Kowalski</td>
<td>1984</td>
<td>( 0.48 \leq n \leq 0.66 )</td>
</tr>
<tr>
<td>Zych</td>
<td>1985</td>
<td>( n = 0.55 )</td>
</tr>
<tr>
<td>Drzęgli</td>
<td>1989</td>
<td>( 0.50 \leq n \leq 0.70 )</td>
</tr>
<tr>
<td>Preusse</td>
<td>1990</td>
<td>( n = 0.54 )</td>
</tr>
</tbody>
</table>

**Verification of the Analytical Model on the Basis of Hitherto Experience**

These considerations show that the value of \( \lambda \) factor depends on the following:

- average value of the Poisson’s ratio \( \nu \) for the near-surface layer of soil,
- main mining influence area factor in the rock mass \( n \), and
- angle of main influences \( \beta \).

It seems obvious that the use of \( \nu \) and \( n \) will permit better estimation of \( \lambda \) factor. On the other hand, however, it enables the calculation of \( \lambda \) values, which are close to reality.

The obtained equation in the form of the general formula for determining the value of parameter \( B \) (formulas 9 and 16) was confronted with the results of the identification of this parameter value based on in situ measurements for two cases. The first concerns surface deformation in case of Etzel cavern and the second deformation will be by the exploitation of hard coal deposits in the Upper Silesian Basin.

For the characteristic data for the Etzel caverns field:

- Poisson’s ratio: \( \nu = 0.1 \),
- main mining influence area factor in the rock mass: \( n = 0.5 \),
- angle of main influences: \( \beta = 34.8^\circ \),

when using formula (16), we receive the \( \lambda \) factor value of 1.03.

This result is consistent with the calculations made by Quasnieta (1988) and more recently by Sodmann (verbal information 2016 – \( \lambda = 1.06 \)).

When assuming the following values for the Upper Silesian Coal Basin:

- \( \nu = 0.11 \),
- \( n = 0.5 \),
- \( \tan \beta = 2.0 \),

the resulting value of \( \lambda = 0.32 \) corresponds well with the observed actual conditions (i.a. Popiolek and Ostrowski, 1978; Knothe and Sroka, 2010).
CONCLUSION

The considerations of this article enable us to provide a general formula for determining the value of the horizontal deformation factor \( B \) for various applications of the Knothe’s theory. This applies to the calculations of surface deformations not only in the case of hard coal deposits mining, but also for borehole salt mining, deformations caused by the convergence of caverns used to store liquid and gaseous fuels or the deformations related to the fluidized bed extraction. According to formula (16) formulated based on the Knothe’s theory, the value of the \( \lambda \) factor (for \( B = \lambda/R \)) depends on the following values:

- angle of main influences \( \beta \),
- Poisson’s ratio for the near-surface layer of soil \( \nu \),
- main influence area factor in the rock mass \( n \).

REFERENCES


Sroka, A., Knothe, S., Tajduš, K. and Misa, R.: 2015, Point movement trace vs. the range of mining exploitation effects in the rock mass. Archives of Mining Sciences, 60, 4, 921–929. DOI: 10.1515/amsc-2015-0060
Tajduš, K. and Tajduš, A.: 2015, Some considerations on horizontal displacement and horizontal displacement coefficient B. Studia Geotechnica et Mechanica, 37, 4, 75–82. DOI: 10.1515/sgem-2015-0047