



## ORIGINAL PAPER

## AN ADVANCED ASSESSMENT OF MECHANICAL FRACTURE PARAMETERS OF SANDSTONES DEPENDING ON THE INTERNAL ROCK TEXTURE FEATURES

Leona VAVRO<sup>1)</sup>, Lucie MALÍKOVÁ<sup>2)</sup>, Petr FRANTÍK<sup>2)</sup>, Pavel KUBEŠ<sup>2)</sup>,  
Zbyněk KERŠNER<sup>2)</sup> and Martin VAVRO<sup>1)\*</sup>

<sup>1)</sup> The Czech Academy of Sciences, Institute of Geonics, Studentská 1768/9, 708 00 Ostrava-Poruba, Czech Republic

<sup>2)</sup> Brno University of Technology, Faculty of Civil Engineering, Institute of Structural Mechanics,  
Veveří 331/95, 602 00 Brno, Czech Republic

\*Corresponding author's e-mail: [martin.vavro@ugn.cas.cz](mailto:martin.vavro@ugn.cas.cz)

## ARTICLE INFO

## Article history:

Received 13 June 2018

Accepted 4 March 2019

Available online 28 March 2019

## Keywords:

Sandstone

Mode I fracture toughness

Bending Young's modulus

Fracture energy

Load–displacement diagram

Fracture test

Chevron type notch

## ABSTRACT

In this paper, sandstones from three Czech localities were subjected to mechanical fracture tests in order to obtain their properties. Carboniferous sandstone from the Staříč site was primarily different from the two other Cretaceous sandstones from Podhorní Újezd and Javorka localities in the type of grain contact, as well as in their mineralogical composition of the rock matrix and cement. These differences were primarily reflected in different rock porosities. An advanced assessment of the fracture response of the chevron notch specimens made of sandstones subjected to three-point bending test was carried out by means of the GTDiPS program suggested for processing the loading diagrams. Bending Young's modulus, mode I fracture toughness and fracture energy were subsequently calculated for all tested sandstone samples. Obtained outcomes show that the sandstone from the Staříč mine exhibits several times higher values of investigated properties than the Podhorní Újezd and Javorka sandstones. This was a result of a higher degree of rock compaction, siliciferous rock cement and, therefore, relatively low total porosity. Internal rock texture and mineralogical composition of matrix or cement are thus one of the most important factors influencing the values of mechanical fracture parameters of sandstones.

## LIST OF SYMBOLS

$A_{\text{lig}}$	Undamaged area ahead of the notch (area of the ligament) [mm <sup>2</sup> ]
$a_0$	Initial notch length [mm]
CB	Chevron bend
$CMOD$	Crack mouth opening displacement [mm]
$d$	Diameter of chevron bend specimen [mm]
$\delta$	Displacement [mm]
$E$	Bending Young's modulus [GPa]
$F$	Loading force [N]
$F_{\text{max}}$	Maximum load [N]
$g_0$	Geometrical factor [–]
$G_F$	Fracture energy [J·m <sup>–2</sup> ]
GTDiPS	General Transformation of Discrete Point Sequence
$h$	Depth of cut in notch flank [mm]
$K_{\text{IC}}$	Mode I fracture toughness [MPa·m <sup>1/2</sup> ]
$L$	Length of chevron bend specimen [mm]
$M_d$	Average grain size [mm]
MIP	Mercury intrusion porosimetry
$M_{\text{max}}$	Maximum grain size [mm]
$S$	Support span [mm]
$\theta$	Chevron angle [°]
$t$	Notch width [mm]
UCS	Uniaxial compressive strength [MPa]
$Y_I$	Geometrical factor [–]
$W_F$	Work of fracture [N·mm]

## 1. INTRODUCTION

Sandstones represent a very wide group of siliciclastic sedimentary rocks with a variable mineralogical composition and many kinds of sedimentary textures and structures, which is reflected in a broad range of values of their physical and mechanical properties. They are, therefore, an ideal study material also for observation of mechanical fracture parameters, and how these are affected by their composition and microtextural characteristics. For these reasons, three different types of sandstones have been deliberately selected for this study. Detail petrographic description and determination of basic physical and mechanical properties of chosen sandstones were undertaken before the study of their mechanical fracture parameters.

Generally, three kinds of fractures can be distinguished: brittle, elastic-plastic and quasi-brittle. Although several fracture mechanics approaches have been derived and successfully applied to the assessment of crack behaviour in some materials, there is always a large group of materials where the classical theories are not valid. For example, the procedures suggested in the linear elastic fracture mechanics cannot be used when elastic-plastic or quasi-brittle fracture occurs in the material.



**Fig. 1** Schematic map of the Czech Republic showing the location of studied sandstones. Explanation of used abbreviations and symbols: USC – Upper Silesian Coal Basin, BCB – Bohemian Cretaceous Basin, 1 – Staříč hard coal mine, 2 – Podhorní Újezd quarry, 3 – Javorka quarry.

Sandstone and rocks generally exhibit quasi-brittle behaviour (see Shah et al., 1995; Bažant and Planas, 1998, etc.) which is typical through the large zone ahead of the crack tip, where more complex non-linear fracture processes occur. Thus, fracture description needs to be carried out based on non-linear fracture models involving the cohesive nature of the crack propagation; often the fracture energy and/or other softening parameters are utilised. Unfortunately, those parameters are dependent on the specimen size/shape and configuration of the experiment. This phenomenon has been investigated extensively in recent works (see Bažant, 1996; Karihaloo et al., 2003; Hu and Duan, 2008).

Consequently, the quasi-brittle nature of the fracture response causes specific results of fracture tests, which are evaluated in this paper for selected sandstones. A specific data processing methodology is used for correction of the dependences obtained from experiments in order to obtain as precise values of fracture parameters as possible. In addition to advanced assessment of fracture tests, emphasis is also placed on the question how internal sandstone textural features such as grain contacts, cementing materials and porosity control these mechanical fracture parameters.

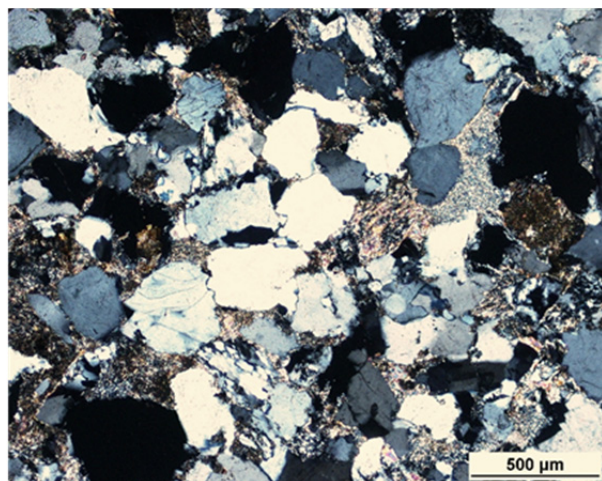
## 2. GEOLOGY AND PETROGRAPHY OF STUDIED SANDSTONES

Studied sandstones used for mechanical fracture measurements came from two different regional geological units of the Bohemian Massif (Fig. 1). Sandstone from the Staříč site stratigraphically belongs to the Upper Carboniferous Petřkovice

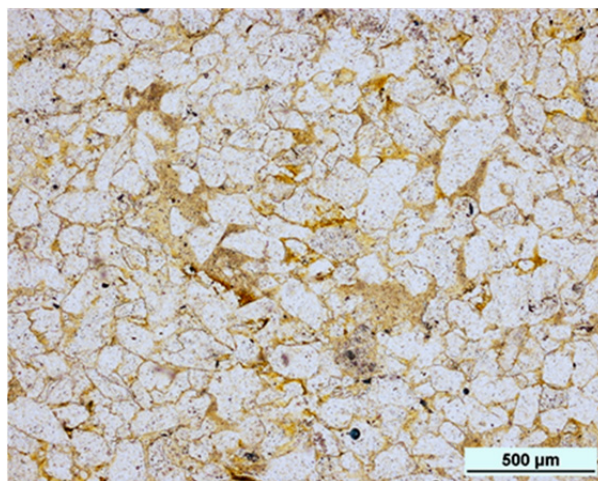
Member (Ostrava Formation, Mississippian, Early Namurian) of the Czech part of the Upper Silesian Coal Basin (Hýlová et al., 2013, 2016). Against it, sandstones from the Podhorní Újezd and Javorka quarries are the parts of Upper Cenomanian Korycany Member of the Peruc-Korycany Formation of the Bohemian Cretaceous Basin (Čech, 2011).

Petrographic analyses of sandstones under study were performed on cover-slipped thin sections using an optical polarising microscope NIKON Eclipse LVDIA-N with the aid of the NIS-Elements microscope imaging software platform (Nikon Metrology NV). Planimetry of sandstone thin sections was performed using a point counter of Eltinor type. In total, about 1500 point measurements were made on each thin section.

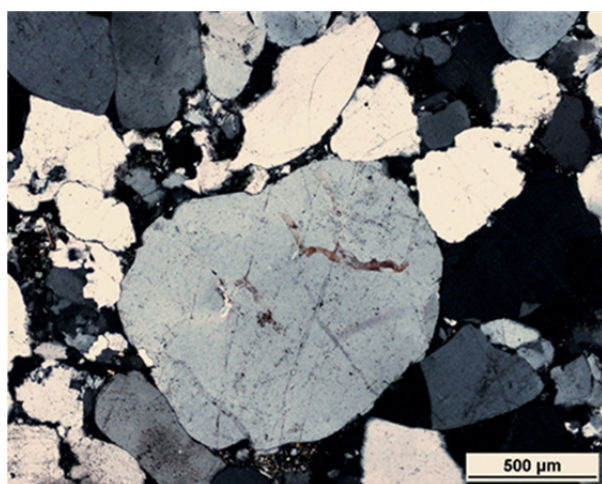
Medium-grained, moderately to poorly sorted sandstone (Fig. 2) from Staříč hard coal underground mine (approximately 15 km S from Ostrava) is predominantly composed of subangular to angular monocrystalline quartz grains with average grain size ( $M_d$ ) of 0.36 mm. Polycrystalline clasts of fine-grained quartzite ( $M_d = 0.25$  mm) occur less frequently. Non-stable fragments are represented by subangular K-feldspar grains (orthoclase and microcline) with  $M_d$  0.20 mm, polysynthetically twinned plagioclase, flakes of muscovite and biotite and subangular to subrounded clasts of lutite. Both feldspars are at different stages kaolinised and sericitised. Accessory minerals are represented by rutile, tourmaline and zircon. The rock matrix is formed of clay minerals. Cement is siliciferous, the growth of authigenic quartz around detrital quartz grains is very common.



**Fig. 2** Subangular quartz clasts with frequently occurring sutured grain contacts in the texture of the Staříč sandstone. Optical microscopy in transmitted light, crossed polarizers (XPL).



**Fig. 3** Porous to basal (matrix-supported) texture of the Podhorní Újezd sandstone. Limonite pigment finely dispersed in clay matrix is well visible. Optical microscopy in transmitted light, plane-polarised light (PPL).



**Fig. 4** Subangular to subrounded quartz clasts with bimodal grain size distribution in sandstone from Javorka quarry. Optical microscopy in transmitted light, crossed polarizers (XPL).

The so-called Hořice sandstone is a well-known building, sculpture and decorative stone material that has been used in what is now the Czech Republic for centuries (Rybařík, 1994, 2010; Vavro and Vavro, 2001; Příkryl et al., 2004). Whilst in the past the Hořice sandstone was mined in more than 150 larger or smaller quarries (Rybařík, 2010), currently it is exploited only at the Podhorní Újezd deposit (about 30 km NW from Hradec Králové). The studied sandstone (Fig. 3) is fine-grained, moderately sorted and in particular composed of subangular, exceptionally to angular quartz grains (about 75 % of rock volume) with a low degree of sphericity and  $M_d$  0.22 mm. Other stable fragments consist of

subangular clasts of quartzite with a grain size similar to monocrystalline quartz. Non-stable rock fragments are composed of K-feldspar with abundant perthitic lamellae and plagioclase. Feldspar grains (ca. 10 % of rock volume,  $M_d = 0.18$  mm) are subangular to subrounded in shape and are often sericitised. Muscovite, tourmaline and zircon occur as accessory minerals. The clay matter in the rock matrix is dominantly kaolinite with very low contents of illite-smectite mixed-layers. Fe oxyhydroxides (“limonite”) are very finely dispersed in the matrix or rarely fill the pores.

A very wide variability in colour is a typical macroscopical feature of Upper Cretaceous, fine- to medium-grained sandstone from the Javorka quarry (about 15 km W from the city of Dvůr Králové nad Labem). The rock is composed of dominant (80–85 vol. %) subangular to subrounded monocrystalline quartz grains with bimodal grain size distribution ( $M_d = 0.26$  mm,  $M_{max}$  up to 0.7–1.2 mm, see Fig. 4). Quartz grains often contain small needle-like inclusions of rutile. Polycrystalline quartzite clasts are extremely scarce. Subangular grains of sericitised and kaolinised feldspars (orthoclase), illitised flakes of muscovite and heavy minerals (titanite and zircon) also occur very rarely. The clay rock matrix is composed of kaolinite, which predominates over the illite. Matrix and open pores are often penetrated and filled by secondary oxyhydroxides of Fe and Mn, which cause often occurring pinkish and purplish colour of the rock (Vavro et al., 2008).

Comparison of basic rock texture characteristics (e.g. grain size, sorting, shape of grains, grain sphericity and type of contact among the grains) of all studied sandstones is presented in Table 1.



**Table 1** Microscopic characterisation of internal texture features of tested sandstones.

Locality	Average grain size of quartz grains [mm]	Sorting	Shape of grains	Sphericity	Texture	Grain contacts (according to terminology of Taylor, 1950)
Staříč	0.36	fair to poor	subangular to angular	low	psammitic, porous	long to concavo-convex contacts, in some cases even up to sutured contacts
Podhorní Újezd	0.22	fair to poor	subangular to angular	low	psammitic, porous to basal	point (tangential), rarely long contacts; some grains also without contact
Javorka	0.26 (bimodal grain size distribution)	fair	subangular to subrounded	medium	psammitic, porous	long contacts, rarely point (tangential) contacts

### 3. BASIC PHYSICAL AND MECHANICAL PROPERTIES OF THE STUDIED ROCKS

Tested rocks which originated from active localities (Podhorní Újezd and Javorka sandstones) were taken in open quarries in the form of blocks of irregular shape and with a side length of about 0.3–0.4 m. These blocks were provisionally selected in the field as non-weathered and free of macroscopically visible joints, failures and fissures, which would represent planes of discontinuity. Such discontinuities could influence the rock specimen properties in the process of laboratory experiments, especially during their loading. Cylindrical samples measuring 48 mm in diameter were subsequently drilled from the blocks in laboratories; the drilling was carried out in a direction perpendicular to the bedding planes. The ends of the cylindrical cores were finally cut perpendicularly to the length, so that the length-to-diameter ratio (slenderness ratio) of prepared rock samples was about 2 and 0.7, respectively.

Rock samples of 48 mm diameter prepared from sandstone from the Staříč coal mine originated from the drill core of the borehole Nb. III-1184/07. The drill core was taken in the laboratories of the Green Gas DPB, a.s. company.

For all samples from the aforementioned localities, some fundamental physical and mechanical rock properties were determined, since these were assumed to influence the fracture mechanical behaviour of rocks. Specifically, these properties were:

- mass (real) density, measured by pycnometer according to ČSN EN 1936,
- bulk density, calculated from the mass of the specimen and the bulk volume (the bulk volume of the specimen prepared in the form of regularly shaped cylinders was calculated by means of vernier calliper measurements),
- total porosity, calculated from the measurements of mass and bulk densities,

- porosity studied by high-pressure mercury intrusion porosimetry (MIP), carried out on cut samples with volumes of approximately 2 000 mm<sup>3</sup> (10 × 10 × 20 mm) and using an AUTOPORE 9500 mercury porosimetry analyser (Micromeritics Instrument Corporation, USA),
- water absorption capacity, measured after 48 hours of water absorption under atmospheric pressure based on a previously valid ČSN 72 1155 procedure,
- velocity of ultrasonic waves, measured according to internal methodical procedure of the Institute of Geonics,
- uniaxial compressive strength (UCS) on cylinder-shaped samples with height/diameter ratio close to 2:1; testing procedure and calculation was performed according to the suggested method of Bieniawski and Bernede (1979),
- splitting tensile strength, this was determined by the Brazilian test on disc-shaped samples with the thickness-to-diameter ratio approximately 2:3 according to suggested methodology described by Bieniawski and Hawkes (1978).

Aforementioned physical and mechanical properties were tested before observation of mechanical fracture ones in order to obtain a more complete picture about material parameters of the sandstones under study. Calculated and experimentally determined parameters of the physical and mechanical properties of studied sandstone are shown in Table 2. It is evident that the Carboniferous Staříč sandstone significantly differs from the sandstones of the Czech Cretaceous Basin in terms of their physical and mechanical properties. It is characterised by high values of bulk density, low water absorption capacity, low total porosity and high to very high values of strength properties. The values of its material properties are very similar, for example, to Godula sandstone from Outer Western Carpathian

**Table 2** Basic physical and mechanical properties of tested sandstones.

Parameter / Locality	Staříč		Podhorní Újezd		Javorka	
	Sample quantity	Average value (min–max)	Sample quantity	Average value (min–max)	Sample quantity	Average value (min–max)
Mass density [ $\text{kg}\cdot\text{m}^{-3}$ ]	12	2672 (2664–2676)	12	2640 (2631–2646)	28	2647 (2619–2699)
Bulk density [ $\text{kg}\cdot\text{m}^{-3}$ ]	15	2520 (2444–2667)	29	1892 (1760–1955)	79	1990 (1897–2142)
Total porosity [%]	12	4.1 (2.4–5.0)	12	28.1 (27.9–28.4)	28	25.7 (24.6–27.8)
Water absorption capacity [%]	13	1.5 (1.2–1.9)	23	9.1 (7.3–11.6)	26	6.7 (5.0–8.6)
Ultrasonic wave velocity (dry sample) [ $\text{km}\cdot\text{s}^{-1}$ ]	15	3.56 (3.34–3.69)	11	2.51 (2.39–2.55)	6	2.31 (2.13–2.49)
UCS (dry sample) [MPa]	9	151.3 (79.5–180.2)	9	27.9 (26.1–32.6)	14	37.1 (31.2–43.2)
Splitting tensile strength (dry sample) [MPa]	9	7.00 (5.40–10.99)	8	1.44 (1.23–1.56)	9	2.30 (1.59–2.84)

(Vavro et al., 2016). On the contrary, Podhorní Újezd and Javorka sandstones, like the other Cretaceous sandstones of the Czech Cretaceous Basin, are distinguished by low bulk densities, high values of total porosity and water absorption capacity and related low strength values. Determined values of physical and mechanical properties of tested sandstones are in very good accordance with previously published data (e.g. Rybařík, 1994; Müller et al., 1997; Vavro and Vavro, 2001; Vavro et al., 2008, etc.).

#### 4. CHEVRON-NOTCH THREE-POINT BEND TECHNIQUE PROCEDURE, METHODOLOGY OF STUDY AND USED EQUIPMENT

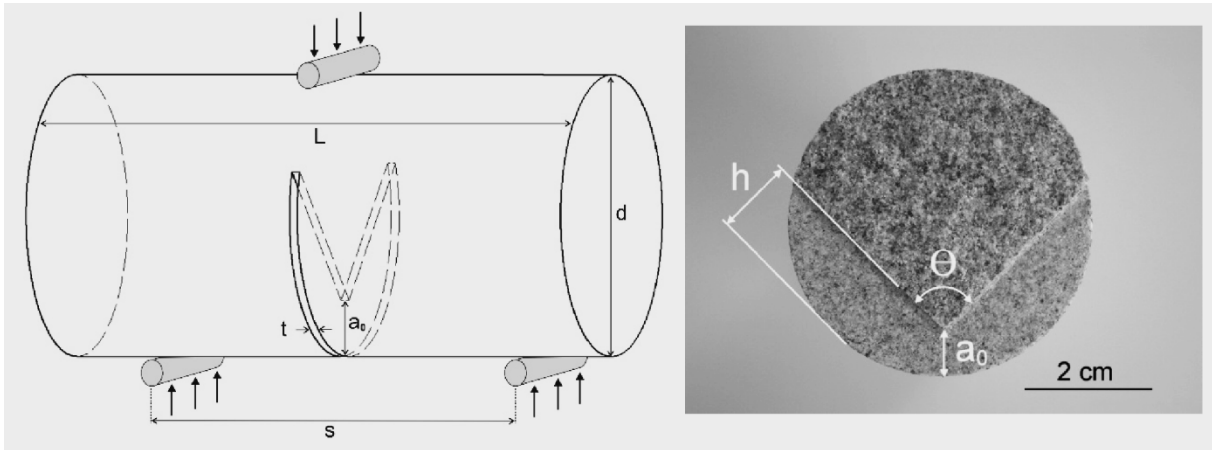
In order to estimate the additional properties of the materials that are necessary for assessment of their fracture behaviour, the chevron bend (CB) test was performed and the mode I fracture toughness and other important mechanical fracture properties of the selected sandstones were evaluated. For this test, long cylindrical specimens with the chevron (V-shaped) notch perpendicular to the specimen axis (Fig. 5) are used. At the mouth of the chevron notch the clip-on gage type of extensometer can be attached. Due to this extensometer use, the relative crack face opening (*CMOD* – crack mouth opening displacement) can be measured (Vavro and Souček, 2013).

Cylindrical test specimens of 48 mm in diameter and about 190 mm in the length were prepared by drilling from the rock blocks or drill core. The chevron notch with the internal angle of  $90^\circ$  was produced using a diamond blade with a thickness of 1.5 mm perpendicularly to core body axis and positioned in the centre of each sample. The direction of the notch was simultaneously perpendicular to the bedding plane of all tested sandstones. After chevron

notch cutting the test specimens were dried to a constant weight. Fracture toughness was measured using three-point bending test carried out at room temperature on an FPZ 100 power press with displacement control at a constant loading rate of  $0.1 \text{ mm}\cdot\text{min}^{-1}$ . The load versus load point displacement curve for each tested sandstone sample was recorded. The specimen geometry and configuration of the test have been chosen with regards to the previous experience and of course to the laboratory equipment, while in order to respect the relevant International Society for Rock Mechanics recommendation (ISRM, 1988). However, it should be noted that the resulting values of mechanical fracture properties are affected by the size of the rock specimens. It is important to recognise that the properties of the rocks determined in laboratory on intact samples cannot be generalized in a simple way and extended, for example, to the properties of the whole rock mass because of its inhomogeneity. Unfortunately, the size effect of rock material has not been studied sufficiently yet. A study on this phenomenon would be extensive and hard. A nice summary of general conclusions on size effect issue can be found in Bažant (2000).

The advantage of the chevron notch test is taken: the chevron notch causes crack initiation at the tip of the “V”; the crack subsequently proceeds in the notch plane transverse to the core axis in a stable manner. In the straight edge notch, the crack initiates and then propagates rapidly just after initiation. During the experiment, as mentioned, the dependence of the loading force  $F$  on the displacement  $\delta$  and *CMOD* could be recorded (Fig. 6).

The obtained curves were then subjected to a special procedure in the program GTDiPS (General Transformation of Discrete Point Sequence) created at



**Fig. 5** CB fracture toughness test configuration and geometry of the chevron bend specimen. Explanation of used symbols:  $d$  – diameter of chevron bend specimen ( $> 10$ -fold of maximal grain size),  $L$  – length of chevron bend specimen ( $4d$ ),  $t$  – notch width ( $\leq 0.03d$ ),  $S$  – support span ( $3.33d$ ),  $a_0$  – initial notch length (i.e. chevron tip distance from specimen surface) ( $0.15d$ ),  $h$  – depth of cut in notch flank,  $\theta$  – chevron angle ( $90^\circ$ ).



**Fig. 6** Test set-up used for *CMOD* measurements.

the Institute of Structural Mechanics, Faculty of Civil Engineering, Brno University of Technology (Frantik and Mašek, 2008). The software enables several data rearrangements such as data reduction and data approximation, etc. More details about the data processing can be found e.g. in Havlikova et al. (2016) and in the following sections. Furthermore, several relationships are presented that were used for subsequent calculation of several mechanical fracture parameters, including bending Young's modulus, fracture toughness and fracture energy.

The first investigated parameter was the bending Young's modulus. This was obtained from the initial linear part of the processed diagrams. Note that

sometimes this kind of elastic modulus is referred to as the tangential modulus. Its value was determined from the relationships (Whittaker et al., 1992; Shah et al., 1995):

$$E = g_0 \frac{F}{CMOD} \frac{1}{d} \quad (1)$$

$$g_0 = 20.8 + 19.4 \frac{a_0}{d} + 142.3 \left( \frac{a_0}{d} \right)^2 \quad (2)$$

The meaning of the symbols in Eq. (1) and (2) is the following:  $a_0$  represents the initial notch length,

$d$  is the diameter of chevron bend specimen and  $F$  with  $CMOD$  are the loading force with corresponding opening displacement determined from the linear part of the processed  $F$ – $CMOD$  diagram.

The mode I fracture toughness  $K_{IC}$  in the CB test is calculated by Eqs. (3) and (4) (ISRM, 1988).

$$K_{IC} = Y_1 \frac{F_{\max}}{d^{1.5}} \quad (3)$$

$$Y_1 = \left[ 1.835 + \frac{7.15 \cdot a_0}{d} + 9.85 \cdot \left( \frac{a_0}{d} \right)^2 \right] \cdot \frac{S}{d} \quad (4)$$

where  $F_{\max}$  is the maximum load and  $d$  is again the diameter of the sample.  $Y_1$  is dimensionless geometrical factor dependent on the specimen geometry and given by a function of the specimen dimensions ( $a_0$ ,  $d$ ) and the support span  $S$  (ISRM, 1988); see Figure 5 for more details about the geometry and loading configuration. Note that Eqs. (3) and (4) were originally derived for brittle materials and they do not take into account the phenomenon that the maximal load  $F_{\max}$  can be reached for different displacement depending on the test configuration, etc. This effect should be studied/eliminated within the following research.

The last parameter determined is the fracture energy  $G_F$  that can be calculated by means of the following equations:

$$G_F = \frac{W_F}{A_{lig}} \quad (5)$$

$$W_F = \int F(\delta) d\delta \quad (6)$$

In Eqs. (5) and (6),  $W_F$  represents the work of fracture that is estimated from the area below the curve in the corrected  $F$ – $\delta$  diagram. Note that the area below the curve was calculated by means of numerical integration.  $A_{lig}$  denotes the area of the ligament, it means the undamaged area ahead of the notch.

Remind that it can be expected that all the fracture mechanical properties are dependent on the size of the specimen used for their estimation, but any useful conclusions based on tests on rock masses are not available.

## 5. CORRECTION OF CB TEST DATA

The raw output data from the fracture tests are not ideal for further processing for several reasons and they must be adjusted. The first source of inaccuracies in measuring of the parameters during the tests is sitting and movement of the supports. The second reason for editing the data is the error caused by the sensitivity of the measuring instruments (some sections of individual characteristics have the same value for more points in time and that makes it impossible to describe a point sequence as a function or further processing). Therefore, it is necessary to

process the point sequences. Preservation of the course of the function and its values remains crucial.

Two programs were used to edit the data obtained from the fracture tests – MS Excel and GTDiPS (František and Mašek, 2008). GTDiPS, created in the Java programming language, allows the processing of extensive point sequences. It contains dozens of transformation methods, ranging from simpler ones, such as replacing two selected dimensions or removing points at the beginning/end of a sequence, to advanced transformations or the entire chain of these operations. The program works with text files that must have a given structure. The output after editing is also a .txt file.

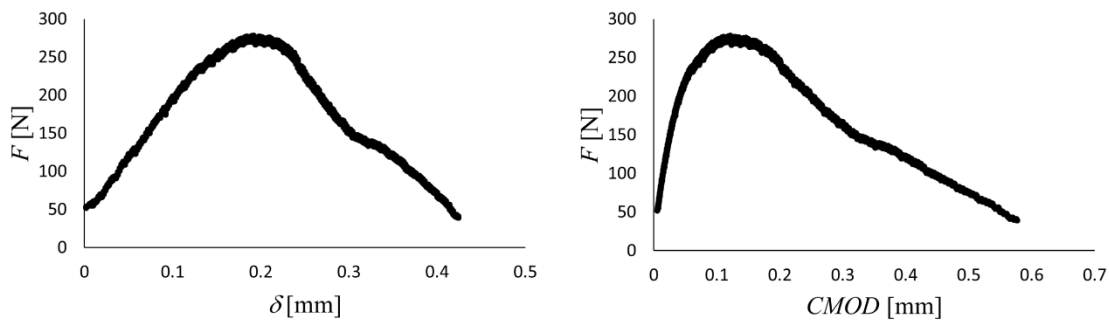
The typical editing procedure is described along with graphs (Figs. 7 to 11) for clarity. The sample used is the specimen 11972/22 from the Javorka sandstone. The editing procedure consists of the following steps:

- *Dimension Swap*  
Basic adjustment for rearrangement of dimensions so that the diagram displayed selected parameters (Fig. 7).
- *Constant Weight, Delta Weighted Moving Average*  
These two transformations reduce the number of points in the sequence and then smoothing the course of the sequence (Fig. 8).
- *Soft Start Replacement*  
This procedure removes the initial part of the sequence ending with the point where the maximum derivative value of the function is. This operation uses the Quadratic Difference transformation, this is also included in the GTDiPS program.
- *Polynomial Gap Filler*  
Filling the initial gap created by the previous operation by first degree polynomial approximation with new points with a given distance (Fig. 9).
- *Quadratic Difference, Delta Weighted Moving Average*  
Finding the derivative at the end of the sequence and smoothing its course (using last 3–7 points of the point sequence) if the derivative does not have a smooth course (Fig. 10).
- *Completion of the end of the sequence with new points using the obtained derivative* (Fig. 11).

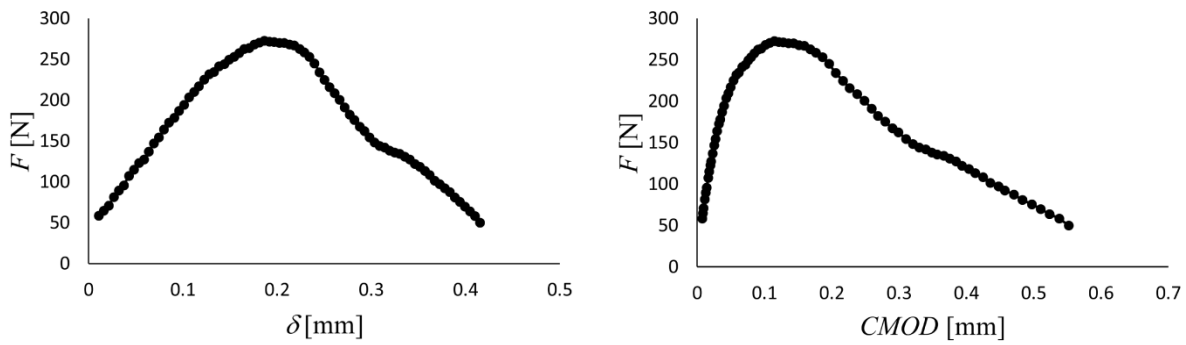
## 6. RESULTS OF MECHANICAL FRACTURE PARAMETERS ASSESSMENT AND DISCUSSION

The corrected load–displacement curves derived from several samples of each material system (locality) are presented in Figure 12. Differences in both the pre- and post-peak part of the curves for various microtextures are obvious.

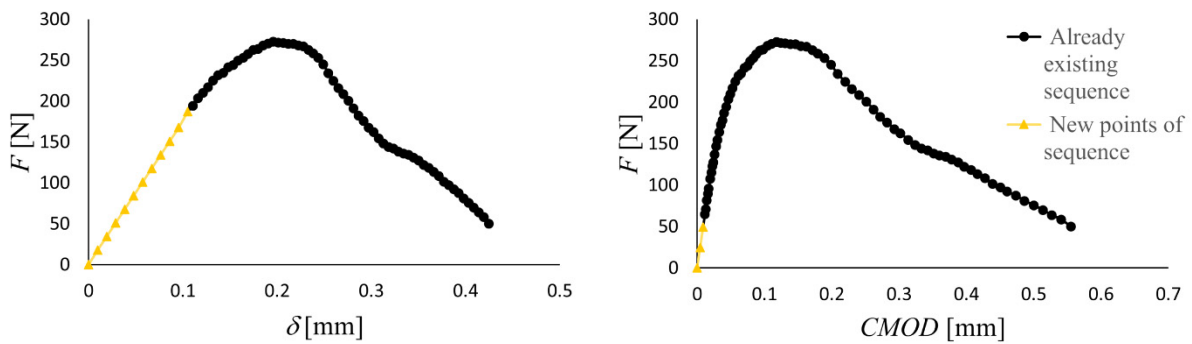
Data of bending Young's modulus, fracture toughness and fracture energy of all tested rock samples, measured and subsequently calculated on the



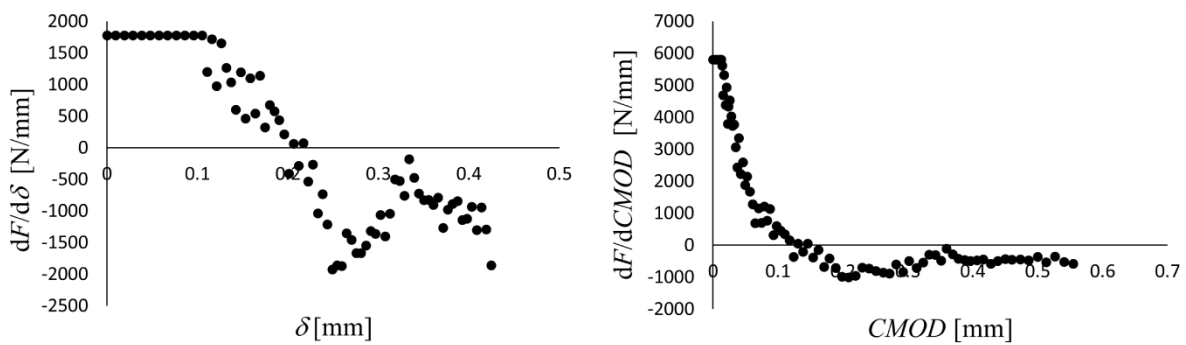
**Fig. 7** Inputs:  $F$ - $\delta$  and  $F$ - $CMOD$  diagrams.



**Fig. 8** Diagrams after reduction and smoothing.

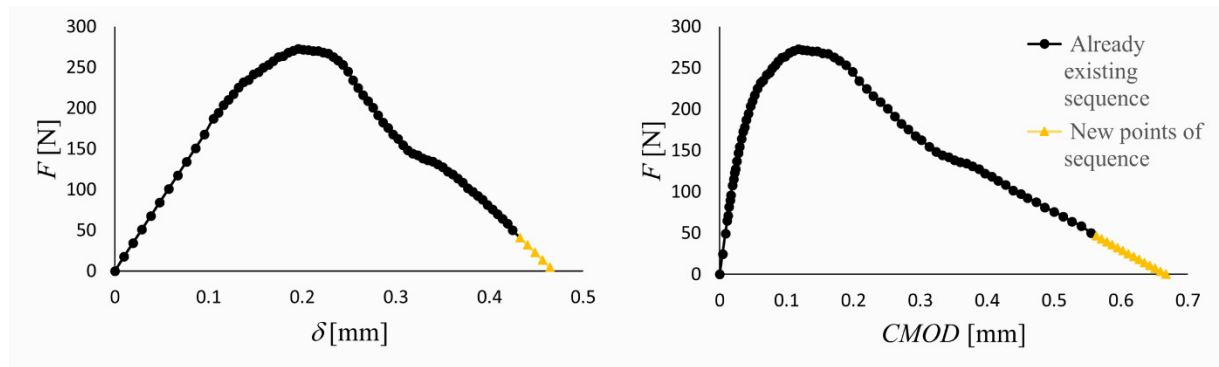


**Fig. 9** Diagrams after modifying the ascending part of the sequence.

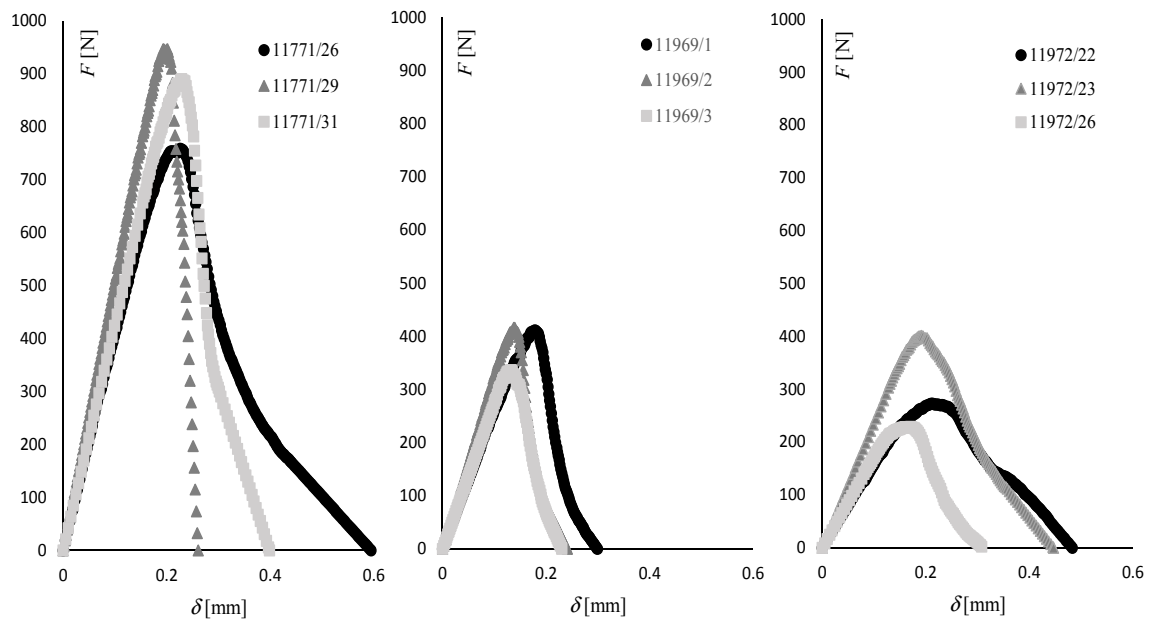


**Fig. 10** The course of derivative of the loading force by  $\delta$ ,  $CMOD$ .





**Fig. 11** Completion of the descending part of the diagrams.



**Fig. 12** Corrected load-displacement diagrams on the specimens 11771/26, 29 and 31 from Stařič mine (left), 11969/1, 2 and 3 from Podhorní Újezd quarry (middle) and 11972/22, 23 and 26 from Javorka quarry (right).

basis of corrected load-displacement curves, together with values of other important parameters, are shown in Tables 3, 4 and 5. Furthermore, Table 6 presents the comparison between the average values of fracture toughness, bending Young's modulus and fracture energy of studied sandstones on one hand, and total porosity and mineralogical composition of their interstitial material on the other.

Evaluating the results obtained from measurements on three types of sandstones show that the highest bending Young's modulus, as well as fracture toughness and fracture energy were found on samples from the Stařič locality. Average values of bending Young's modulus, fracture toughness and fracture energy of Cretaceous sandstones from Podhorní Újezd and Javorka quarries reached only several tens of percents of average values determined on Carboniferous Stařič sandstone. Specifically, average bending Young's modulus of the Podhorní Újezd sandstone is more than two times lower than the one found in Stařič sandstone; in case of the

Javorka sandstone the average value is even more than five times lower. Fracture toughness of both Podhorní Újezd and Javorka sandstones is approximately two and a half or three times lower. The fracture energy of these sandstones is even then three or four times lower as in case of Stařič sandstone. This was, in particular, due to a significant differences in internal rock texture as well as in mineralogical composition of rock interstitial components in sandstone from Stařič site when compared with Podhorní Újezd and Javorka ones. As can be seen from Tables 1 and 6 and Figure 2, the Stařič sandstone is characterised by a relatively high degree of rock compaction (diagenesis), of which the concavo-convex to sutured grain contacts, authigenic silica overgrowths onto the detrital quartz grains or even intensive silicification of clay matrix are the microscopic indicators. These microscopic features of the Stařič sandstone are then reflected in a relatively low value of its porosity (total porosity ca. 3–5 % and porosity determined by MIP ca. 2 % respectively).

**Table 3** Values of selected mechanical fracture parameters of individual tested rock specimens from Staříč mine.

Parameter	Unit	Specimen number		
		11771/26	11771/29	11771/31
$F_{\max}$	N	759	947	890
$K_{\text{Ic}}$	$\text{MPa}\cdot\text{m}^{1/2}$	0.82	1.03	0.97
$E$	GPa	13.2	25.0	19.8
$A_{\text{lig}}$	$\text{mm}^2$	1029.10	1022.20	1013.19
$W_{\text{F}}$	$\text{N}\cdot\text{mm}$	196.81	134.36	168.23
$G_{\text{F}}$	$\text{J}\cdot\text{m}^{-2}$	191.2	131.4	166.0

**Table 4** Values of selected mechanical fracture parameters of individual tested rock specimens from Podhorní Újezd quarry.

Parameter	Unit	Specimen number		
		11969/1	11969/2	11969/3
$F_{\max}$	N	411	416	336
$K_{\text{Ic}}$	$\text{MPa}\cdot\text{m}^{1/2}$	0.40	0.40	0.33
$E$	GPa	8.8	8.9	6.9
$A_{\text{lig}}$	$\text{mm}^2$	1184.99	1189.76	1175.55
$W_{\text{F}}$	$\text{N}\cdot\text{mm}$	59.13	46.21	37.78
$G_{\text{F}}$	$\text{J}\cdot\text{m}^{-2}$	49.9	38.8	32.1

**Table 5** Values of selected mechanical fracture parameters of individual tested rock specimens from Javorka quarry.

Parameter	Unit	Specimen number		
		11972/22	11972/23	11972/26
$F_{\max}$	N	273	401	232
$K_{\text{Ic}}$	$\text{MPa}\cdot\text{m}^{1/2}$	0.27	0.39	0.23
$E$	GPa	2.5	4.5	3.9
$A_{\text{lig}}$	$\text{mm}^2$	1175.85	1175.85	1169.98
$W_{\text{F}}$	$\text{N}\cdot\text{mm}$	70.84	90.60	36.64
$G_{\text{F}}$	$\text{J}\cdot\text{m}^{-2}$	60.2	77.0	31.3

**Table 6** Average values of individual mechanical fracture parameters of individual tested rock specimens compared with rock porosity and mineralogical composition of fine-grained rock matter constituents.

Specimen number	Locality	Bending Young's modulus $E$ [GPa]	Fracture toughness $K_{\text{Ic}}$ [ $\text{MPa}\cdot\text{m}^{1/2}$ ]	Fracture energy $G_{\text{F}}$ [ $\text{J}\cdot\text{m}^{-2}$ ]	Total porosity / Porosity determined by MIP [%]	Mineralogical composition of rock matrix and/or cement
11771/26 11771/29 11771/31	Staříč	19.3	0.94	162.9	4.7 / 2.2	clay matrix with intense silicification (siliciferous rock cement)
11969/1 11969/2 11969/3	Podhorní Újezd	8.2	0.38	40.3	28.3 / 25.9	clay matrix with secondary Fe-oxyhydroxide (limonite) pigment
11972/22 11972/23 11972/26	Javorka	3.6	0.30	56.2	25.4 / 19.6	clay matrix rich in pigment formed by oxyhydroxide of Fe and Mn

In contrast, high values of total porosity of both Podhorní Újezd and Javorka sandstones (at a level of around 25–30 %) resulting from point or at most long grain contacts, occasionally even up to basal (matrix-supported) rock texture and absence of secondary siliciferous rock cement. Substantial differences between fracture toughness of Staříč sandstone on one side and Cretaceous sandstones from Podhorní Újezd and Javorka quarries on the other side, as described above, are visible also in the case of other physical and mechanical properties such as ultrasonic waves velocity or strength parameters (see Table 2). It also follows from the above that a very good congruency between physical, strength and mechanical fracture properties of studied sandstones was found. Staříč sandstone, characterised by the highest values of bulk density and strength and, conversely, of the lowest porosity and water absorption within the studied rocks logically has the highest values of bending Young's modulus, fracture toughness and fracture energy. On the contrary, Podhorní Újezd and Javorka sandstones characterised by higher porosity and water absorption capacity and significantly lower bulk density and strength have adequately lower values of mechanical fracture parameters compared to Staříč sandstone. However, there is something of a mutual discrepancy between some mechanical fracture properties determined at Podhorní Újezd and Javorka sandstones. Samples from the Javorka locality exhibit substantially lower values of bending Young's modulus for practically the same fracture toughness and fracture energy as Podhorní Újezd sandstone. The reasons for this have not been studied in detail yet. One can assume at this moment that it may be probably connected again with differences in sandstone texture for each of these localities and possibly also with often very high content of secondary Fe and Mn oxyhydroxides in Javorka sandstone.

## 7. CONCLUSIONS

This paper focuses on advanced assessment of the response of the sandstone specimens with chevron notch under three-point bending test. Loading  $F-\delta$  and  $F-CMOD$  diagrams were corrected and evaluated in order to obtain fundamental fracture mechanical parameters such as bending Young's modulus, fracture toughness and fracture energy. On one hand, obtained results show that values of the parameters investigated are dependent on parameters of internal rock texture – in particular grain size, degree of grain sorting and kind of grain contacts; whilst on the other hand, the mineralogy of interstitial materials between framework grains is also important. Sandstone from the Staříč locality is several times stiffer and more fracture-resistant than Podhorní Újezd as well as Javorka samples, which corresponds to higher degree of compaction and presence of siliciferous cement. It may be also noted that mechanical fracture parameters of the sandstones

under study correspond very closely with their basic physical and mechanical properties. The reason why the samples from the Javorka locality exhibit substantially lower values of bending Young's modulus for practically the same fracture toughness and fracture energy values in comparison to the Podhorní Újezd sandstone has not been identified at this moment. It is probably caused again by differences in both sandstone texture and mineralogical composition of rock interstitial constituents for each of these sites.

## ACKNOWLEDGEMENTS

This article was written in connection with the project of the Institute of Clean Technologies for Mining and Utilization of Raw Materials for Energy Use – Sustainability program (ICT-PU, identification code: LO1406). This project is supported by the National Programme for Sustainability I (2013–2020) financed by the state budget of the Czech Republic. The presented work was also supported by a project for the long-term conceptual development of research organisations (RVO: 68145535). The financial support provided by the Czech Science Foundation (GACR) under project No. 19-09491S (MUFRAS) is also gratefully acknowledged.

The authors wish to thank the companies Kámen Ostroměť, s.r.o., Kamenolom Javorka, s.r.o. and Green Gas DPB, a.s. for providing the sandstone samples required for our study.

## REFERENCES

- Bažant, Z.P.: 1996, Analysis of work-of-fracture method for measuring fracture energy of concrete. *Journal of Engineering Mechanics*, 122, No. 2, 138–144. DOI: 10.1061/(ASCE)0733-9399(1996)122:2(138)
- Bažant, Z.P.: 2000, Size effect. *International Journal of Solids and Structures*, 37, 69–80. DOI: 10.1016/S0020-7683(99)00077-3
- Bažant, Z.P. and Planas, J.: 1998, *Fracture and size effect in concrete and other quasibrittle materials*. CRC Press, Boca Raton and London, 640 pp.
- Bieniawski, Z.T. and Hawkes, I.: 1978, Suggested methods for determining tensile strength of rock materials. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, 15, No. 3, 99–103.
- Bieniawski, Z.T. and Bernede, M.J.: 1979, Suggested methods for determining the uniaxial compressive strength and deformability on rock materials. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, 16, No. 2, 135–140. DOI: 10.1016/0148-9062(79)91451-7
- Čech, S.: 2011, Palaeography and stratigraphy of the Bohemian Cretaceous Basin (Czech Republic) – an overview. *Geological Research in Moravia and Silesia*, 18, No. 1, 18–21.
- ČSN 72 1155: 1984, Determination of water absorption and apparent porosity of natural building stone. Office for Standards and Testing, Prague, 7 pp., (in Czech).
- ČSN EN 1936: 2007, Natural stone test methods - Determination of real density and apparent density, and of total and open porosity. Czech Standards Institute, Prague, 12 pp., (in Czech).

- Frantík, P. and Mašek, J.: 2008, GTDiPS Application. [www.kitnarf.cz/gtdips](http://www.kitnarf.cz/gtdips). Accessed 18 September 2015.
- Havlikova, I., Frantík, P., Mašek, J., Sobek, J., Simonova, H., Vesely, V., Kersner, Z., Seitzl, S., Merta, I. and Schneemayer, A.: 2016, Evaluation of fracture tests of concrete specimens via advanced tool for experimental data processing. *Applied Mechanics and Materials*, 821, 585–590.  
DOI: 10.4028/www.scientific.net/AMM.821.585
- Hu, X.Z. and Duan, K.: 2008, Size effect and quasi-brittle fracture: the role of FPZ. *International Journal of Fracture*, 154, No. 1–2, 3–14.  
DOI: 10.1007/s10704-008-9290-7
- Hýlová, L., Jureczka, J., Jirásek, J., Sivek, M. and Hotárková, J.: 2013, The Petřkovice Member (Ostrava Formation, Mississippian) of the Upper Silesian Basin (Czech Republic and Poland). *International Journal of Coal Geology*, 106, 11–24.  
DOI: 10.1016/j.coal.2013.01.004
- Hýlová, L., Jirásek, J., Sivek, M. and Jureczka, J.: 2016, Coal-bearing capacity of the Petřkovice Member (Ostrava Formation, Serpukhovian, Mississippian) of the Upper Silesian Basin (Czech Republic and Poland). *Geological Quarterly*, 60, No. 3, 637–649.  
DOI: 10.7306/gq.1283
- ISRM Testing Commission (co-ordinator: Ouchterlony, F.): 1988, Suggested methods for determining the fracture toughness of rock. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, 25, No. 2, 71–96.
- Karihaloo, B.L., Abdalla, H.M. and Imjai, T.: 2003, A simple method for determining the true specific fracture energy of concrete. *Magazine of Concrete Research*, 55, No. 5, 471–481.  
DOI: 10.1680/macr.2003.55.5.471
- Müller, K., Martinec, P., Ptáček, J., Konečný, P., Fialová, V. and Kožušníkova, A.: 1997, Physical and technical properties of rocks from coal-bearing Carboniferous and its sedimentary cover. In: Dopita, M. (Ed.): *Geology of the Czech part of the Upper Silesian Basin*. Ministry of the Environment of the Czech Republic, Prague, 208–215, (in Czech).
- Přikryl, R., Svobodová, J. and Siegl, P.: 2004, Historical dimension stone resources in the Czech Republic. *Roc Maquina*, 53, 28–31.
- Rybařík, V.: 1994, Noble building and sculptural stone of the Czech Republic. *Industrial Secondary School of Stonework and Sculpture in Hořice*, Hořice, 218 pp., (in Czech).
- Rybařík, V.: 2010, Hořice sandstone from Podhorní Újezd. *Kámen*, 16, No. 1, 7–14, (in Czech).
- Shah, S.P., Swartz, S.E. and Ouyang, C.: 1995, *Fracture mechanics of structural concrete: applications of fracture mechanics to concrete, rock, and other quasi-brittle materials*. John Wiley & Sons, Inc., New York, 588 pp.
- Taylor, J.M.: 1950, Pore-space reduction in sandstones. *American Association of Petroleum Geologists Bulletin*, 34, 701–716.
- Vavro, L. and Vavro, M.: 2001, Study of physico-mechanical properties of Hořice sandstone in the Podhorní Újezd quarry with respect to opening of new part of deposit. *Transactions of the VŠB-Technical University Ostrava, Mining and Geological Series*, 47, No. 2, 69–76, (in Czech).
- Vavro, L. and Souček, K.: 2013, Study of the effect of moisture content and bending rate on the fracture toughness of rocks. *Acta Geodynamica et Geomaterialia*, 10, No. 2 (170), 247–253.  
DOI: 10.13168/AGG.2013.0025
- Vavro, M., Vavro, L., Petrů, J., Martinec, P. and Ščučka, J.: 2008, Bělohrad sandstone from Javorka quarry. *Kámen*, 14, No. 3, 17–25, (in Czech).
- Vavro, M., Vavro, L., Martinec, P. and Souček, K.: 2016, Properties, durability and use of glauconitic Godula sandstones: a relatively less known building stone of the Czech Republic and Poland. *Environmental Earth Sciences*, 75, No. 22 (1437), 1–17.  
DOI: 10.1007/s12665-016-6248-3
- Whittaker, B.N., Singh, R.N. and Sun, G.: 1992, *Rock fracture mechanics: Principles, design and application*. Elsevier, Amsterdam, 570 pp.