A DECORRELATION FILTER BASED ON THE FULL ERROR COVARIANCE OF GRACE GRAVITY FIELD SOLUTIONS

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ABSTRACT

We propose a decorrelation filter (noise-reduction filter) constructed by using the full error covariance matrix information of the spherical harmonic solutions derived from the observations of the Gravity Recovery and Climate Experiment (GRACE) mission. To construct the noise-reduction filter, the filter factors are inversely proportional to the eigenvalues of the covariance matrix. Using the designed noise-reduction filter, we can significantly reduce the north-south stripes in monthly GRACE gravity field solutions. Our study shows that the noise-reduction filter can achieve higher signal-to-noise ratio as compared to other filtering methods under consideration, i.e., P40 and P60 as well as Gaussian smoothing. Using the noise-reduction filter, the estimated mass rates over the entire Antarctica, East Antarctica and West Antarctica (including the Antarctic Peninsula) are -107.38±41.06 Gt/yr, 36.42±14.40 Gt/yr and -147.45±16.78 Gt/yr respectively, consistent with the results from the P40 filter and P60 combined with Gaussian filter.

1. INTRODUCTION

The twin-satellite mission of the Gravity Recovery and Climate Experiment (GRACE) was launched in March 2002 and stopped operation in October 2017 (Tapley et al., 2004). With over 15 years of GRACE observations, the monthly gravity solutions have been widely used to analyze the global mass changes with about 400 km resolution (Tapley et al., 2004) and about 1-cm equivalent water height (Wahr et al., 2004), especially over the Antarctic Ice Sheet (AIS) (Shum et al., 2008; Velicogna et al., 2013; Velicogna et al., 2014; Ju et al., 2014; Chen et al., 2009; Loomis et al., 2020; Zou et al., 2019; Gao et al., 2019).

Due to the instrument errors and polar orbital configuration, there exist obvious north-south stripes in the GRACE time-variable gravity field models. Such stripe errors are successfully eliminated or decreased via some filtering methods, including the Gaussian filter (Jekeli, 1981; Sasgen et al., 2016; Chen et al., 2006; Zhang et al., 2009; Han et al., 2005), the decorrelation and approximately decorrelating filter technique (Chambers, 2006; Swenson and Wahr, 2006; Chen et al., 2008; Duan et al., 2009), the Empirical Orthogonal Functions (EOF) filter (Rangelova et al., 2007; Wouters and Schrama, 2007), the Singular Spectrum Analysis (SSA) (Wang et al., 2011) and the Multichannel Singular Spectrum Analysis (MSSA) (Zotov and Shum, 2010; Guo et al., 2018; Prevost et al., 2019; Wang et al., 2020), the Slepian method (Simons et al., 2006) and the wavelet analysis (Panet et al., 2007; Fengler et al., 2007), as well as the regularization method by using the prior error information of the gravity field model (Kusche, 2007). Kusche et al. (2009) proposed the DDK filter that using no signal covariances in the spherical harmonics (SH) domain, which propagates to a full covariance matrix in the spatial domain. This DDK filter can be downloaded from the website ICGEM (http://icgem.gfz-potsdam.de/ICGEM/). Klees et al. (2008) developed the filtering method using signal variances but no covariances in the spatial domain and propagated them to the full matrix in the SH domain (Klees et al., 2008). Davis (2008) designed a filter in the spatial domain, which has similar properties with the Gaussian smoothing (Davis, 2008). When constructing the filtering matrix, the two methods (Kusche, 2007; Klees et al., 2008) need not only the covariance matrix of the GRACE solution but also a priori covariance information of the spatial domain signal. However, using the prior covariance of the signal is essentially a constraint solution, and unreasonable prior information can distort the mass change signal. Thereby, we try to develop a noise-reduction filter by only using the error covariance matrix of the GRACE solutions.

Cite this article as: Ju X, Shen Y, Chen Q: A decorrelation filter based on the full error covariance of GRACE gravity field solutions. Acta Geodyn. Geomater., 19, No. 2 (210), 49–60, 2023. DOI: 10.13168/AGG.2023.0005
The rest of this paper is organized as follows. In Section 2, the noise-reduction filter is developed based on the error covariance matrix of GRACE spherical harmonic solutions. In Section 3, the effectiveness of the noise-reduction filter is demonstrated by comparing the noise-reduction filter to the traditional \( P \cdot M_\text{no} \) decorrelation filter (Swenson and Wahr, 2006; Chen et al., 2008). In Section 4, the temporal and spatial characteristics of AIS mass changes are analyzed. The discussion and conclusions are given in Sections 5 and 6 respectively.

2. NOISE-REDUCTION FILTER METHOD

Since the GRACE Spherical Harmonic (SH) coefficients up to degree and order 8 are dominated by signals (Swenson and Wahr, 2006), the noise-reduction filter is designed to filter the SH coefficients beyond degree and order 8. The GRACE SH coefficients are truncated to degree and order 60 and the effects of atmosphere, ocean and tides have been removed from estimating the SH coefficients (Bettadpur, 2012). When solving for the GRACE SH coefficients, the corresponding covariance matrix is also computed and used to evaluate the formal errors of the estimated SH coefficients. The covariance matrix \( Q \) can be orthogonally decomposed as,

\[
Q = P \Lambda P^T
\]

where \( \Lambda \) is a diagonal matrix sorted in ascending order with \( k \)-th diagonal element \( \lambda_k \), and \( P \) are the eigenvectors. The eigenvalues of the error covariance matrices of all months are shown in Figure 1 for Tongji-GRACE2018 models (Chen et al., 2019). In Figure 1, the eigenvalues of September 2004 and January 2008 are presented in green and red respectively, while the eigenvalues of the remaining 153 months are shown in gray. The solution of September 2004 suffers from a short repeat orbit (with bad condition) (Chen et al., 2019) and that of January 2008 is in the middle of the GRACE period (with good condition). Here we need to point out that the orbital altitude of GRACE became very low at the end of GRACE lifetime. As we know, the decreased orbital altitude is more sensitive to the gravity field signals at high degrees and orders, leading to improved accuracy of geopotential coefficients at high degrees and orders.

Furthermore, the formal errors of the unconstrained gravity field solutions for the months September 2004 and January 2008 are both presented in the form of order and the degree in Figure 2. It indicates that the noise of the unconstrained solutions increases with degree and order.

The SH coefficients are usually expressed as \( \{C_{lm}, S_{lm}\} \), where \( l \) and \( m \) denote the degree and order. After removing the average SH coefficients of all months \( \{\bar{C}_{lm}, \bar{S}_{lm}\} \), the differences of SH coefficients \( \{\Delta C_{lm}, \Delta S_{lm}\}_t \) month are expressed as,

\[
U = \{\cdots, \Delta C_{lm}, \Delta S_{lm}, \cdots\}
\]

Using the decomposed orthogonal matrix \( P \) in Eq. (1), we can transform the SH coefficients \( U \) to the uncorrelated coefficients \( R \) with,

\[
R = P^T U
\]

Obviously, the error covariance matrix of the transformed coefficients \( R \) is just the diagonal matrix \( \Lambda \). Since large noise exists in the SH coefficients of high degrees, a reasonable filter is to suppress the noise at high degrees and keep the signals at low degrees. In other words, the higher the degree SH coefficients are, the smaller the filter factors should be applied to. While for the coefficients which are dominated by signals, the corresponding filter factors should be close to 1 to avoid signal loss. For GRACE SH coefficients, the first 8 degrees and orders are not filtered since the coefficients are dominated by signals (Swenson and Wahr, 2006). Because the error covariance matrix can reflect the actual noise of the
SH coefficients, designing a filter function based on the full covariance matrix is reasonable and necessary. Since the transformed SH coefficients $R$ with Eq. (3) are uncorrelated, the diagonal matrix $A$ just indicates the covariance matrix of the coefficients $R$. Thus, the filter factors are designed as $F = \sqrt{A^{-1}}$, the filtered transformed SH coefficients $R_{\text{filter}}$ are,

$$R_{\text{filter}} = \sqrt{\lambda_1} \sqrt{A^{-1}} R$$  

(4)

where $\lambda_1$ is the first eigenvalue of the error covariance matrix of the SH coefficients, after the first 8 degrees and orders are removed. The final SH coefficients $U_{\text{filter}}$ can be expressed as,

$$U_{\text{filter}} = PR_{\text{filter}}$$  

(5)

By substituting Eq. (3) into Eq. (4) and then into Eq. (5), we have

$$U_{\text{filter}} = P\sqrt{\lambda_1} \sqrt{A^{-1}} P^T U = F^\prime U$$

(6)

where the filter factors $F^\prime = P\sqrt{\lambda_1} A^{-1} P^T$ are symmetric matrixes directly applied to the original SH coefficients $U$.

Figure 3 demonstrates the characteristics of the designed filter factors $F$, which decrease with degree and order. Since there are so many months concentrated together that we cannot clearly identify the two highlighted months (i.e., September 2004 and January 2008), an additional sub-figure is provided at the top-right corner of Figure 3. The noise of the SH coefficients at higher degrees and orders is gradually suppressed, while the signals at lower degrees and orders are mostly retained. The solutions of September 2004 and January 2008 are considered as a bad and a good one respectively. The corresponding filter factor of the former is smaller, especially at high degrees and orders.
Figure 4 presents the diagonal elements of the filter factors $F$ in the form of order and degree for September 2004 and January 2008. In Figure 4 the filter factors only have slight differences for the coefficients at the same degree, but decrease fast with the increase of degrees, indicating the noise-reduction filter is like a degree-dependent, i.e., isotropic filter.

Figure 5 shows the geoid degree height for the monthly solution for September 2004 (solid line) and January 2008 (dotted line) without (black) and with the noise-reduction filter (red) applied. We can see that the geoid degree height after filtering is much smaller than that without filtering. The high degree noises are significantly suppressed, while the signals at low degrees are retained.

3. METHOD VALIDATION

The $P_nM_m$ decorrelation filter (Swenson and Wahr, 2006; Chen et al., 2008) combined the Gaussian filter has been successfully applied in analyzing ice sheet mass balance (Chen et al., 2008; Chen et al., 2009), land-water mass changes (Wang et al., 2011), and seismic monitoring (Chen et al., 2007), also DDK filter is a successful way in the GRACE mass changes analysis (Kusche, 2007). The decorrelation filter (called $P_nM_m$) is applied to suppress the longitudinal stripes. That is, to each GRACE solution, at spherical harmonic orders $n$ and above, a degree $m$ polynomial is fitted by least squares and is removed from even and odd coefficient pairs. Figure 6 shows the geoid degree height for September 2004 and January 2008 after applying the $P_3M_8$ and $P_4M_6$ filter and DDK1-DDK8 filters. The used GRACE data are the Tongji-Grace2018 monthly gravity solutions released on the ICGEM website. We can see that the geoid degree height from the noise-reduction filter is close to that from DDK7 for the case of September 2004 and that from DDK4 for the case of January 2008. For September 2004 and January 2008, the corresponding geoid degree height from the noise-reduction filter is both smaller than that from $P_nM_m$. Compared to the
DDK filter, the noise-reduction filter still has noise at high degrees and orders. In this case, applying an additional Gaussian filter to further suppress the remaining noise at high degrees and orders is necessary.

We choose $P_{3}M_{8}$ ($P_{3}M_{8}$ and $P_{4}M_{6}$) filter combined with Gaussian filter and compare it to the noise-reduction filter combined with Gaussian filter. Figures 7 and 8 show the global surface mass changes based on the noise-reduction filter combined with Gaussian filter. The radius of the Gaussian filter is selected as 0 km, 200 km and 300 km, respectively. We can find that in the case of using the same Gaussian filter radius, the striping errors of the global surface mass changes derived by applying the noise-deduced filter are significantly reduced.

The global surface mass changes derived from Tongji-GRACE2018 solution processed without any filtering and with different filtering are presented in Figures 7(d), (g), (j) and 8(d), (g), (j) for September 2004 and January 2008, respectively. In the case without any filtering, as shown in Figures 7(a) and 8(a), the north-south stripes are obvious and only limited signal can be found in some particular areas (e.g., Greenland and Antarctic Ice Sheets). We can see from Figure 7(b) (200 km Gaussian filter) and Figure 7(d) (noise-reduction filter) that the noise-reduction method suppresses much more noise as compared to the Gaussian filter. When the noise-reduction filtering or $P_{3}M_{8}$ or $P_{4}M_{6}$ decorrelation filtering is applied alone, the corresponding map of global surface mass changes is shown in Figures 7(d), 7(g) and 7(j) and 8(d), 8(g) and 8(j), where the noise still exists but it is significantly reduced as compared to that in Figures 7(a) and 8(a), especially over oceans at low latitudes. The remaining noise in Figure 7(d), especially at the medium and low latitudes is probably caused by the repeat ground track in September 2004, which leads to insufficient data sampling. As such, the unconstrained normal equation for geopotential coefficients constructed in the case of insufficient data sampling is ill-conditioned, which degrades the estimated geopotential coefficients. As a consequence, the corresponding variance-covariance matrix for the estimated geopotential coefficients, the basic for
constructing our filtering matrix in this paper, lacks insufficient sampling information. Due to the insufficient sampling information at the medium and low latitudes for constructing the filtering matrix, the remaining noise can be found in September 2004 when the noise-reduction filter is applied. Nevertheless, when we compare Figure 7(d) to Figures 7(h) and 7(k), we find that the noise-reduction filter can remove the north-south stripes more effectively than P3M8 and P4M6 filters, particularly in the case of September 2004. When a Gaussian filter with a radius of 200km and 300km is individually combined with the noise-reduction filter or P3M8 or P4M6 decorrelation filter, the corresponding global surface mass changes are presented in Figures 7(e), 7(h), 7(k), 7(f), 7(i), 7(l) and 8(e), 8 (h), 8 (k), 8 (f), 8(i), 8(l), respectively, where the north-south stripes are further reduced.

The global surface mass change signals in Figure 8(e) are slightly weaker than those in Figures 8(h) and 8(k) in the Gangguo River basin located in Africa. The probable reason is the Gaussian smoothing radius used for combined filtering is determined by maximizing the signal-to-noise ratio at a global scale, which depends on the way of computing the signal-to-noise ratio. To maximize the signal-to-noise ratio at a global scale may ignore the characteristics of either signals or noise over some particular regions, which may lead to signal damping regionally. In the future, further investigations need to be conducted to discuss the way of determining the optimal Gaussian smoothing radius for combined filtering.
A DECORRELATION FILTER BASED ON THE FULL ERROR COVARIANCE OF ...

Fig. 8  Same as Figure 7, in the caption of Figure 8 but for January 2008.

4. MASS CHANGE ANALYSIS OF ANTARCTIC ICE SHEET BASED ON NOISE-REDUCTION FILTER

Based on Tongji-Grace2018 data, we analyze the mass changes over the Antarctic Ice Sheet using the noise-reduction filter and Gaussian filter compared to the combination of the P4M6 and P3M8 decorrelation and Gaussian filter. We get the signal/error ratios for AIS (The terms of trend, annual, semi-annual are composed of the signal part, while the residuals are the error part), which are 1.1376, 1.1652, 1.2009 and 1.1943 under 0 km, 100 km, 200 km and 300 km Gaussian filter radius respectively. So, the radius of 200 km is chosen. In AIS, since the low degrees have a large impact on the mass estimates (Su et al., 2020), the C20 coefficients are replaced by those from satellite laser ranging data (Cheng and Ries, 2017; Chen et al., 2021) and the degree-one coefficients provided in GRACE technical note 13 (TN13) (A file that provides the degree-1 coefficients (Geocenter) corrections using the GRACE-OBP method) are added back (Swenson and Wahr, 2008; Sun et al., 2016). What’s more, the GIA model IJ05_R2 (Ivins et al., 2013) are applied and the leakage errors are corrected with the scale factor determined by using the GLDAS model (1.14 for Antarctic and East Antarctica, 1.2 for West Antarctica) (Loomis et al., 2020).

The time series of monthly mass changes are fitted with the terms of bias, trend and other periodical terms as follows (Földváry, 2012),

\[ \Delta h(\theta, \lambda, t) = \beta_0(\theta, \lambda) + \beta_1(\theta, \lambda)(t-t_0) + \beta_2(\theta, \lambda) \cos(2\pi(t-t_0)+\varphi_1(\theta, \lambda)) + \beta_3(\theta, \lambda) \cos(4\pi(t-t_0)+\varphi_2(\theta, \lambda)) + \beta_4(\theta, \lambda) \cos(4.5342\pi(t-t_0)+\varphi_3(\theta, \lambda)) \]  

(7)

where \( \beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \varphi_1, \varphi_2, \varphi_3 \) are the parameters to be solved, \( \beta_1 \) denotes the change rate, \( \beta_2, \beta_3 \) and \( \beta_4 \) are the annual, semi-annual and 161-day term respectively, \( \varphi_1, \varphi_2 \) and \( \varphi_3 \) are the initial phases, \( t \) is the epoch of time series in a unit of year and \( t_0 \) is the referenced epoch.
Fig. 9  The mass changes over the Entire, East and West Antarctica.

Table 1  The Antarctic ice surface mass change by different filter methods.

<table>
<thead>
<tr>
<th>Methods (+Gaussian 200km)</th>
<th>Entire AIS(Gt/yr)</th>
<th>East AIS(Gt/yr)</th>
<th>West AIS(Gt/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise-reduction filter</td>
<td>-107.38±41.06</td>
<td>36.42±14.40</td>
<td>-147.45±16.78</td>
</tr>
<tr>
<td>P3M8 filter</td>
<td>-106.50±41.04</td>
<td>35.90±14.39</td>
<td>-142.41±16.75</td>
</tr>
<tr>
<td>P4M6 filter</td>
<td>-111.55±41.05</td>
<td>38.35±14.44</td>
<td>-149.90±16.78</td>
</tr>
</tbody>
</table>
Figure 9 shows the time series of monthly mean surface mass changes of Entire Antarctica, East Antarctica, and West Antarctica (including the Antarctic Peninsula throughout this paper) after applying the noise-reduction filter. The uncertainty of mass change includes three parts: one is the fitting error (68.3% confidence interval) in the least-squares solution of Eq. (7), and the other two parts are the GIA model error given by Velicogna and Wahr (2013) and leakage error (Ivins et al., 2013) which are about 35 Gt/yr and 2 Gt/yr respectively. Using the Tongji-GRACE2018 data, the mass changes over the entire Antarctica and West Antarctica show an obvious decline trend, while over the East Antarctica the mass change is increasing. The mass change rates over the entire, East and West Antarctica are given in Table 1 together with the estimated uncertainties. Over the Entire Antarctica, the mass change rate is $-107.38 \pm 41.06$ Gt/yr, which is close to that from the $P_3 M_8$ filter ($-106.50 \pm 14.39$ Gt/yr) and $P_4 M_6$ filter ($-111.55 \pm 14.44$ Gt/yr). Over the East Antarctica, the mass changes increase with a trend of $35.90 \pm 14.44$ Gt/yr ($P_3 M_8$ filter), and $38.35 \pm 14.44$ Gt/yr ($P_4 M_6$ filter). In the case of West Antarctica, the mass change trend is estimated to be $-144.75 \pm 16.78$ Gt/yr for the noise-reduction filter, $-146.00 \pm 16.75$ Gt/yr for the $P_3 M_8$ filter, and $-153.73 \pm 16.78$ Gt/yr for the $P_4 M_6$ filter. In general, the results from different filtering methods are consistent within their uncertainties.

The spatial distribution of mass change rates is demonstrated in Figure 10 in the form of equivalent water height. The mass change increase mainly locates in the East Antarctica, including Coats Land (CL), Queen Maud Land (QML), Enderby Land (EL), and the Siple Coast (SC), while the main mass loss occurs in the Amundsen Sea Embayment (ASE) and Antarctic Peninsula (AP). The mass loss over the entire Antarctica is mainly contributed by the West Antarctica. Figures 10(d) and 10(e) show the difference between the mass change trends derived from the noise-reduction filter and the decorrelation filter. We can see there are slight differences (mainly between -0.4 cm/yr to 0.4 cm/yr) occurring in coastal regions, such as the QML, the Wilkes Land, Victorid Land in EA and ASE in WA, but for the inland regions, less difference can be found.

5. DISCUSSION

In Section 2, we propose the noise-reduction method. While in the processing of building the filter factor, for each solution of spherical harmonic coefficients, the filter factor corresponding to the first eigenvalue is equal to 1. In a mathematic sense, the filter factors should vary in different monthly
solutions. For the high-quality monthly solution, the value should be 1, while for the worse one the value is better smaller than 1. Theoretically, the worse solution should be filtered stronger if a reasonable value can be determined to replace the first eigenvalues for the corresponding month. In general, further investigations need to be done to determine the optimal first eigenvalues for each month on the basic of careful analysis in the future.

The mass changes over Antarctica are analyzed based on the three filter methods in Section 4. Since the error of mass changes over Antarctica are smoothing to a great extent due to the large area of Antarctica, we further analyze the mass changes over Sahara Desert with weaker signals. The mass change and RMS distribution maps of Sahara desert area are shown in Figures 11 and 12 respectively. We can see from Figure 11 that the noise-reduction filter achieves the least mass changes over the Sahara among the three filtering methods. The noise-reduction filter also has the smallest mean RMS value, which is 2.13 cm, and the remaining two methods are 6.58 cm (P3M8 filter combined with 200 km Gaussian filter) and 7.94 cm (P4M6 filter combined with 200 km Gaussian filter). When we get the signal-to-error ratio, the values are 0.80 (noise-reduction filter combined with 200 km Gaussian filter), 0.45 (P3M8 filter combined with 200 km Gaussian filter) and 0.45 (P4M6 filter combined with 200 km Gaussian filter) respectively. This finding supports that the noise-reduction filter can achieve higher signal-to-noise ratio in an area with weaker signal.

6. CONCLUSIONS

In this paper, a noise-reduction filter is proposed to suppress the north-south stripes based on the error covariance matrix information of the GRACE SH solutions. The efficiency of the proposed filter is
obvious during the mass change analysis over AIS and Sahara. The main conclusions can be summarized as follows:

1. Using the full error covariance matrix information of the GRACE SH coefficients, we construct the noise-reduction filter, which can suppress the noise of gravity field solutions at high degrees and effectively reduce the north-south stripes.

2. The noise-reduction filter is compared to the $P_{1}M_{0}$ and $P_{2}M_{0}$ decorrelation filters in terms of degree geoid height of GRACE SH coefficients. The results show that the degree geoid height from the noise-reduction filter is smaller than that from both $P_{1}M_{0}$ and $P_{2}M_{0}$.

Based on the combination of noise-reduction filter and Gaussian smoothing with a radius of 200 km, the mass changes over the Antarctic Ice Sheet, East and West Antarctica are analyzed. The mass change trend is estimated to be $-107.38 \pm 41.06$ Gt/yr over the Entire Antarctica, $36.43 \pm 14.40$ Gt/yr over the East Antarctica and $-147.45 \pm 16.78$ Gt/yr over the West Antarctica.

AUTHOR CONTRIBUTIONS

Y.S. conceived and designed the experiments; X.J. and Q.C. performed the experiments; X.J. analyzed the data; X.J. wrote the paper; Y.S. revised the manuscript.

FUNDING

This research was funded by the Natural Science Foundation of Shandong Province (ZR2017ZB2210134; ZR2019QD003) and Shandong Jiaotong University Research Fund (Z202113).

ACKNOWLEDGMENTS

We thank ISDC for providing GRACE data.

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