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GRAVIMETRIC GEOID MODEL DETERMINATION ON CENTRAL UKRAINE AREA USING COMBINATION OF LSC AND TRUNCATED SHA METHODS

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ABSTRACT

In this paper we investigate the possibility of combining least squares collocation (LSC) and spherical functions (spherical harmonic analysis SHA) with real eigenvalues methods for calculating the geoid model. A comparison of local analytical covariance functions calculated on the basis of Legendre polynomials and polynomials with real eigenvalues is carried out. We computed geoid model for the region of Central Ukraine using the proposed method and assessed its accuracy. The input data for the computation of model are gravity anomalies, obtained after filtering the results of digitization of gravimetric maps by the 3σ criterion, functionals of the Earth's gravity field from the global gravity model EGM2008 and GNSS leveling data. The standard deviation was found between the model values of the geoid height and the values, obtained from GNSS leveling at the points of the state geodetic network. The accuracy of the geoid model obtained using the proposed method is approximately 2 cm. Such accuracy is primarily conditioned by the quality of initial data. To increase the accuracy of the model it is necessary to carry out a complex of gravimetric works in the studied region.

INTRODUCTION

An integral part of solving modern scientific and applied problems of geodesy, geophysics, global geodynamics etc. is the modeling of the Earth's gravitational field, the interpretation of the obtained results and their direct use. The active development of new geodetic and geophysical technologies, increasing the accuracy of measuring devices, technological challenges of modern times require the improvement of methods of the Earth's gravity field modeling. In particular, networks of permanent GNSS stations actively develop in Ukraine today (Tretyak and Brusak, 2022). We need to compute a high-precision geoid model for determination of gravity-dependent heights by satellite methods.

On a global scale Legendre's spherical functions are mostly used as the basic system of functions to calculate gravity field models. Such parameterization is considered standard for representing the global gravitational field of the Earth.

On a regional scale other methods are used to solve similar problems based on model and operational approaches of physical geodesy. Such methods include least square collocation (LSC) (Tscherning, 2015), calculation of geoid heights based on the Stokes formula (Sideris, 2005), use of radial basis functions (Marchenko et al., 2015) etc.

Another method of modeling the regional gravity field of the Earth is spherical cap harmonic analysis, which is based on the model approach of physical geodesy. This method is proposed in (Haines, 1985). It involves transforming the input data into a spherical cap and using Legendre's spherical functions of real degree as a base system of functions. Such functions are solutions of Laplace equation and form two orthogonal systems of functions on the cap of the sphere. However, in general these functions are non-orthogonal. They do not have recurrence relations and can be found by expansion into an infinite hypergeometric series (Haines, 1988). Based on the SCHA method, a number of similar methods have been developed, for example ASHA (De Santis, 1992), R-SCHA (Thébault et al., 2006), STHA (Dzhuman, 2017), TOSCHA (De Santis, 1991) etc.

The aim of this work is to compare the LSC methods and spherical functions with real eigenvalues, namely the use of the corresponding polynomials as a basic system of functions to represent the local analytical covariance function, as well as the approbation of this method for calculating a high-precision geoid model.

METHOD

The use of the SCHA method for modeling the regional gravity field of the Earth involves the

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expansion of the gravity potential in a series by spherical functions with real eigenvalues (De Santis and Torta, 1997):

$$V = \sum_{k=1}^{K_{max}} \sum_{m=0}^{k} \left(\bar{a}_{km}\bar{R}_{km}(\theta,\lambda) + \bar{b}_{km}\bar{S}_{km}(\theta,\lambda)\right), \tag{1}$$

where $\bar{a}$ and $\bar{b}$ are normalized unknown coefficients of the model, K_{max} is the maximum order of the model, functions R and S are next:

$$\begin{aligned} R_{km}(\theta,\lambda) &= P_{n_k m}(\cos\theta)\cos(m\lambda), \\ S_{km}(\theta,\lambda) &= P_{n_k m}(\cos\theta)\sin(m\lambda). \end{aligned} \tag{2}$$

Formulas for finding the norm of these functions can be found in (Haines, 1985; Hwang and Chen, 1997). Functions $P_{n_k m}(\cos\theta)$ are obtained after using boundary conditions on the differential equation

$$\frac{d}{d\mu}\left[(1-\mu^2)\frac{dP_{mn}(\mu)}{d\mu}\right] + \left[n(n+1) - \frac{m^2}{1-\mu^2}\right]P_{mn}(\mu) = 0, \tag{3}$$

In the case of a spherical trapezium (STHA method) functions (2) will take the form (Dzhuman, 2017):

$$\begin{aligned} R_{km}(\theta,\lambda) &= P_{km}(\cos\theta)\cos\left(2\pi m\frac{\lambda-\lambda_{min}}{\lambda_{max}-\lambda_{min}}\right), \\ S_{km}(\theta,\lambda) &= P_{km}(\cos\theta)\sin\left(2\pi m\frac{\lambda-\lambda_{min}}{\lambda_{max}-\lambda_{min}}\right). \end{aligned} \tag{4}$$

and functions $P_{n_k m}(\cos\theta)$ can be found from the formula

$$P_{km} = \sin^m(\theta-\theta_{min})\, F\left(m-n_k, n_k+m+1, 1+m, \frac{1-\cos(\theta-\theta_{min})}{2}\right),\ \ if\ \theta_{min} \le \theta \le \theta_{mean}$$

$$P_{km} = (-1)^{k+m}\sin^m(\theta_{max}-\theta)\, F\left(m-n_k, n_k+m+1, 1+m, \frac{1-\cos(\theta_{max}-\theta)}{2}\right),\ \ if\ \theta_{mean} \le \theta \le \theta_{max} \tag{5}$$

where k and m are integer numbers, θ_{mean} is average value, that is $\theta_{mean} = (\theta_{min} + \theta_{max})/2$.

Then the basic system of functions will be orthogonal. We can use quadrature formulas similar to Neumann's second method (Dzhuman, 2018) for finding the coefficients of the model. In this case, all elements of the matrix of normal equations can be neglected except the elements of the main diagonal. For a grid with node coordinates θ_i $(i = \overline{1,\dots,N})$, λ_j $(j = \overline{1,\dots,M})$ any element of the main diagonal of the matrix of normal equations can be found as follow

$$n_{qq} = \frac{M}{2-\delta_m}\sum_{i=1}^{N}\omega_i P_{km}^2(\theta_i), \tag{6}$$

where n_{qq} is element of the main diagonal of the matrix of normal equations, δ_m is the Kronecker symbol:

$$\delta_m = \begin{matrix} 1,\ if\ \ m = 0, \\ 0,\ if\ \ m \neq 0. \end{matrix} \tag{7}$$

Let us consider the possibility of using spherical functions with real eigenvalues in the LSC method. The optimal way to represent the local analytic covariance function (ACF) between points P and Q is its series expansion by Legendre polynomials (Moritz, 1980):

$$\tilde{K}(P,Q) = \sum_{n=N_{max}+1}^{\infty}\tilde{k}_n\left(\frac{R^2}{rr'}\right)^{n+1}P_n(\cos\psi), \tag{8}$$

where R is mean radius of the Earth, ψ is spherical distance between points P and Q, r and r' are spherical coordinates of points P and Q respectively, P_n are Legendre polynomials, $\tilde{k}_n$ are expansion coefficients. To determine the coefficients $\tilde{k}_n$ the well-known Cherning-Rapp model was used (Tscherning, 2015):

$$\tilde{k}_n = \frac{A}{(n-1)(n-2)(n+B)}s^{n+2}, \tag{9}$$

and s is given by the formula

$$s = \frac{R_B^2}{R^2}, \tag{10}$$

where R_B is Bjerhammar sphere radius.

In the Cherning-Rapp model four parameters are unknowns: N_{max}, A, B and s. They are determined by the fitting based on the empirical covariance function (ECF).

Usually, the covariance function of the disturbance potential is taken as the function of the reproducing kernel $K(P, Q)$:

$$K(P,Q) = \sum_{n=N_{max}+1}^{\infty} k_n \left(\frac{R^2}{rr'}\right)^{n+1} P_n(cos\psi). \quad (11)$$

Practically all functionals of the gravity potential can be represented through the function of the disturbance potential using linear functionals. In this case the arbitrary covariance function C can be represented as (Moritz, 1980):

$$C_{ij}(P,Q) = L_i^P L_j^Q K(P,Q), \quad (12)$$

where L_i^P is i-th linear functional at point P, L_j^Q is j-th linear functional at point Q. There is a dependence between the coefficients of the covariance functions of the geoid heights and the disturbance potential:

$$k_n = \gamma_0^2 \tilde{k}_n. \quad (13)$$

It is not difficult to find expressions for covariance functions (Moritz, 1980)

$$\begin{aligned} \tilde{C}(P,Q) &= cov[T(P), \Delta g(Q)], \\ C(P,Q) &= cov[\Delta g(P), \Delta g(Q)]. \end{aligned} \quad (14)$$

They can be obtained in the form

$$\begin{aligned} \tilde{C}(P,Q) &= \sum_{n=N_{max}+1}^{\infty} \frac{n-1}{r'} k_n \left(\frac{R^2}{rr'}\right)^{n+1} P_n(cos\psi), \\ C(P,Q) &= \sum_{n=N_{max}+1}^{\infty} \frac{(n-1)^2}{rr'} k_n \left(\frac{R^2}{rr'}\right)^{n+1} P_n(cos\psi). \end{aligned} \quad (15)$$

For the expansion of the considered covariance functions in a series by STHA-polynomials, we first set a hard condition $\theta_{min} = 0$. In this case the function of the reproducing kernel (the covariance function of the disturbance potential) $K(P, Q)$ we can present as

$$K(P,Q) = \sum_{l=L_{min}}^{L_{max}} k_l \left(\frac{R^2}{rr'}\right)^{n_l+1} P_{n_l}(cos\psi). \quad (16)$$

The coefficients of the expansion into a series of the covariance function of the geoid heights $\tilde{k}_l$ can be found using the formula (13). Formulas (15) will be next:

$$\begin{aligned} \tilde{C}(P,Q) &= \sum_{l=L_{min}}^{L_{max}} \frac{n_l-1}{r'} k_l \left(\frac{R^2}{rr'}\right)^{n_l+1} P_{n_l}(cos\psi), \\ C(P,Q) &= \sum_{l=L_{min}}^{L_{max}} \frac{(n_l-1)^2}{rr'} k_l \left(\frac{R^2}{rr'}\right)^{n_l+1} P_{n_l}(cos\psi). \end{aligned} \quad (17)$$

In formula (15) the expansion is carried out using Legendre polynomials. In turn in formula (17) the expansion is performed using polynomials with real eigenvalues. Accordingly, n_l is a real eigenvalue, and l is an integer for ordering n_l.

Calculation of the coefficients of the covariance function of the disturbance potential is also possible using the Cherning-Rapp model in the following version:

$$k_l = \frac{A}{(n_l-1)(n_l-2)(n_l+B)} s^{n_l+2}. \quad (18)$$

Comparison of covariance and cross-covariance functions, decomposed into a series by Legendre polynomials and polynomials with real eigenvalues, was carried out to test the proposed method as well as geoid model was calculated using the proposed functions and the accuracy of the obtained model was evaluated.

DATA

We chose the Central Ukraine, namely Odesa and Vinnytsia regions, as the research area. Input data consists of number of heterogeneous information:

- terrestrial gravity anomalies Δg, obtained after filtering the results of digitization of gravimetric maps by the 3σ criterion (Marchenko et al., 2015);
- functionals of the Earth's gravity field from the global gravity model EGM2008;
- GNSS leveling data.

Map of gravity anomalies Δg, obtained after filtering the results of digitization of gravimetric maps, is shown in Figure 1.

Researched region (spherical trapezoid) must be completely and uniformly covered by the input data for a reliable determination of geoid heights. Therefore, in places where gravity anomalies Δg are represented in insufficient quantities (mostly the Black Sea area and Moldova area), we used anomalies, calculated from the model of the global gravitational field of the Earth EGM2008 (Barthelmes and Köhler, 2016; Pavlis et al., 2008).

We computed gravity anomalies Δg_{sys} and contribution of geoid heights N_{sys}, corresponding to the long-wave features of the gravity field, as a systematic component according to the global model of the Earth's gravity field EGM2008 up to 360 degrees/order (Ince et al., 2019) for using the procedure "Remove-Compute-Restore".

Since the main type of input data for building geoid model for the area of Vinnytsia and Odesa regions is terrestrial gravimetry data, the use of the EGM2008 model gives the best results precisely in the case of 360 degrees/order, which corresponds to the field resolution of (30'×30'). Applying its version to a higher degree and order does not improve the solution for the geoid in long and medium waves.

Residual differences Δg_{res} were calculated between gravity anomalies, obtained as a result of digitization of gravimetric maps, and gravity anomalies according to the EGM2008 model up to 360 degree/order:

$$\Delta g_{res} = \Delta g - \Delta g_{sys}. \quad (19)$$

Residual differences Δg_{res} are shown in Figure 2.

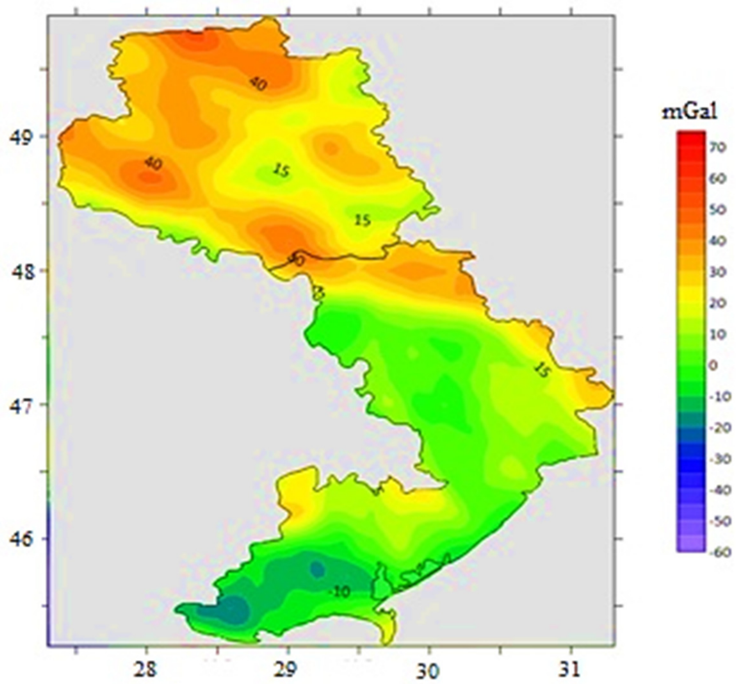

Fig. 1 Map of gravity anomalies Δg, obtained after filtering the results of digitization of gravimetric maps.

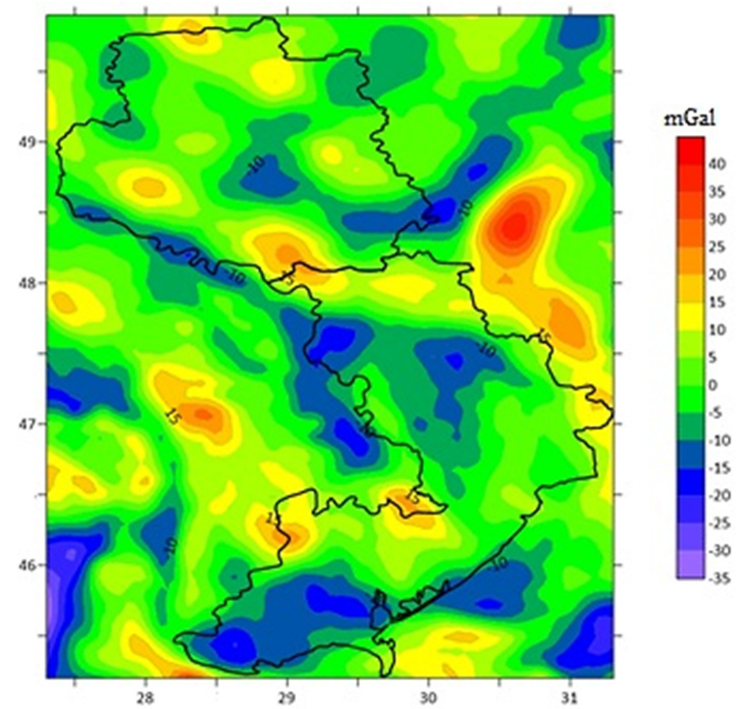

Fig. 2 Residual differences of gravity anomalies Δg_{res}.

The input data are reduced to the uniform grid for determining the elements of the normal equation matrix according to formula (6) using the least square prediction. We found the covariance functions $cov[T(P),T(Q)]$, $cov[N(P),N(Q)]$, $cov[T(P),\Delta g(Q)]$ та $cov[\Delta g(P),\Delta g(Q)]$ according to the Cherning-Rapp model (Tscherning and Rapp, 1974) using Legendre polynomials and polynomials with real eigenvalues by the ECF, constructed on the basis of gravity anomalies. The series expansion is from 361 to 1000 orders in the case of Legendre polynomials (15), and from 3 to 12 orders in the case

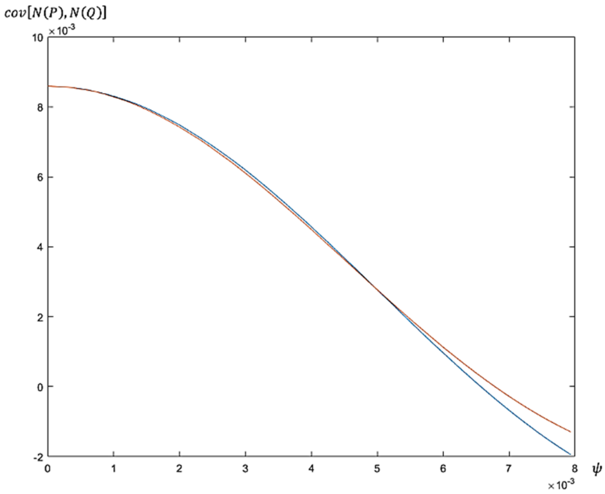

Fig. 3 Covariance function $cov[N(P), N(Q)]$, expanded in a series by Legendre polynomials (blue curve) and polynomials with real indices (red curve), $[m^2]$.

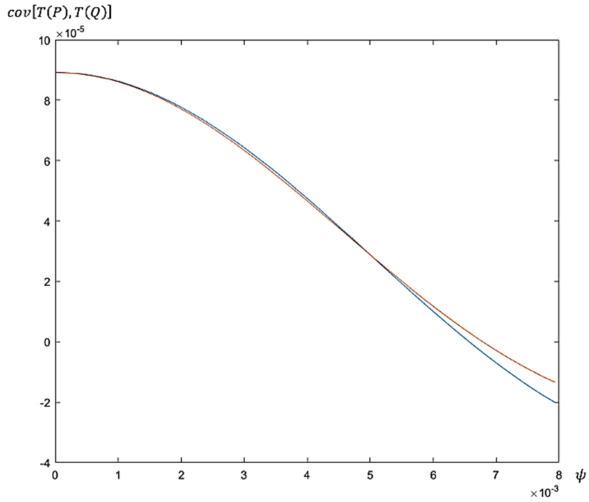

Fig. 4 Covariance function $cov[T(P), T(Q)]$, expanded in a series by Legendre polynomials (blue curve) and polynomials with real indices (red curve), $[m^2 \cdot mGal^2]$.

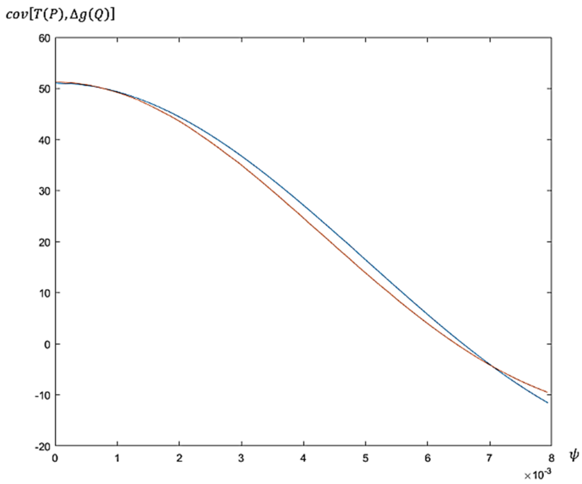

Fig. 5 Covariance function $cov[T(P), \Delta g(Q)]$ expanded in a series by Legendre polynomials (blue curve) and polynomials with real indices (red curve), $[m \cdot mGal^2]$.

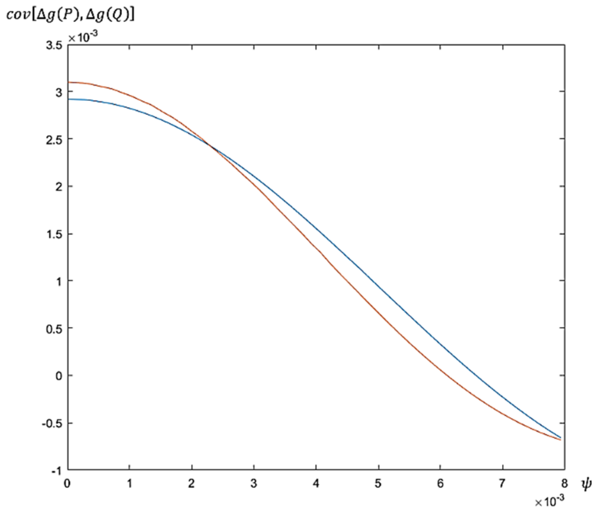

Fig. 6 Covariance function $cov[\Delta g(P), \Delta g(Q)]$, expanded in a series by Legendre polynomials (blue curve) and polynomials with real indices (red curve), $[mGal^2]$.

of polynomials with real indices (17). Figures 3-6 show the obtained covariance functions.

We investigated that the difference between the covariance functions is no more than 5 %. This approach will make it possible to significantly decrease the number of coefficients of analytical covariance function expansion in the. For example, we can use for the studied region only ≈10 coefficients of polynomials with real eigenvalues instead ≈640 coefficients of Legendre polynomials for constructing the ECF (see Figures 3-6).

Data from GNSS leveling at the points of the State Geodetic Network of Ukraine were used to assess the accuracy of the geoid model. This data set consisted of 485 points, including 184 points of the first class, 82 points of the second class and 219 points of the third class (Fig. 7). The accuracy of heights obtained from GNSS-leveling is 10-15 mm.

The use of GNSS leveling data in the construction of a geoid model leads to the need of determination the accuracy of GNSS leveling results in an absolute measure taking into account their connection with geometric leveling of various classes. It is sufficient to estimate the total error of the measured geoid heights using the Molodensky formula if there are mean square errors of the geodetic height according to GNSS measurements and mean square errors of normal heights according to the results of leveling.

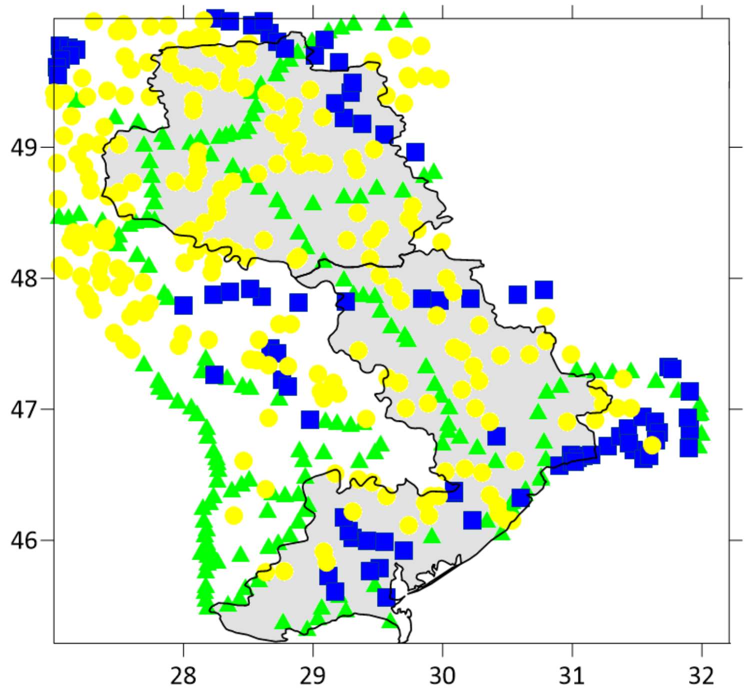

Fig. 7 Data set of GNSS leveling points: I class (green triangles), II class (blue squares) and III class (yellow circles).

Table 1 Eigenvalues n_k of basic functions.

k/m	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0.0												
1	58.1	44.4											
2	92.9	92.9	74.0										
3	134.1	129.5	124.7	101.9									
4	170.5	170.5	163.0	155.1	129.2								
5	210.5	207.6	204.7	194.9	184.5	155.9							
6	247.5	247.5	242.6	237.5	225.8	213.4	182.4						
7	287.0	284.9	282.8	276.1	269.3	256.0	241.8	208.7					
8	324.3	324.3	320.6	316.8	308.7	300.3	285.6	269.8	234.8				
9	363.5	361.9	360.2	355.1	349.9	340.5	330.8	314.8	297.6	260.7			
10	401.1	401.1	398.1	395.1	388.7	382.3	371.8	360.9	343.7	325.1	286.5		
11	440.1	438.7	437.4	433.2	429.0	421.6	414.1	402.5	390.5	372.2	400.6	312.3	
12	477.8	477.8	475.2	472.7	467.5	462.3	453.9	445.4	432.9	419.9	703.8	379.7	889.0

However the situation is significantly complicated by the fact that the average square errors of the leveling results is traditionally estimated depending on the length of the leveling. For this reason, an independent assessment of the total error of the measured geoid heights was carried out using the standard 3σ criterion by comparison with independent models of gravimetric geoids (Marchenko et al., 2015).

RESULTS AND DISCUSSION

We calculated the STHA-model of residual values of geoid heights N_{res} up to the 12th degree/order using the least squares method. The procedure for calculating the STHA-model of geoid is described in detail in (Dzhuman, 2018). For this purpose, the eigenvalues n_k of spherical functions with real degrees (see Table 1) were first calculated according to the formulas given in (Haines, 1988). Figure 8 shows the model of residual values of geoid heights.

The next stage was the "Restore" of the geoid model (Fig. 9).

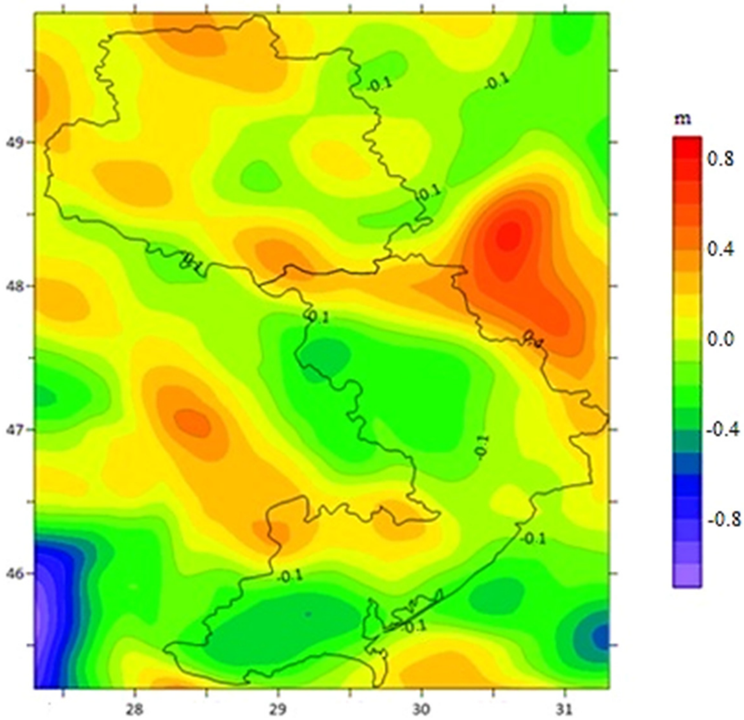

Fig. 8 Model of residual values of geoid heights N_{res}.

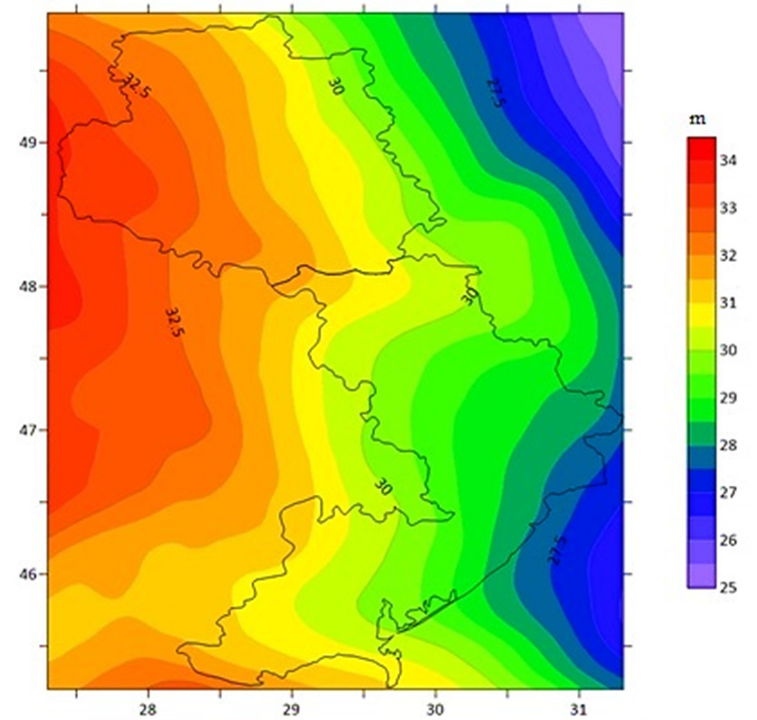

Fig. 9 The model of the gravimetric geoid heights N_{mod}.

Geoid models are mostly characterized by minimum, maximum, average and standard deviations. All the above statistics we calculated for the geoid model EGM2008 up to 360 degree/order N_{sys}, the model of residual values of geoid heights N_{res} and the model of gravimetric geoid heights N_{mod}. These statistics are obtained relative to the reference surface of the global ellipsoid. The results are shown in Table 2.

Table 2 Main statistics of geoid models.

Model	Minimum deviation, *m*	Maximum deviation, *m*	Average deviation, *m*	Standard deviation, *m*
N_{sys}	25.141	33.518	30.364	1.949
N_{res}	-1.056	0.749	0.006	0.232
N_{mod}	25.054	33.730	30.370	1.980

The standard deviation of the residual values of the geoid is ≈0.23 m (see Table 2), which is fully consistent with the accuracy of the EGM2008 model up to 360 degrees/order for the studied region. Usually there is a significant systematic component in the differences between gravimetric and geometric geoids. It is well known that for the territory of Europe such difference is in absolute value about half meter in average. In our case the difference between gravimetric and geometric (obtained from GNSS-leveling) geoids was ΔN = -0.390 *m*. The standard deviation between the model values of the geoid height and the values obtained from GNSS leveling is ≈2.1 *sm*. Such accuracy is primarily conditioned by the quality of initial data. To increase the accuracy of the model it is necessary to carry out a complex of gravimetric works in the studied region.

Since in Ukraine today the official reference system is USK2000, which is based on the Krasovskyi ellipsoid, we recalculated the obtained geoid model to this ellipsoid. The geoid model relative to the Krasovskyi ellipsoid for the territory of the Vinnytsia region is shown in Figure 10 and on the territory of Odesa region is shown in Figure 11.

It should be noted that in general the heights of the geoid for the territory of Ukraine, including the territory of the Vinnytsia and Odesa regions, are significantly smaller relative to the Krasovskyi ellipsoid compared to the GRS-80 ellipsoid. The heights of the geoid relative to the Krasovskyi ellipsoid range from +0.8 m (southeast Odesa region) to +3.6 m (west Vinnytsia region). The heights of the geoid above the GRS-80 ellipsoid range from +28 m to +33 m.

CONCLUSIONS

We proposed to use polynomials with real degrees as the basic system of functions for representing the local analytic covariance function. Covariance and cross-covariance functions were constructed using Legendre polynomials and polynomials with real degrees and compared to test this approach. We investigated that the difference between the covariance functions is no more than 5 %. This approach will make it possible to significantly decrease the number of coefficients of analytical covariance function expansion in the series.

High-precision geoid model was computed for the territory of Central Ukraine (Vinnytsia and Odesa regions) relative to the GRS80 ellipsoid and the Krasovskyi ellipsoid using the proposed method. The accuracy of the obtained geoid model was evaluated in relation to the GNSS leveling data at 485 points. The standard deviation is ≈2.1 *cm*.

We investigated that for the studied region the systematic difference between the heights of the gravimetric geoid and the geoid heights, determined from GNSS leveling, is -0.390 *m*, while for the territory of Europe such difference is on average about half a meter.

The proposed method can also be used to calculate regional models of the Earth's magnetic field and *VTEC* ionospheric parameter models.

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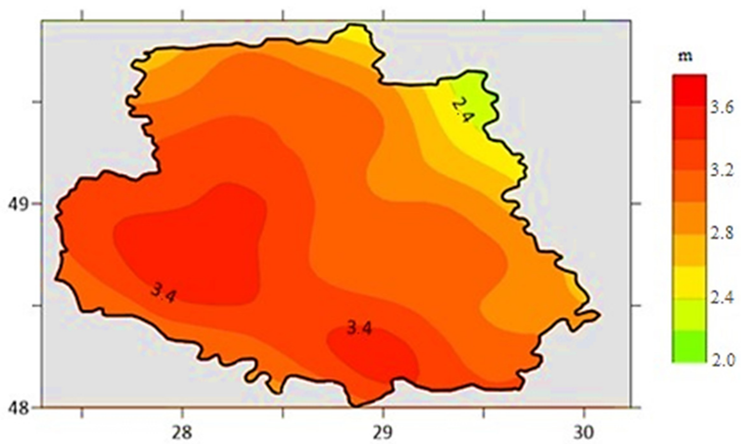

Fig. 10 Geoid model relative to the Krasovskyi ellipsoid on the Vinnytsia region area.

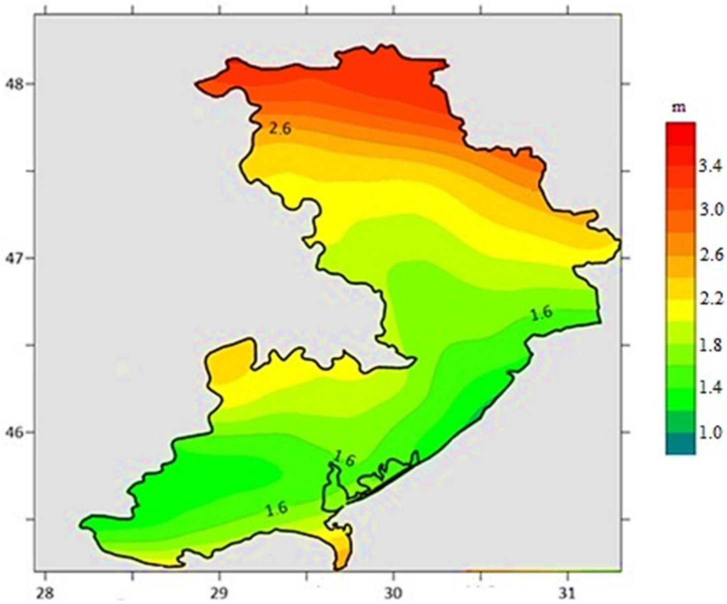

Fig. 11 Geoid model relative to the Krasovskyi ellipsoid on the Odesa region area.

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