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ORIGINAL PAPER

COMPARISON AND ASSESSMENT OF FLOAT, FIXED, AND SMOOTHED PRECISE POINT POSITIONING

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ABSTRACT

Precise Point Positioning (PPP) has been considered a powerful method for GNSS data processing. The essential input products, such as precise satellite orbits and clocks, are provided within the International GNSS Service (IGS) with a sufficient quality for estimating receiver coordinates with centimeter level accuracy. However, the IGS satellite clocks enable users to estimate ambiguities only as float values. An additional product for satellite phase biases is necessary for an integer ambiguity resolution (PPP AR). Another approach is the backward smoothing algorithm utilizing already precise and converged parameters for improving those parameters estimated at previous epochs. All the three approaches for ambiguity estimation are compared and assessed in terms of advantages and disadvantages, achieved coordinates precision, and flexibility. The comparison are performed through a processing of GNSS data from selected IGS permanent stations during 30 days in 2018, and a processing of high rate GNSS observations of the station STRF in Greece collected during the seismic event occurred on October 25, 2018. The backward smoothing improved the float solution similarly like the PPP AR, and therefore can be considered an alternative approach providing easier implementation and no dependency on additional satellites products.

We utilized two different products for phase biases in the PPP AR, namely Integer Recovery Clocks (IRC) provided by the Centre National d'Études Spatiales/Collecte Localisation Satellites (CNES/CLS) analyses center and Fractional Cycle Biases (FCB) which were estimated at the Geodetic Observatory Pecny (GOP) analyses. The IRC is based on the assimilation phase biases into satellite clocks, while the FCB products are distributed in terms of wide-lane and narrow-lane biases. A similar accuracy obtained from our comparison indicates an interoperability of products when using different strategies and even different software.

1. INTRODUCTION

Data from Global Navigation Satellite Systems (GNSS) can be processed either in a relative or in an absolute sense. Relative processing is based on forming double differenced observations between two satellites and two receivers, and site coordinates are estimated in a centimeter level accuracy with respect to a reference station position. On the other hand, site coordinates can be calculated autonomously on the basis of measurements from a single receiver if precise satellite orbits and clocks are available. The autonomous positioning is beneficial particularly for monitoring station displacement in a large area, where stable positions of reference stations are not guaranteed. Recently, an autonomous strategy called VADASE (Variometric Approach for Displacements Analysis Stand-alone Engine) has been developed by Colosimo et al. (2011). It is based on processing epoch differenced code and carrier-phase GNSS observations, and it is primarily designed for site velocity and displacement estimation, which can be applied particularly in seismology. Another autonomous strategy called Precise Point Positioning (PPP) (Malys et al., 1990; Zumberge et al., 1997) is based on processing zero differenced GNSS observations. It has been well established during the last decade, and it plays an important role besides traditional methods like Real Time Kinematics (RTK) or double-difference (DD) network processing. Essential precise products for satellite orbits and clocks are provided for instance by the International GNSS Service (IGS) for offline as well as for real-time applications.

When using precise orbits and clocks fulfilling the IGS standards, initial ambiguity parameters can be estimated in the PPP as float values only. The reason originates in assimilation of hardware-induced delays of carrier-phase observations into estimated ambiguities destroying their integer nature. Therefore, additional product for carrier-phase biases is necessary for integer ambiguity resolution (AR). This product is not available within the IGS at the moment, but it is already provided by several research centers or commercial services. Original biases of different carrier-phase observation signals are not estimable, thus reparametrization is necessary. Three main approaches have been developed for the AR during

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recent years: Decoupled Clocks (DC) (Collins et al., 2008), Integer Recovery Clocks (IRC) (Laurichesse and Mercier, 2007), and Fractional Cycle Biases (FCB) (Ge et al., 2008). In case of the DC model, code observations contribute only to receiver and satellite code clock parameters, and carrier-phase observations contribute only to receiver and satellite phase clock parameters. The reason for this clock decoupling is that code and phase observations are not synchronized, and introducing a common clock parameter leads to a contamination of ambiguity parameters by code biases. Phase biases are then absorbed by satellite phase clocks. An accompanying parameter to the code and phase clocks is the time varying wide-lane phase bias derived directly from the Melbourne-Wuebbena combination (MW) (Melbourne, 1985; Wuebbena, 1985). The concept of the IRC model is similar, but differs in the way that code and phase clock products are aligned each other on the level of a half cycle of the narrow-lane wavelength. As a result, the estimated satellite clocks can contribute to the IGS because it fulfills the IGS standards. In case of the FCB model, carrier-phase biases are not absorbed by the estimated satellite clocks, but wide-lane and narrow-lane phase fractional biases are estimated and provided to users. Different assumptions and parameter definitions implemented on a server side have to be applied on the user side correspondingly. Using different products by different users can be difficult due to a limited documentation of used models and conventions implemented for products generation, missing products accuracy assessment, and unclear parameter definition. Nevertheless, Teunissen and Khodabandeh (2015) showed that all the mentioned models are equivalent, and a mutual transformation can be found.

A most critical disadvantage of the PPP is a long convergence period. The AR is a technique for shortening the convergence, however, up to 30 minutes is still necessary for obtaining receiver coordinates with a centimeter level accuracy. Fast ambiguity resolution is possible only if precise local ionosphere and troposphere corrections are available (Banville et al., 2014). When real-time processing is not required, an elegant backward smoothing algorithm can be employed (Rauch et al., 1965; Zhang et al., 1996; Vaclavovic and Dousa, 2015). The backward smoothing requires the same products like the PPP solution, and it can be used globally without any additional support. The principal consists in exploiting already stable and precise float ambiguity parameters and recalculation of estimated parameters in the backward direction. The Kalman filter can use only past observations for estimating epoch-wise parameters. The backward smoothing however enables a contribution of all measurements from all the epochs to the estimation of such parameters at every single epoch. This approach works with float ambiguities only, therefore, the solution is not sensitive to an incorrect integer ambiguity fixing. Additional biases such as phase hardware delays are also not necessary. A precision of estimated state vector should theoretically correspond to a precision gained when using the traditional least squares adjustment (LSQ). However, a processing of high-rate data can be easily supported by the smoothing contrary to the LSQ. Another disadvantage, and similar to the LSQ, is that the smoothing is applicable in post-processing or near real time mode only.

All the mentioned strategies for the AR have advantages, disadvantages, and some limitations. There is a little research presenting assessment and comparison of estimated stations coordinates from PPP, PPP AR, and PPP with the backward smoothing. Therefore, we estimated pseudokinematic coordinates from several IGS permanent stations from 30 days and compared achieved repeatability from all three strategies. To resolve integer ambiguities, we have implemented the FCB model for a phase biases estimation when processing data from the global IGS permanent network. Moreover, the PPP client has been enhanced to apply also the IRC products provided by Centre National d'Études Spatiales/Collecte Localisation Satellites (CNES/CLS) (Loyer et al., 2012) to demonstrate the interoperability when using carrier-phase biases from different providers and to compare their quality. Montenbruck et al. (2018) successfully applied CNES/CLS IRC product for PPP with integer ambiguity resolution when processing data from Low Earth Orbit (LEO) altimetry satellite Sentinel-3A. A special procedure based on differencing observations from corresponding GPS satellite arcs was applied. This approach can be used for LEO orbit determination, when a satellite pass duration typically ranges from 10 to 40 minutes, but it is impractical for processing data collected by a receiver on the Earth.

We compared PPP float, PPP AR, and PPP smoothed solutions also on the bases of processing 10Hz GPS observations collected at the station STRF in Greece. The station is located close to the epicenter of the earthquake detected on the October 25, 2018, therefore the results present a potential application in geohazard warning systems or in various geophysical studies.

We start with repeating fundamentals of PPP and PPP AR based on processing ionosphere-free linear combinations of code and carrier-phase dual frequency observations. We then summarize theoretical background of the estimation wide-lane and narrow-lane phase biases. Properties of the CNES/CLS IRC products and important instructions for their correct using will be presented afterwards. Following chapters focus on assessment of results from different strategies. Finally, conclusion sums up achievements and recommendations.

2. PRECISE POINT POSITIONING

Fundamental equations for code and carrier-phase observations forming ionosphere-free (IF) linear combinations are expressed in units of length as follows:

$$P^s_{r.IF} = \rho^s_r + c(\delta_r - \delta^s) + m \cdot ZTD + b_{r,IF} - -b^s_{IF} + e^s_{P,r,IF} \quad (1)$$

$$L^s_{r.IF} = \rho^s_r + c(\delta_r - \delta^s) + m \cdot ZTD + N^s_{r,IF} + B_{r,IF} - -B^s_{IF} + e^s_{L,r,IF} \quad (2)$$

where, upper scripts *r, s* denote receiver and satellite, ρ is the geometry distance between a receiver and a transmitter, *c* is the speed of light, δ is a clock correction, *ZTD* is the zenith total delay projected to the line-of-sight direction by a mapping function m, *b* stands for a code bias, and *e* characterizes remaining unmodeled errors such as multipath. Carrier-phase observation equation contains also an initial ambiguity *N* and phase bias *B* both expressed in units of length.

The receiver IF code bias is assimilated into the receiver clock parameter. When a common clock parameter is introduced in code as well as in carrier-phase equations, the IF code bias will appear also in phase observations. According to the IGS standard, satellite clock corrections publicly available, include the IF code bias formed from P1 and P2 signals defined in RINEX 2 notation. If such satellite clock corrections are applied in the carrier-phase observation model, ambiguities are additionally contaminated by such IF code bias. When all input products and models are moved to the left hand side of the equation, the code and carrier-phase observation can be reformulated as:

$$P^s_{r.IF} = \rho^s_r + c\overline{\delta_r} + m \cdot ZTD + e^s_{P,r,IF} \quad (3)$$

$$L^s_{r.IF} = \rho^s_r + c\overline{\delta_r} + m \cdot ZTD + \overline{N^s_{r,IF}} + e^s_{L,r,IF} \quad (4)$$

where $\overline{\delta_r}$ and $\overline{N^s_{r,IF}}$ are new parameters for receiver clock and ambiguities:

$$c\overline{\delta_r} = c\delta_r + b_{r,IF} \quad (5)$$

$$\overline{N^s_{r,IF}} = N^s_{r,IF} + B_{r,IF} - B^s_{IF} + b^s_{IF} - b_{r,IF} \quad (6)$$

Initial ambiguities are contaminated by phase and code biases and are estimated as constant float values for one continuous satellite arc without cycle slip.

3. MODIFIED EQUATIONS FOR PPP AR

When initial ambiguities are supposed to be resolved as integer values, they must be isolated from code and phase biases. Receiver based biases can be eliminated by forming between-satellites single differences, and satellite-specific biases are then required as an input product. Ionosphere-free linear combination is generated using real-value coefficient, therefore IF ambiguity cannot be expressed as an integer value. Easy modification of the equation (6) leads to IF ambiguity decomposition into wide-lane (WL) and narrow-lane (NL) linear combinations, which can be estimated as integer values. The decomposition is defined as follows:

$$N^{p,s}_{r,IF} = \frac{c}{f_1+f_2}(N^{p,s}_{r,n} + B^{p,s}_{r,n}) + \frac{cf_2}{f_1^2-f_2^2}(N^{p,s}_{r,w} + B^{p,s}_{r,w}) \quad (7)$$

Upper scripts *p* and *s* represent satellites used for single differences. All satellite-based biases included in the IF ambiguity are decomposed into the narrow-lane phase bias $B^{p,s}_{r,n}$ and wide-lane phase bias $B^{p,s}_{r,w}$ expressed in cycles.

The wide-lane ambiguity defined as N1 – N2 can be directly calculated from the MW linear combination, which eliminates all geometrical errors, ionosphere and troposphere delay, and clocks corrections. Since code measurements are utilized, the combination is noisy, and the success of ambiguity resolution depends on the code observations accuracy. Therefore, several consecutive epochs are used for averaging the MW observables for improving their accuracy. Due to stability of wide-lane biases, only one set of values are usually suitable for daily processing (Ge et al., 2008). When remaining fractional wide-lane cycle is less than a defined threshold (0.25 cycles in our case), simple rounding to the nearest integer value can be utilized, because the wavelength of the wide-lane carrier-phase observation is 86 cm in case of L1 and L2 GPS measurements, which is long enough.

Float ionosphere-free and fixed wide-lane ambiguities are then used for estimating narrow-lane ambiguities. The narrow-lane biases need to be applied to recover integer nature of corresponding ambiguities. Since the narrow-lane wavelength is already very short (about 11 cm in case of GPS L1 and L2), simple rounding is not suitable, and more reliable strategy is essential, e.g. the LAMBDA algorithm (Teunissen, 1995).

When both wide-lane and narrow-lane ambiguities are fixed to integer values, IF ambiguities are corrected and used in fixed solution. A sufficient accuracy of IF ambiguities are critical for correct NL fixing, therefore an initial convergence is reduced only but not fully eliminated.

4. FRACTIONAL CYCLE BIASES ESTIMATION

The phase biases estimation method proposed by Ge et al. (2008) was modified to process observations of different signals. Since the satellite clocks absorb IF code delays, all pseudorange observations were first converted to the reference signals using differential code biases (DCB). For a maximum flexibility, we transformed the differential biases to undifferenced values. The bias datum was set up by assuming that the IF code bias of the reference signals is zero:

$$b_{P1} - b_{P2} = DCB_{P1P2} \quad (8)$$

$$\alpha \cdot b_{P1} - \beta \cdot b_{P2} = 0 \quad (9)$$

b_{P1} and b_{P2} indicate the undifferenced code biases of the reference signals, α and β are the coefficients of the IF combination. After applying the undifferenced code biases of the reference signals, the code delays of the other signals can be obtained by combining with other DCB. The absolute values of code biases are directly added to raw pseudorange observations for the subsequent FCB estimation.

Float wide-lane ambiguities are derived from the MW combination, which can be expressed as:

$$MW_r^s = \frac{1}{f_1 - f_2}(f_1 L_1 - f_2 L_2) - \frac{1}{f_1 + f_2}(f_1 P_1 + f_2 P_2) \tag{10}$$

where r and s represent the receiver and satellite, λ is the wavelength of the corresponding carrier-phase observation. The MW observations from a moving time window are averaged to obtain wide-lane float ambiguities with a better precision:

$$\overline{N_{r,w}^s} = \left\langle \frac{MW_r^s}{\lambda_w} \right\rangle = N_{r,w}^s + B_{r,w} - B_w^s \tag{11}$$

The bracket ⟨ ⟩ denotes average over epochs, λ_w denotes the wide-lane wavelength, $N_{r,w}^s$ is the integer wide-lane ambiguity, $B_{r,w}$ and B_w^s represent the receiver- and satellite-specific wide-lane hardware biases. To eliminate receiver hardware delays, one reference satellite with the highest elevation is selected, and the single differences between satellites are formed. The equation is then written as:

$$\overline{N_{r,w}^{s,REF}} = N_{r,w}^{s,REF} + B_w^{REF} - B_w^s \tag{12}$$

Single-differenced wide-lane observations can be directly introduced into the FCB estimation engine or resolved as an integer value if external wide-lane phase biases are available.

The ionosphere-free float ambiguities $N_{r,IF}^s$ are derived from the standard PPP solution. Single differences of the ionosphere-free ambiguities are formed for deriving the narrow-lane float ambiguities $\overline{N_{r,n}^{s,REF}}$:

$$\overline{N_{r,n}^{s,REF}} = \frac{f_1 + f_2}{f_1} N_{r,IF}^{s,REF} + \frac{f_2}{f_2 - f_1} N_{r,w}^{s,REF} = N_{r,n}^{s,REF} + B_n^{REF} - B_n^s \tag{13}$$

where ionosphere-free ambiguities are expressed in cycles with the wavelength of L1. Looking at the equations (12) and (13), the wide-lane and narrow lane float ambiguities can be expressed in the same formula:

$$R_r^{s,REF} = N_r^{s,REF} + B^{REF} - B^s \tag{14}$$

where $R_r^{s,REF}$ indicates the float ambiguities feeding to the phase biases estimation engine, B^{REF} and B^s are the satellite phase biases, which will be solved for. The integer part $N_r^{s,REF}$ of the float ambiguities should be first removed and adjusted by properly considering the one-cycle inconsistency problem (Xiao et al., 2018). The equation (14) leads to a singular normal equation system, therefore, the mean of all estimated phase delays are constrained to zero.

Finally, the wide-lane and narrow-lane phase biases are estimated on the basis of analyzing data from a permanent network using the Kalman filter. The wide-lane biases are very stable within a day, therefore, a zero signal process noise was used in the Kalman filter time prediction. On the other hand, the narrow-lane biases were estimated dynamically epoch by epoch.

5. CNES/CLS IRC PRODUCT

The CNES/CLS provides precise products for orbits and clocks within the Multi-GNSS Experiment and Pilot project (MGEX) using the GRM acronym for the solution. The MGEX project has been set up by the IGS to collect and analyze all available multi-GNSS signals. The GRM products include integer recovery clocks distributed in the standard RINEX clocks format and wide-lane biases stored in the RINEX header. The provided clocks are phase clocks, but aligned to the code clocks with differences less than half a cycle of L1 wavelength (Loyer et al., 2012). The wide-lane biases estimation is based on the MW combination like in case of the FCB model. However, narrow-lane biases are not estimated but assimilated into reparametrized satellite clock corrections.

Equation for code IF observation is reformulated as:

$$P_{r,IF}^s = \rho_r^s + c\overline{\delta_{r,P}} - c\overline{\delta_P^s} + m \cdot ZTD + e_{P,r,IF}^s \tag{15}$$

where $\overline{\delta_{r,P}}$ and $\overline{\delta_P^s}$ are receiver and satellite code clocks corrections including corresponding IF code bias:

$$\overline{\delta_{r,P}} = \delta_{r.P} + b_{r,IF} \tag{16}$$

$$\overline{\delta_P^s} = \delta_P^s + b_{IF}^s \tag{17}$$

The equation for the carrier-phase IF observation is formulated as:

$$L_{r,IF}^s = \rho_r^s + c\overline{\delta_{r.L}} - c\overline{\delta_L^s} + m \cdot ZTD + \frac{c}{f_1 + f_2}\overline{N_{r,n}^s} + \frac{cf_2}{f_1^2 - f_2^2}\overline{N_{r,w}^s} + e_{L,r,IF}^e \tag{18}$$

where $\overline{N_{r,n}^s}$ and $\overline{N_{r,w}^s}$ are integer narrow-lane and wide-lane ambiguities, respectively. Receiver and satellite phase clocks are reformulated and defined as:

$$c\overline{\delta_{r,L}} = c\delta_{r,L} + B_{r,IF} - \frac{cf_2}{f_1^2 - f_2^2} B_{r,w} - \lambda_1 k_r \tag{19}$$

$$c\overline{\delta_L^s} = c\delta_L^s + B_{IF}^s - \frac{cf_2}{f_1^2 - f_2^2} B_w^s - \lambda_1 k^s \tag{20}$$

where $B_{r,IF}$ and B_{IF}^s are IF phase biases, $B_{r,w}$ and B_w^s are receiver and satellite wide-lane biases, and k is an integer value used for estimating phase-clock correction as much as close to code-clocks correction, actually with differences less than half a cycle of L1. As a result, such satellite clock products can contribute to the IGS, because IGS standards are practically

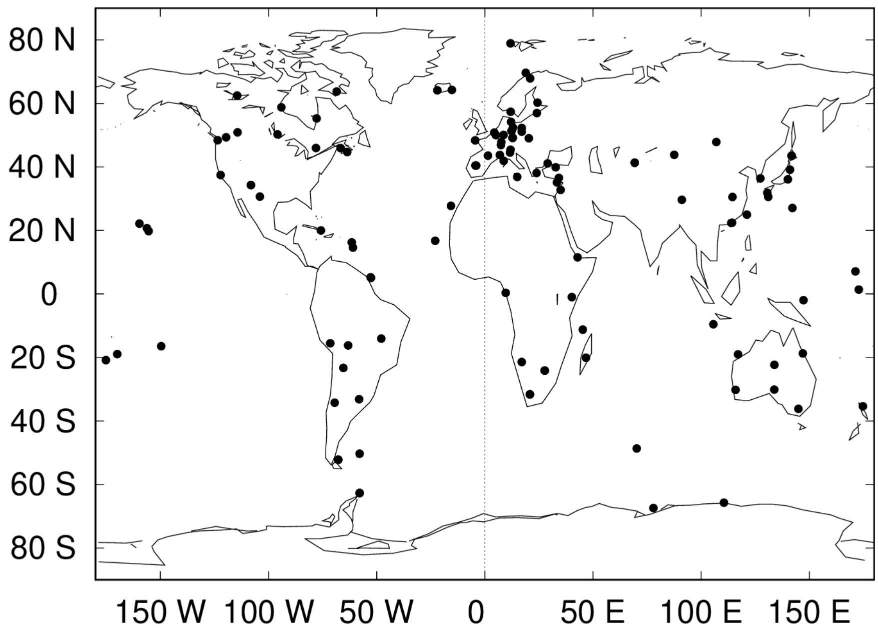

Fig. 1 Globally distributed permanent IGS stations used for the FCB estimation.

fulfilled. Receiver clock of a single station is used as a reference to resolve the system singularity.

Only phase clocks are provided to users, hence uncertainty of up to half a cycle of L1 occurs when applying for code observations. Considering code observation accuracy, this issue can be accepted, and clock errors increase pseudorange residuals only. The wide-lane satellite bias is applied in satellites clocks estimation, therefore, the same biases must be used also at a user side to adhere to the consistency.

6. PHASE BIASES PRODUCT ASSESSMENT

We estimated wide-lane and narrow-lane biases for GPS satellites at the Geodetic Observatory Pecny (GOP) for days 001-030 of the year 2018. Observations from globally distributed IGS permanent stations (Fig. 1) were utilized together with precise orbits and clocks provided by the GeoForschungsZentrum Potsdam (GFZ), which contributes to the MGEX with the GBM acronym. One set of wide-lane biases was estimated on a daily basis and used as a priori values for next day. Narrow-lane biases were estimated from a separate process when using already known WL biases.

The FCB product was then exploited for the PPP AR pseudo kinematic solution for twelve IGS stations: ZIM200CHE, WTZR00DEU, POTS00DEU, HOFN00ISL, BRUX00BEL, KIRU00SWE, GOP600CZE, JFNG00CHN, CUT000AUS, WROC00POL, YELL00CAN, ISTA00TUR. Remaining fractional parts of the wide-lane ambiguities were calculated after applying the wide-lane biases and when fulfilling following criteria: satellite elevation is greater than 20 degrees, and at least 20 epochs were used for averaging the MW combination.

We calculated the same residuals also when applying wide-lane biases provided by the CNES/CLS GRM accompanying their satellite clock product. Figure 2 demonstrates histograms for both residuals of all stations from the 30 days. We can see a very good agreement between residuals based on both wide-lane products. Standard deviation of both remaining fractional parts of wide-lane ambiguities are equal 0.11 cycle, which proves that a simple rounding to the nearest integers is applicable. It should be noted, that a direct comparison of wide-lane biases is not possible due to a different datum defined by a reference receiver. The comparison can be done only on the basis of between-satellites single differences.

Fixed integer wide-lane and float IF ambiguities were used for generating float narrow-lane ambiguities. The narrow-lane phase biases of our GOP GBM products were applied to recover their integer nature. When using CNES/CLS GRM, the NL ambiguities were not corrected because the essential biases were already included in the satellite's clocks. Remaining fractional parts of the narrow-lane ambiguities were calculated using both kind of precise PPP AR products, and Figure 3 shows the achieved histogram. We can see again very good agreement indicating a comparable accuracy of both products. A standard deviation of fractional parts of narrow-lane ambiguities is equal to 0.12 cycles, therefore a narrow-lane integer ambiguity resolution is possible.

Utilizing products for PPP AR from different providers can be a challenging task. Different strategies with many assumptions are implemented on

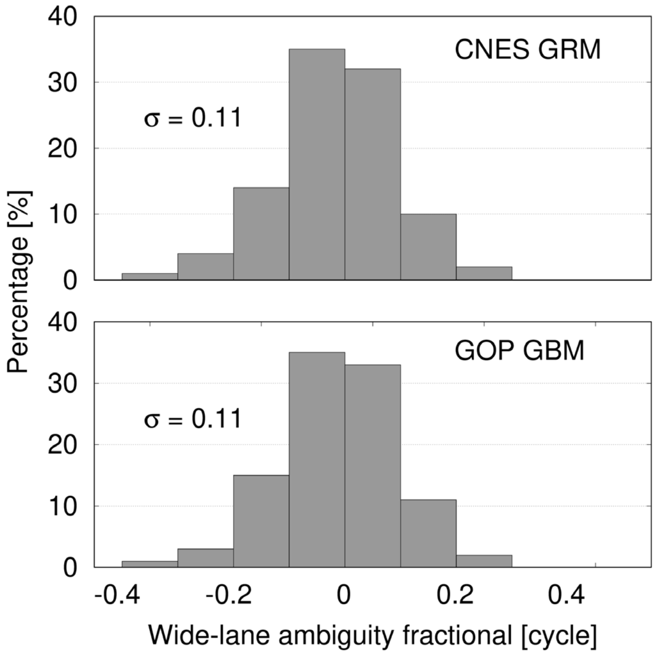

Fig. 2 Histogram of remaining fractional parts of wide-lane ambiguities after applying the wide-lane biases provided by CNES/CLS (top plot) and those estimated at GOP (bottom plot).

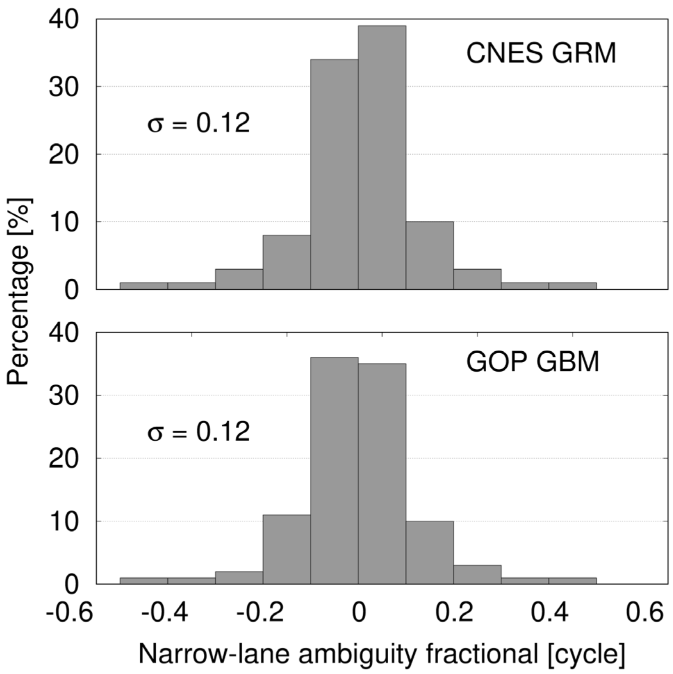

Fig. 3 Histogram of remaining fractional parts of narrow-lane ambiguities after applying the satellite integer recovery clock products provided by CNES/CLS (top plot) and the narrow-lane biases estimated et GOP (bottom plot).

Table 1 Processing models and used precise products.

Precise orbits/clocks	GFZ from the MGEX project (GBM)
Code biases	German Space Operations Center (DLR/GSOC)
Ionosphere	Eliminated using the ionosphere-free combination from L1/L2
Troposphere	Zenith hydrostatic delay (ZHD): Saastamoinen model using atmospheric pressure from Global Pressure and Temperature model (GPT) Zenith wet delay (ZWD): estimated Random walk process noise for ZWD: 6mm/sqrt(hour)
Phase center	igs08_1915.atx ANTEX file

a server side, because phase biases in their original form cannot be estimated due to several singularities in normal equations. To exploit products accuracy completely, the same models must be implement on the user side, and provided corrections must be applied correctly. Satisfying these requirements can be difficult because of necessary documentation, and limited information are usually available only in scientific papers. In commercial domain, providers of PPP corrections distributes usually also a client software, which is currently the best way to provide an internal consistency within implemented models. The standardization for phase biases estimation and utilization has become very desirable, and it should be solved optimally within the IGS.

When comparing our GOP GBM phase biases with the CNES/CLS GRM products, we had to be sure that attitude modeling and the axes orientation of the spacecraft body are defined correspondingly. We have implemented nominal yaw attitude model without checking beta and orbit angles. Satellites in eclipsing and post shadow periods were excluded, because we did not have any information about models used on the CNES/CLS side. Due to the good agreement between remaining fractional parts of ambiguities, we expect that used models were implemented similarly on both GOP and CNES/CLS analyses centers sides.

7. BACKWARD SMOOTHING

When phase biases are not available, and data can be processed in an offline mode, a backward smoothing algorithm can be applied. The standard PPP float solution using the Kalman filter is performed in the first step. Predicted and updated state vectors together with corresponding covariance matrices are stored every epoch for a further sequential reprocessing. Results from the last epoch is used for initializing parameters, and the smoothing algorithm is employed for recalculating the state vector in the backward direction. The benefits of such algorithm are that ambiguities are initialized precisely on the basis of the previous Kalman filter, and also that observations from all epochs contribute to estimating the state vector at every epoch. In other words, not only previous but also past observations are utilized in a solution at a particular epoch. As a result, the achieved parameters time series are stable and without any initial convergence.

The smoothing algorithm for a state vector can be expressed in the recursive formula:

$$x_{k|N} = x_{k|k} + C_k(x_{k+1|N} - x_{k+1|k}) \quad (21)$$

$$C_k = \Sigma_{k|k} \Phi \Sigma_{k+1|k}^{-1} \quad (22)$$

where $x_{k|N}$ is smoothed state vector, $x_{k|k}$ and $\Sigma_{k|k}$ are updated state vector and covariance matrix, and $x_{k+1|k}$, $\Sigma_{k+1|k}$ are predicted state vector and covariance matrix. C is usually called smoother gain. Φ is the identity in our study because an equation of receiver motion was not defined. The recursion starts at N-th epoch and continues down to the first one.

The backward smoothing is not applicable in real time, but it can be used in many applications as an alternative method for the traditional least squares adjustment (LSQ). In case of LSQ, all observations from a particular period usually contribute to a one set of parameter. However, when using the backward smoothing, observations from the whole processing period are utilized for estimating parameters at every epoch. As a result, a high-rate processing is still highly efficient compared to the LSQ algorithm.

8. COMPARISON OF PPP FLOAT, FIXED, AND SMOOTHED SOLUTIONS

The three strategies described in the previous chapters differ in a number of aspects, such as achieved accuracy, essential input products, or dependency on GNSS observations from permanent stations. We have focused on a precision of estimated pseudokinematic coordinates of several permanent stations to demonstrate a potential of the strategies. The list of stations is the same as we used in the Chapter 6 for the phase biases assessment.

Table 1 summarizes used models and products in the processing. The precise products were used in the PPP solution as well as in the network solution for fractional cycle biases estimation. The GBM clocks fulfills the IGS standards providing that IF code bias from P1 and P2 code signals are included in the satellite clocks. To be consistent with this standard, we applied differential code biases. This study is intended for demonstrating a precision of PPP solution when

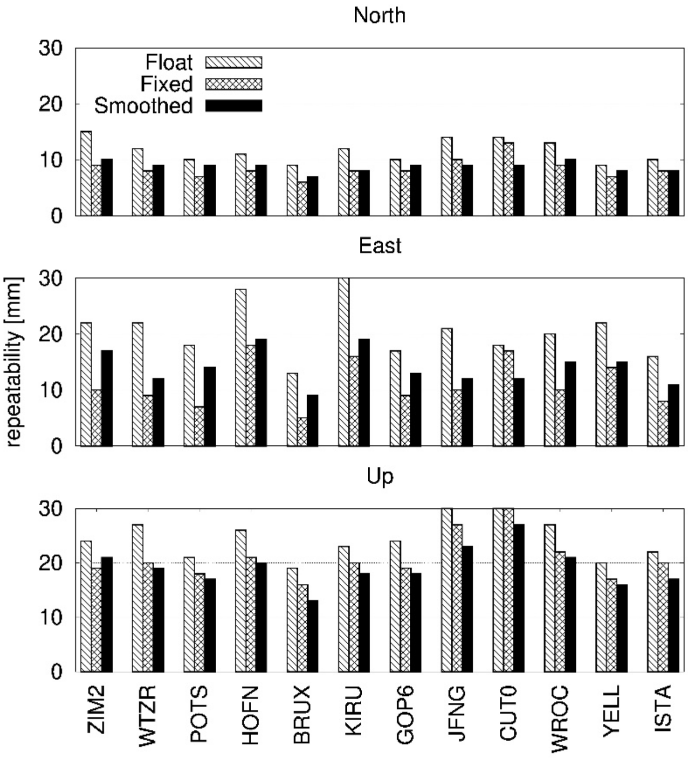

Fig. 4 Repeatability of the North, East, and Up coordinates components of all included stations from kinematic solutions when ambiguities are estimated as float values, resolved as integer values, and smoothed by the backward filter

using different strategies for ambiguity handling. Therefore, only GPS observations were used.

Pseudokinematic coordinates of the twelve IGS stations were estimated on a daily basis during 001-030 days of the year 2018. A precision is represented by the repeatability in North (N), East (E), and Up (U) coordinates components and calculated from the whole processing period. The repeatability is defined as a standard deviation of coordinates expressing precision with respect to their mean values. The stations are mounted in the permanent positions, therefore, the solution stability and precision can be expressed using such parameter. Figure 5 demonstrates resulted statistics as mean values from all stations and all days. The precision of positioning of individual stations can differ depending on a data quality and environmental conditions. Such dependency is visible in Figure 4, which shows achieved coordinates precision of individual included stations. An impact of the integer ambiguity resolution can be observed mainly on the East coordinates component, which was improved by 48 % compared to the PPP float solution. The North and Up components are improved by 33 % and 16 %, respectively. If carrier-phase biases are not available, or their calculation is not possible, a technically simpler strategy such as the backward smoothing can be used. The precision of the kinematic coordinates of the PPP float solution was improved by 25 %, 33 %, and 24 % of N, E, U coordinates, respectively. As a result, the precision of the kinematic coordinates estimated by the backward smoothing are comparable to those obtained by the PPP AR solution. Nevertheless, the most significant advantage is that no additional carrier-phase bias products are necessary.

The site coordinates precision improvement is gained due to reducing a convergence period and making time series more stable. An example can be seen in Figure 6 showing coordinates of the station JFNG in China on the doy 002 of the year 2018. Comparing the float and fixed solutions, convergence is shorter, and the East coordinate is more stable from 5 hours until 9 hours of the day in case of the fixed ambiguities. If ambiguity resolution fails, the fixed and float solution coincide as can be seen around 2:15 at the East coordinate. The smoothed solution is very precise and stable during the whole presented interval.

We analyzed also the length of the initial convergence period. We divided the first two hours of every daily coordinate solution into 10 minutes long subintervals, and we calculated standard deviations of coordinates from every interval. The precision in the

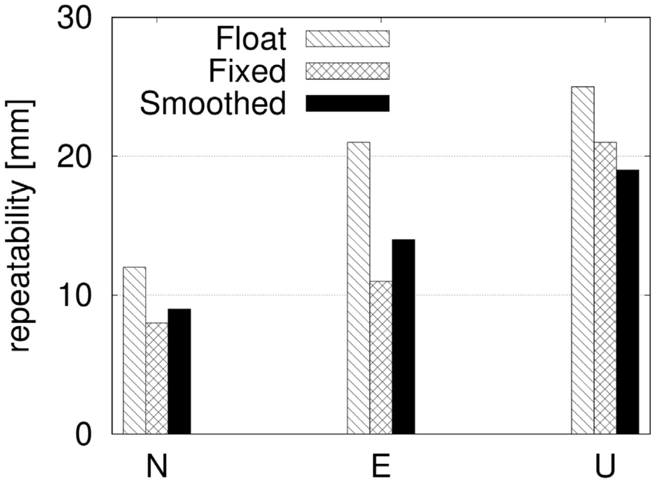

Fig. 5 Mean repeatability of the North, East, and Up coordinates components from kinematic solutions when ambiguities are estimated as float values, resolved as integer values, and smoothed by the backward filter.

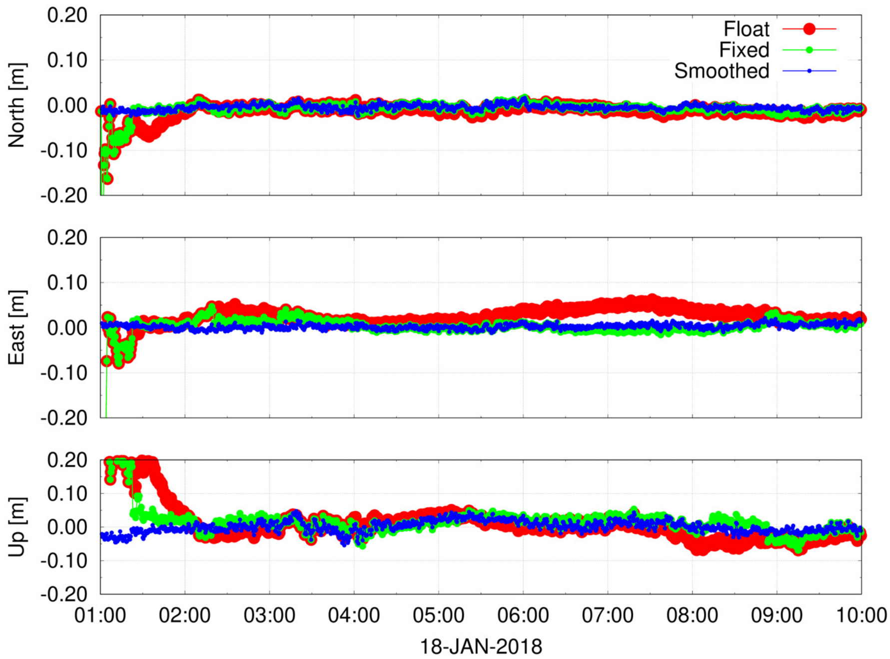

Fig. 6 Coordinates time series of the station JFNG achieved from the standard PPP float solution, PPP with integer ambiguity resolution, and PPP improved by the backward smoothing.

first 10 minutes is the worst one, and it is being increased in the following intervals due to the initial convergence. Figure 7 shows improving N, E, U coordinates standard deviations during the first two hours of daily solutions. The mean values from all stations and all days are used in this figure. Figure 7 shows that the backward smoothing eliminated the convergence, therefore, the standard deviation is very stable from the first minutes. When ambiguities were resolved as integers, the coordinates convergence was accelerated after 20 minutes when comparing with the float solution. The convergence time was most significantly improved in the East coordinates component, which corresponds with Figure 5.

Figure 8 shows the standard deviations of East coordinates of individual stations after 30 minutes of processing. The precision improvement of fixed solution compared to the float one after this 30 minute

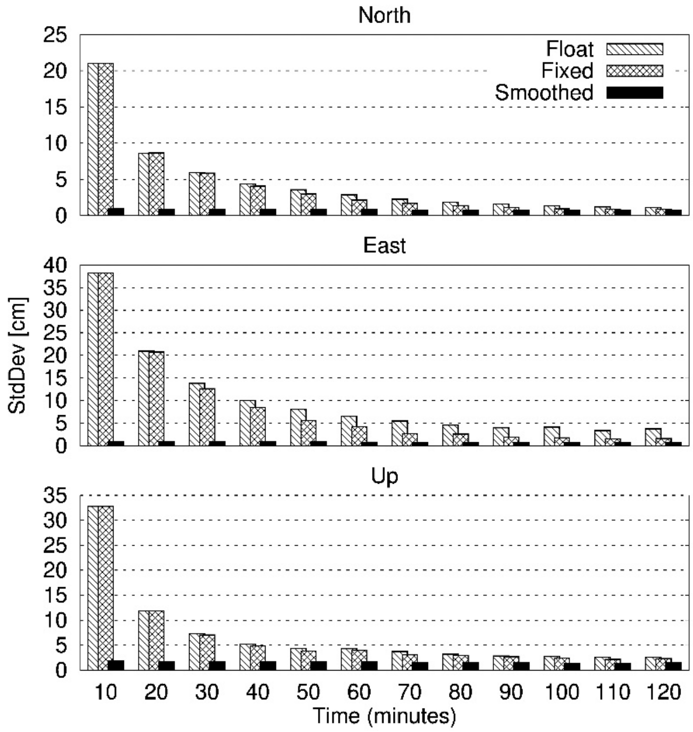

Fig. 7 The standard deviations of coordinates from the first two hours of daily processing. The improving precision demonstrates initial convergence of float, fixed, and smoothed PPP solution.

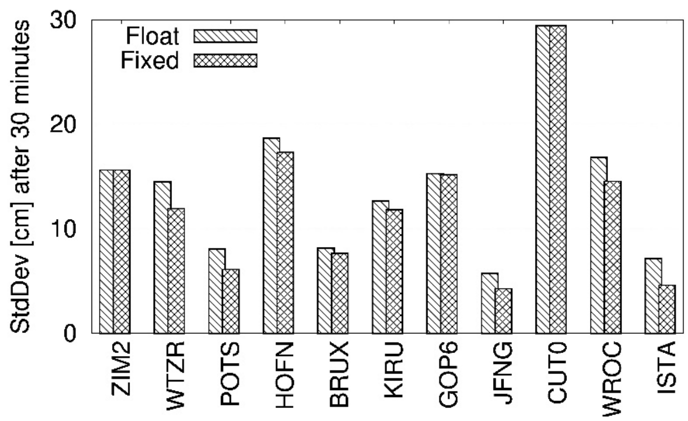

Fig. 8 East coordinate component standard deviations of individual stations after 30 minutes of processing gained by the float and fixed solutions

is different among stations. The plot demonstrates that ZIM2, GOP6, and CUT0 stations needed more than 30 minutes for ambiguity fixing. The reason for longer time to first fix can origin in different quality of code observations, different observation conditions, or different observation biases quality. Nevertheless, it needs further investigation and software improvement.

9. HIGH-RATE POSITIONING

GNSS positioning is frequently used in geodynamics or seismology for site stability monitoring. High-rate observations are usually used for such purposes. To present potential of integer ambiguity resolution and the backward smoothing approach in this field of research, we have processed 10 Hz GPS observations collected at the STRF station situated on the Strofades Island in south Greece and

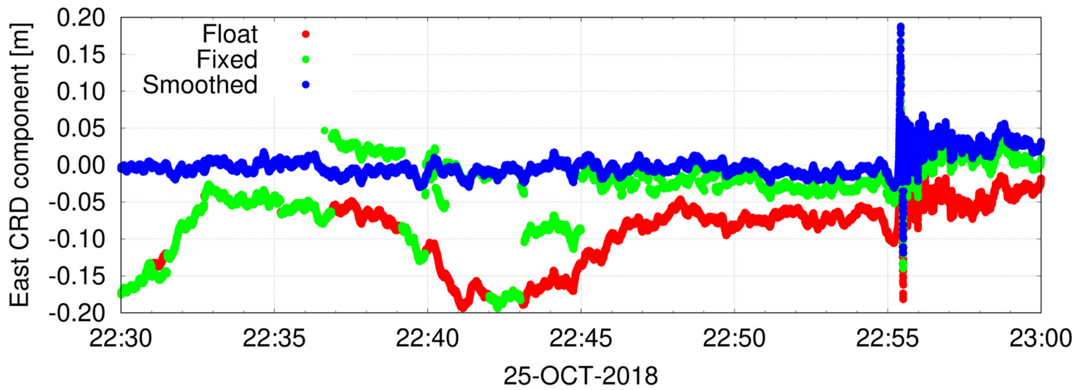

Fig. 9 High-rate coordinates of the STRF station before and during the earthquake detected at about 22:55 on the October 25, 2018. The PPP, PPP AR, and PPP smoothed solutions are compared.

equipped with the GNSS receiver (INGV, 2016). The station was affected by the earthquake which occurred about 50 km away at 22:54:52 UTC on October 25, 2018. The epicenter of the earthquake with M 6.8 was about 30 km south-west of Lithakie and 133 km from the Patras city.

We processed one and half hour of observations starting at 22:00:00 and ending at 23:30:00 and utilizing all the three comparing strategies: PPP float, PPP fixed, and PPP smoothed. Figure 9 demonstrates the East component of estimated coordinates as an example of comparison all approaches. The smoothed solution (blue dots) is very stable until about 22:55, when the station started to move due to the earthquake. On the other hand, the coordinates achieved with the PPP float solution (red dots) fluctuates in time, because the IF ambiguities were still not very precise. When trying to resolve ambiguities as integers, we can see a first attempt to fix after 22:37, however, reliable fixing was reached only after about 22:45. Better agreement with the smoothed solution is visible from this epoch. It corresponds to the statistics obtained from the processing of permanent stations presented above.

The time to first fix was quite long, which needs our further investigation and the software tuning, however, a potential reason can origin in a limited number of tracked satellites resulting in a slower convergence of IF ambiguities.

The achieved time series could be analyzed thoroughly, but it is out of the scope of this paper. We only want to show, that the PPP AR can give comparable results as the backward smoothing, when ambiguities are still estimated as float values only. The AR has many pitfalls, such as distinguishing incorrect ambiguity fixing, accuracy of input products for carrier-phase biases, and their correct application. Moreover, the backward smoothing can eliminate every re-convergences caused by resetting ambiguities due to a long data gap or a high number of cycle slips.

10. CONCLUSION

We have estimated wide-lane and narrow-lane phase biases of the GPS satellites when processing GPS observations from globally distributed IGS permanent stations and exploiting precise orbits and clocks from GFZ. The estimated phase biases were used in the PPP AR solution to estimate pseudokinematic coordinates of the twelve IGS stations. The PPP AR solution was performed also when using the IRC product provided by CNES/CLS. Both kinds of phase biases were assessed on the basis of comparing remaining fractional parts of the estimated wide-lane and narrow-lane ambiguities obtained from the PPP AR solution. Very good agreement was demonstrated by obtaining the same standard deviations of 0.11 and 0.12 cycles for wide-lane and narrow-lane remaining fractional parts, respectively. We can conclude that the publicly available CNES/CLS IRC products are suitable for the AR when using fully independent software and strategy. Nevertheless, a consistency of applied models, such as satellite attitudes, phase centers and variations, axes orientation of spacecraft body, windup effect, etc. is very important.

The coordinates of the IGS permanent stations were estimated with the three PPP solutions using different approaches for the ambiguity estimation. The traditional PPP float solution was compared with the PPP AR and with the PPP improved by the backward smoothing. The improvement of the mean N, E, U coordinates repeatability when resolved ambiguities as integer values was 33 %, 48 %, and 16 %. The reason for this improvement origins in shortening the convergence period and making the time series more stable. After employing the backward smoothing, coordinates precision was enhanced by 25 %, 33 %, and 24 %, respectively. The time series of smoothed coordinates were very stable with completely eliminated initial convergences and any other potential re-convergences. The backward smoothing can improve the float solution similarly to the integer ambiguity resolution, however, without a need of any additional advanced products, which is currently an important advantage of the algorithm.

We also estimated high-rate kinematic coordinates of the station STRF, which was affected by the earthquake. The results correspond to the experience from the processing permanent stations. The fixed and smoothed solution reached significantly better agreement compared to the float solution. In the post processing mode, the smoothing solution can be more suitable in many applications than the PPP AR requiring more complicated implementation and additional products. Moreover, since the initial convergence is completely eliminated, resulting coordinates time series can be studied from a beginning of a processing period. A shortening of the initial convergence in PPP AR is possible only when using precise atmospheric corrections, however, not available for many areas.

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