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ORIGINAL PAPER

LOCAL DISTURBING POTENTIAL MODEL WITH THE USE OF GEOPHYSICAL GRAVITY DATA INVERSION - CASE STUDY IN THE AREA OF POLAND

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ABSTRACT

The presented paper investigates the local quasigeoid modelling based on the geophysical gravity data inversion (GGI method). The calculations performed indicate a very high accuracy of the derived quasigeoid model which is developed when the modelling process is carried out using a global geopotential model. In this case, the local model of disturbing potential (model of type A) is in the form of $T_{GGI} = T_{GM} + T_{dens} + T_{pol}$, where T_{dens} is the part of the model containing the density model, T_{pol} is the polynomial part and T_{GM} is the part determined from a global model. If the global model is not included in the calculations (this is a type B model), the disturbing potential will be $T_{GGI} = T_{dens} + T_{pol}$. The accuracy of the quasigeoid in latter model is significantly worse. Both types of models contain information about Earth's crust density changes; however, only for the type B model we can expect that they correspond to real changes; whereas in the type A model the main information about density changes is contained in the T_{GM} part. So the type B model can be seen as a local, integrated model of both the external gravity field and the density distribution of the Earth's crust. Let us note that the GGI method is only used as a quasigeoid modelling method without geophysical or geological interpretation of the density model parameters. Regarding this, we can notice two problems related to the further development of the GGI method. The first concerns the relation of the GGI density model with geological information. The second problem refers to the possibility of increasing the accuracy of the type B quasigeoid model to the level of the A-type model. For the first problem, we demonstrate that the borders of density changes for the masses lying between the geoid and the Moho surface reflect the geological units surprisingly well. The test calculations relating to second problem, consist of determining the type B model using a certain initial model T_{GGI_0}. There were three initial models T_{GGI_0} adopted in the analysis. Results of the analysis indicates that this procedure increases the accuracy of the type B quasigeoid model, although the accuracy of the A-type model is still better.

INTRODUCTION

The models of the Earth's crust density distribution in physical geodesy are usually used in the regional geoid modelling process. More significant are the regional models of the density of topographic masses, which directly affect the accuracy of geoid or orthometric heights determination (Martinec, 1993; Martinec and Vaníček, 1994; Huang et al., 2001; Kingdon et al., 2009). Density variations of the lower layers of the Earth's crust usually in these issues are represented by gravity data used in calculations.

A well-known approach to the geoid and quasigeoid modelling, using the dependence of the disturbing potential and the Earth's crust density distribution is the method of point masses (Barthelmes and Kautzleben, 1983; Barthelmes and Dietrich, 1991). As a result of the modelling process, we can obtain not only an external model of the disturbing potential but also, as an additional result of the calculations, information on the density distribution of the lower layers of the Earth's crust (Claessens, 2001). As a version of this approach it can be considered, analysed in this paper the GGI method (Geophysical Gravity data Inversion).

In the GGI method, based on a sparse network of points with known GNSS/levelling height anomalies and a dense network of gravity points, a local model of disturbing potential, consisting of three components is built (Trojanowicz, 2012a; Trojanowicz, 2012b).

The first component is the potential T_{Ω} produced by topographic masses, which lies above the geoid, included in volume Ω, with density distribution function ρ. The second component is the potential T_{κ}, produced by disturbance masses, occurring

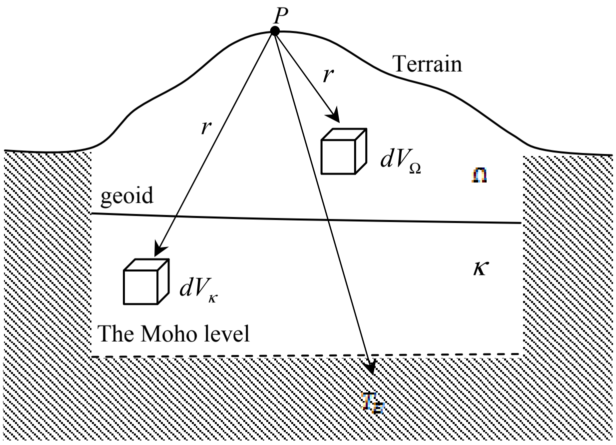

Fig. 1 Components of the disturbing potential model.

between the geoid surface and the compensation level, included in the volume κ, with density distribution function δ. Because volumes Ω and κ are horizontally limited to the area of elaboration, and the data used for calculations contains information about density distribution outside these volumes, the potential T_E (external disturbing potential) is introduced. The role of this potential is to model the mentioned "unwanted" part of the data and to cover long-wavelength errors of gravity data and systematic errors of both levelling and satellite data. It was assumed that the distorting effects mentioned could be modelled using harmonic polynomials of a low degree. So, we can write:

$$T_P = T_\Omega + T_\kappa + T_E \tag{1}$$

Components T_Ω and T_κ are given by Newton's integrals:

$$T_\Omega = G\iiint_\Omega \frac{\rho}{r} dV_\Omega \tag{2}$$

$$T_\kappa = G\iiint_\kappa \frac{\delta}{r} dV_\kappa \tag{3}$$

where r is the distance between the attracting masses and the attracted point P, G is the Newton's gravitational constant and dV_Ω and dV_κ are elements of volume.

In calculations we use component T_E in the form:

$$T_E = a_1 + a_2 X_P + a_3 Y_P + a_4 X_P Y_P + a_5 Z_P \tag{4}$$

Now, we can formulate an inversion task as follows: find the density distribution functions ρ and δ in defined volumes Ω and κ, and the coefficients of the polynomials modelling the potential T_E to satisfy equation (1) for given data. The solution of the task, using linear inversion (Blakely, 1995), requires discretization of the continuous 3D functions ρ and δ. So, the volumes Ω and κ are divided into finite volume blocks and a constant density is assigned to each of the blocks. In studies we use the volume Ω defined by digital elevation model (DEM) in the form of rectangular blocks, grouped into zones of constant, searched density. The κ volume is defined as a slab with thickness nearly the same as the depth of the compensation level and consists of one or many layers of constant density blocks. In previous and current studies a one-layer version was used. In the horizontal plane the volumes Ω and κ exceed the border of data occurrence.

The discrete forms of density distribution functions ρ, δ and the coefficients of the polynomial that defines potential T_E are determined by the least squares method, usually based on surface gravity data and points with measured GNSS/levelling height anomalies.

Calculations are performed in the local Cartesian coordinate system. The Z-axis of the coordinate system is directed towards the geodetic Zenith at the origin point. The X and Y axes lie on the plane of the horizon and are directed toward the North and East, respectively. The definition of the coordinate system enables the determination of the Ω and κ volumes in the form of rectangular prisms for which the solutions of Newton's integrals are presented by Nagy (1966) and Nagy et al. (2001).

Given the above, the equations (2) and (3) can be written as follows:

$$T_\Omega = \sum_{k=1}^{n} \left(\rho_k G \sum_{i=1}^{m_\kappa} \int_{z_{i1}}^{z_{i2}} \int_{y_{i1}}^{y_{i2}} \int_{x_{i1}}^{x_{i2}} \frac{1}{d_i} dx_i\, dy_i\, dz_i \right) \tag{5}$$

$$T_\kappa = \sum_{j=1}^{s} \left(\delta_j G \sum_{i=1}^{m_\kappa} \int_{z_{j1}}^{z_{j2}} \int_{y_{j1}}^{y_{j2}} \int_{x_{j1}}^{x_{j2}} \frac{1}{d_i} dx_j\, dy_j\, dz_j \right) \tag{6}$$

where: ρ_k – the searched constant density of zone k; n – the number of DTM zones; m_k – the number of rectangular prisms of DTM in zone k; $x_{i1}, x_{i2}, y_{i1}, y_{i2}, z_{i1}, z_{i2}$ – the coordinates defining the rectangular prism i of DTM; $d_i = \sqrt{(x_i - X_P)^2 + (y_i - Y_P)^2 + (z_i - Z_P)^2}$; s – the number of rectangular prisms of the κ volume; δ_j – the searched density of rectangular prism j, $x_{j1}, x_{j2}, y_{j1}, y_{j2}, z_{j1}, z_{j2}$ – the coordinates defining rectangular prism j and $d_j = \sqrt{(x_j - X_P)^2 + (y_j - Y_P)^2 + (z_j - Z_P)^2}$.

Denoting in equation (1) $T_\Omega + T_\kappa = T_{dens}$ and $T_E = T_{pol}$ we can write the potential T_P as a sum of density (T_{dens}) and polynomial (T_{pol}) parts.

$$T_P = T_{dens} + T_{pol} \tag{7}$$

The calculations can be performed with the use of a global geopotential model. In this case the remove-compute-restore procedure is used. So, the disturbing potential (5) can be presented as a sum of the global component T_{GM} and residual potential, containing both mentioned parts: the density model (T_{dens}) and polynomial part (T_{pol}).

So we can distinguish two types of GGI models:

The model of type A – with the T_{GM} part:

$$T_{GGI} = T_{GM} + T_{dens} + T_{pol} \tag{8}$$

and the model of type B – without the T_{GM} part:

$$T_{GGI} = T_{dens} + T_{pol} \tag{9}$$

The described approach was proposed with a view to quasigeoid modelling, and more specifically to the modelling of the external gravity field parameters (e.g. disturbing potential (height anomaly) or gravity disturbances (gravity)). However, the GGI model also has components containing information on density distribution, so it can act as an integrated (geodetic and geophysical) model, allowing us to determine the mentioned parameters of the external gravity field and an indication of the Earth's density distribution corresponding to these parameters. Theoretically, it could also allow for the use in the modelling process other geophysical and geological density information like density distribution model of topography (Sheng et al*.,* 2019) or of the Earth's crust (Molinari and Morelli, 2011; Kaban et al., 2010).

Considering the GGI method, it should be noted that while the density distribution of topographic masses has previously been initially analysed in Trojanowicz (2007), the values of the Earth's crust density distribution under the geoid surface, determined from the GGI model, have not been analysed so far. They were regarded as parameters of the model without geophysical or geological interpretation, and the GGI method was used only as a quasigeoid modelling method. Because the accuracy of quasigeoid models calculated using this approach is very high, at least at the level of classical approaches (Trojanowicz, 2015b; Trojanowicz, et al., 2018), it seems to be important to analyse the capabilities of this approach in terms of the Earth’s crust density estimation or inclusion such an information for the modelling process.

Taking this into account, it should be indicated that defined by equations 8 and 9 types of models contain information on the density distribution of the Earth's crust. However, in the model of type A the main information about density is included in the T_{GM} part and depending on the resolution of the global model, density changes will be more or less visible. So, direct obtaining the actual Earth’s crust density from this type of model is complicated or impossible, and inclusion in the modelling process of other geophysical and geological density information is significantly more difficult (each time we should manage with the global model part). The model of type B seems to be better suited for both: determining the actual densities of the Earth's crust and the use in modelling process of other geophysical and geological density information. On the other hand, the calculations performed so far have indicated a very high accuracy of the quasigeoid model developed employing this method when the modelling process is carried out using the type A model and significantly lower accuracy of the quasigeoid model in case of use the model of type B (Trojanowicz, 2012a; Trojanowicz, 2015a; Trojanowicz, 2015b; Trojanowicz et al., 2018). Regarding this we can notice that, there are two problems related to the further development of the GGI method. The first concerns the relation of the GGI density model with geological information. The second problem refers to the possibility of increasing the accuracy of the type B quasigeoid model to the level of the A-type model. Solutions of these problems will indicate the direction of further development of this approach as an integrated approach.

THE USE OF THE GGI METHOD FOR THE AREA OF POLAND

Calculations used to indicate differences between the two types of models were carried out in the area of Poland, using 241 points with known GNSS/levelling height anomalies the same as used for tests by Trojanowicz (2015b) (Fig. 2). These points were divided into two data sets. 111 points were taken as known points (tringles), used to build the model and 130 points as test points, which will be used for assessment of the accuracy of the model (big dots). In the calculations we also used two sets of gravity points, every set in the number of 16665 (approximate density of the points - 1 point for 19 km^2). One of the sets was taken as known points, used to build the model, and the other as test points, used to assess the accuracy of the model.

Volume Ω was defined by DEM with resolution of $1 \times 1\ \text{km}^2$. Volume κ was defined based on the Moho depth model for European plate (Grad et al., 2009). The range of both volumes in the horizontal plane is the limit of Figure 2. It is worth noting that the terrain at the test area is not very varied, Poland is a lowland country. Only in the south are there uplands and mountains areas the Carpathians and the Sudety mountains. The deeper layers of the crust are characterised by greater diversity. The Moho depth varies from about 30 km in the south-west to over 50 km in the north-east. From the north-west to the south-east, the TT zone - the intersection between the Eastern European (Precambrian) Platform and Palaeozoic Platform-runs. Volumes Ω and κ were divided into constant density zones with dimensions

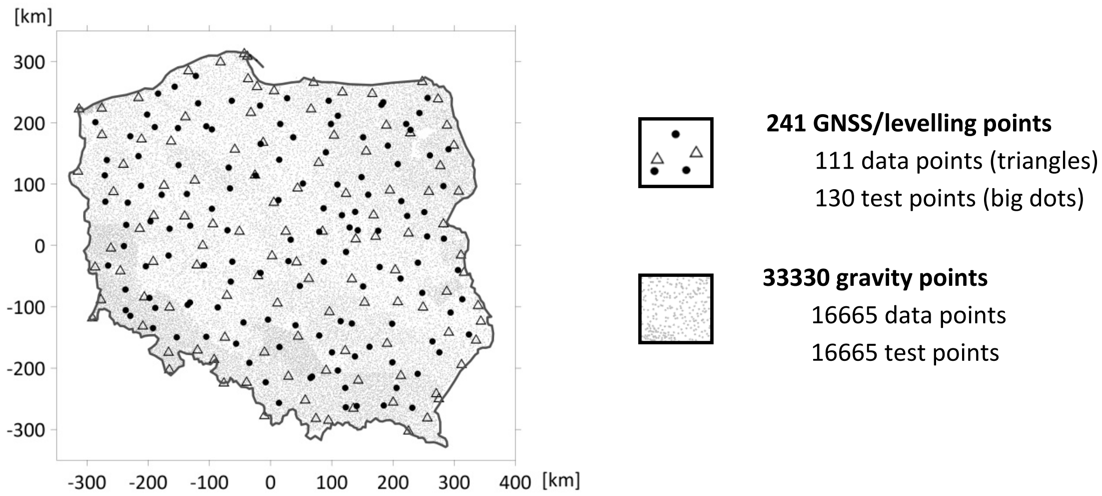

Fig. 2 Location of data and test points used in calculations. Gravity test and data points are marked in the same way due to their large number. Both groups of points cover the whole area evenly.

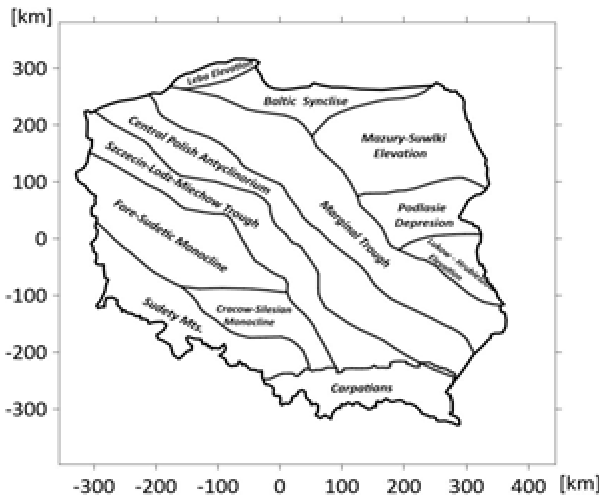

Fig. 3 Main geological units of Poland. Sketch based on Pożaryski (1963).

of 10 × 10 km^2 (5250 for volume Ω and 5250 for volume κ).

For visual validation of the compatibility of the determined density distribution of the κ volume with the geology of the analysed area, we use the sketch made based on the map of geological units of Poland (Pożaryski, 1963) (Fig. 3).

We performed calculations for both types of models. For the A-type model we used two versions of the initial density model $\rho_0 = 0\,\frac{\text{kg}}{\text{m}^3}$ and $\rho_0 = 2200\,\frac{\text{kg}}{\text{m}^3}$ (value close to the average density of topographic masses for the area of Poland (Królikowski and Polechońska, 2005)). For the B-type model only $\rho_0 = 2200\,\frac{\text{kg}}{\text{m}^3}$ as initial density model was adopted. The models were built on the base of above-mentioned GNSS/levelling and gravity data points.

Table 1 Basic statistics of the differences in $\Delta\zeta = \zeta_{GNSS/lev} - \zeta_{GGI}$.

MODEL TYPE	ρ_0 $\left[{}^{\text{kg}}\!/_{\text{m}^3}\right]$	GLOBAL MODEL	$min(\Delta\zeta)$ [cm]	$max(\Delta\zeta)$ [cm]	$mean(\Delta\zeta)$ [cm]	$stdev(\Delta\zeta)$ [cm]
A	0	EGM08	-4.5	2.4	0.0	1.28
A	2,200	EGM08	-3.7	3.0	0.1	1.31
B	2,200	-	-3.7	3.3	-0.1	1.51

Table 2 Basic statistics of the differences in. $\Delta\delta g = \delta g - \delta g_{GGI}$

MODEL TYPE	ρ_0 $\left[{}^{\text{kg}}\!/_{\text{m}^3}\right]$	GLOBAL MODEL	$min(\Delta\delta g)$ [mGal]	$max(\Delta\delta g)$ [mGal]	$mean(\Delta\delta g)$ [mGal]	$stdev(\Delta\delta g)$ [mGal]
A	0	EGM08	-21.2	35.5	0.0	2.45
A	2,200	EGM08	-13.2	14.4	0.0	1.67
B	2,200	-	-16.1	13.8	0.0	1.56

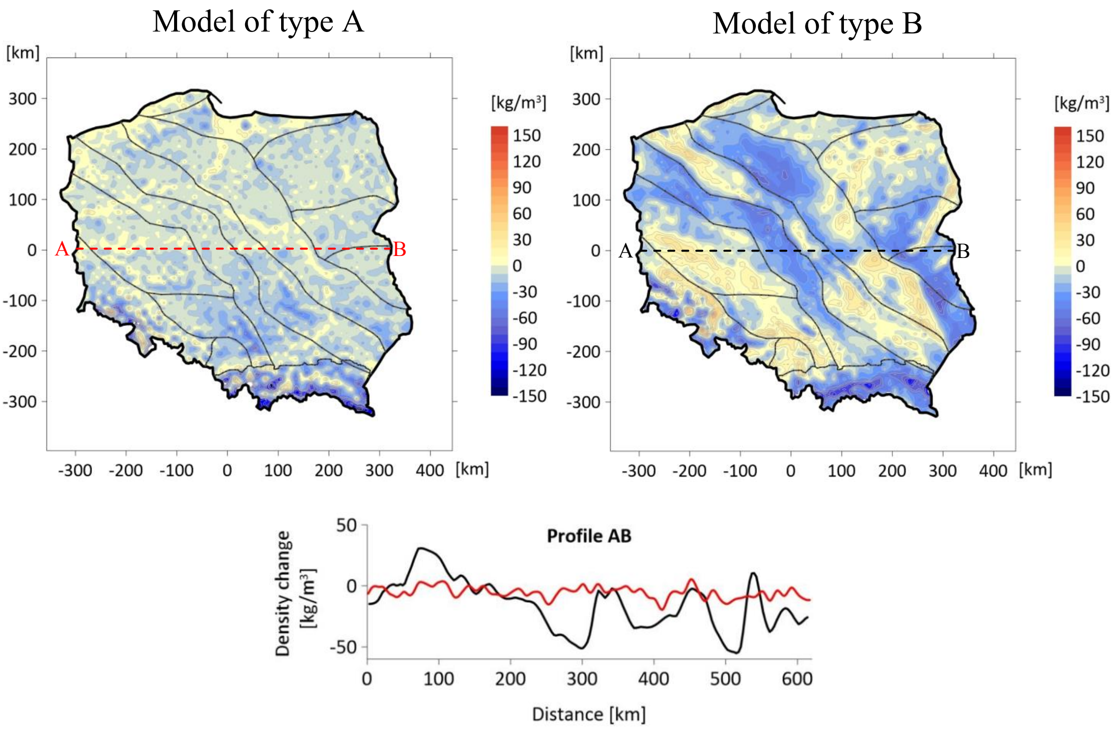

Fig. 4 The maps of density changes of the κ volume with the contours of the main geological units (at the top). Exemplary density profiles (at the bottom).

For each test point there were calculated differences in $\Delta\zeta = \zeta_{GNSS/lev} - \zeta_{GGI}$ and $\Delta\delta g = \delta g - \delta g_{GGI}$, where $\zeta_{GNSS/lev}$ and δg are determined on the basis of measured values while ζ_{GGI} and δg_{GGI} are values calculated from the model. In the Tables 1 and 2 there are basic statistics of these differences for individual model versions. For the A-type model we use EGM08 up to degree 2190 and order 2159 (Pavlis et al., 2012).

If we consider the accuracy of the quasigeoid model, the results are much better for both versions of the type A models. The model of type B gives significantly better results for gravity. As we can see, it is rather related to the use of initial density of topography $\rho_0 = 2200\,\frac{\mathrm{kg}}{\mathrm{m}^3}$, because for this version, in the A-type model, the accuracy of gravity is also better.

Additionally, the contours of geological units shown in Figure 3 were compared with the maps of density changes, prepared on the basis of the density models of the κ volume. The results are presented in Figure 4.

For the type B model, we can observe that the determined borders of density changes for the masses lying between the geoid and the Moho surface reflect the geological units – not all but main surprisingly well. For the model of type A, the main information about density change is included in the global model part, so therefore we do not see the correlation of the determined densities and boundaries of the main geological units. Both types of models allow for preparation of density profiles. They generally indicate small changes in the density in the range of about 100 $\frac{\mathrm{kg}}{\mathrm{m}^3}$ for type B model. For the type A model, these changes are within a few $\frac{\mathrm{kg}}{\mathrm{m}^3}$.

IMPROVEMENT OF THE TYPE B QUASIGEOID MODEL

One of the main goals of this paper is to analyse the possibility of building a type B model that allows the determining of a quasigeoid model with accuracy similar to the model of type A. The presented solution of the problem consisted of determining the type B model using a certain initial model. In the analysed version, the type B model can be written as:

$$T_{GGI} = T_{GGI_0} + d\,T_{GGI} \tag{8}$$

where T_{GGI_0} is the approximate, initial model, while $d\,T_{GGI}$ is the residual model - both mentioned parts have two components in the form of $T_{dens} + T_{pol}$:

$$T_{GGI} = T_{dens_0} + T_{pol_0} + d\,T_{dens} + d\,T_{pol} \tag{9}$$

There were three initial T_{GGI_0} models adopted in the analyses:

1. According to the current practice (also used in the above example), we assumed constant density of topography. For zone i of constant density of volume Ω we accepted $\rho_0^i = 2{,}200\,\frac{\mathrm{kg}}{\mathrm{m}^3}$. For

Table 3 Basic statistics of the differences. $\Delta\zeta = \zeta_{GNSS/lev} - \zeta_{GGI}$

VERSION	ρ_0 $\left[{}^{kg}/_{m^3}\right]$	δ_0 $\left[{}^{kg}/_{m^3}\right]$	$min(\Delta\zeta)$ [cm]	$max(\Delta\zeta)$ [cm]	$mean(\Delta\zeta)$ [cm]	$stdev(\Delta\zeta)$ [cm]
1	2,200	$\delta_0(\rho_0)$	-3.7	3.3	-0.1	1.51
2	ρ_{GGI_0}	δ_{GGI_0}	-3.4	3.2	0.0	1.43
3	ρ_{GGI_0}	δ_{GGI_0}	-3.3	3.1	0.0	1.37

Table 4 Basic statistics of the differences $\Delta\delta g = \delta g - \delta g_{GGI}$.

VERSION	ρ_0 $\left[{}^{kg}/_{m^3}\right]$	δ_0 $\left[{}^{kg}/_{m^3}\right]$	$min(\Delta\delta g)$ [mGal]	$max(\Delta\delta g)$ [mGal]	$mean(\Delta\delta g)$ [mGal]	$stdev(\Delta\delta\zeta)$ [mGal]
1	2,200	$\delta_0(\rho_0)$	-16.1	13.8	0.0	1.56
2	ρ_{GGI_0}	δ_{GGI_0}	-16.1	13.7	0.0	1.56
3	ρ_{GGI_0}	δ_{GGI_0}	-16.1	13.9	0.0	1.59

block j of volume K, situated directly below the zone i, negative, compensating density was determined as $\delta_0^j = -\frac{H_i\rho_0^i}{h_j}$, , where H_i and h_j are the heights of zone i and block j respectively.

2. The initial model $T_{GGI_0} = T_{dens_0} + T_{pol_0}$ was built based on preliminary modelling process using values of the disturbing potential from the EGM08.
3. The initial model $T_{GGI_0} = T_{dens_0} + T_{pol_0}$ is built similarly as in point 2, based on EGM08, but with the use of disturbing potential values and gravity vector components.

The latter two mentioned models of T_{GGI_0} can be considered as a local equivalent of the EGM08 model in the form of the GGI model.Data for calculations for versions 2 and 3 were determined from the EGM08 for 5144 points located 1000 m above the terrain surface. In turn, for this modelling the initial densities were assumed as in version 1.

For each test version, similarly as before, there was calculated difference $\Delta\zeta = \zeta_{GNSS/lev} - \zeta_{GGI}$ and $\Delta\delta g = \delta g - \delta g_{GGI}$. Basic statistics of these differences are presented in the Tables 3 and 4.

As we can see, the changes introduced in the preliminary model improved the accuracy of the final model, in relation to the quasigeoid model. However, we have not achieved the accuracy at the level of the type A model. A worth noting is the high accuracy of gravity disturbances, which remain at a similar level for all versions of the analysed type B models.

CONCLUSIONS

The presented GGI model allows for the determining of height anomalies and gravity disturbances with high accuracy. The model also includes information on density distribution, so it can therefore be seen as a local, integrated model of both: the external gravity field and the density distribution of the Earth's crust. However both analysed types of models (A and B) contain a density part, in the model of type A, the main information about density change is included in the global component. So, the use in the modelling process of geophysical and geological density information, as well as the determination from the model of the actual density distribution, requires the use of a B-type model. For this type of model, the determined borders of density changes for the masses lying between the geoid and the Moho surface reflect the geological units surprisingly well.

On the other hand, determined density model is very simple (one layer under the geoid surface and not included in the model the structures lying below the Moho surface), so perhaps increasing vertical resolution and depth range of the density model domain will cause improvement of the GGI model accuracy.

The carried out analysis also showed that for the type B model, the initial density model plays an important role in the modelling process and clearly affects accuracy of determined height anomalies. In the conducted research the EGM08 model was used to determine a better, more accurate initial density model. Such a procedure allowed us to obtain a significant improvement in height anomalies accuracy, although the highest accuracy was not achieved.

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