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ORIGINAL PAPER

DOWNWARD CONTINUATION OF AIRBORNE GRAVITY DATA BASED ON SPARSE SPHERICAL RADIAL BASIS FUNCTION

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ABSTRACT

Downward continuation (DWC) of airborne gravity data is essential for high-resolution gravity field recovery but is an ill-posed problem sensitive to noise. We propose a novel approach using spherical radial basis functions (SRBFs) with L1-norm regularization (Lasso) to create sparse DWC models, contrasting with traditional Tikhonov (L2) regularization. Simulation experiments using EGM2008 show that the Lasso method achieves accuracy comparable to the Tikhonov method. Crucially, it simultaneously produces parsimonious models: at a 2′ resolution, sparsity rates of 56–70 % were achieved across different flight heights, drastically reducing the number of active parameters. The Lasso-based SRBF method effectively mitigates model complexity without sacrificing precision. This data-driven strategy aligns with the Occam's razor principle, favoring simpler models with greater generalization potential, and presents a viable alternative for airborne gravity DWC.

1. INTRODUCTION

Downward continuation (DWC) of airborne gravity data is one of the most important techniques in geophysical and geodetic data analysis and application. Its results directly impact the quality of further applications, such as the integration of multi-source gravity data, refinement of global or regional geodetic reference surfaces, and construction of regional gravity field models. However, DWC is an ill-posed problem (Schmidt et al., 2007; Rexer et al., 2016), where small changes in the data can cause significant variations in the solution. This leads to a condition number large enough to severely amplify data noise (Li et al., 2022). More importantly, high-frequency noise in airborne gravity data can be significantly amplified during the DWC process, which may lead to physically unrealistic results. Established methods for DWC, such as the inverse Poisson integral approach, are widely employed in the field due to their theoretical soundness and computational efficiency. However, these methods are often sensitive to noise and may require additional smoothing or regularization to mitigate the amplification of high-frequency errors. In contrast, the spherical radial basis function (SRBF) approach, combined with modern regularization techniques such as Lasso, offers a flexible and data-driven framework that can inherently handle the ill-posed nature of the problem while promoting sparsity and interpretability. Therefore, in this study, we explore the use of a sparse SRBF model with L1 regularization as an alternative to conventional methods, aiming to achieve a balance between accuracy, stability, and model simplicity.

In recent years, spherical radial basis function (SRBF) methods have received considerable attention in regional gravity field modeling, see e.g. (Schmidt et al., 2007; Klees et al., 2008; Wu et al., 2018; Li, 2018; Lin et al., 2019; Liu et al., 2020; Yu et al., 2022), such as the point mass method which is a type of kernel function in SRBF based on the Runge-Krarup theorem. Due to its advantage of quasi-localization, the point mass method can be used for DWC once it is fitted to gravity observation data. Furthermore, it is popular in SRBF modeling due to its computational simplicity and efficiency. To construct the SRBF model using the point mass kernel function for DWC, several critical issues must be considered. Specifically, determining the distribution of the kernel function entails controlling the number of layers, their depth, and the number of nodes in each layer. Typically, empirical methods are used to solve this problem. However, researchers such as (Klees et al.,

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2008; Lin et al., 2019) have introduced adaptivity in their work to make the model data-driven to some extent. It is crucial to strike a balance between having too few kernel functions, which may not accurately represent the true gravity properties of the region, and having too many kernel functions, which may cause overfitting problems. Therefore, it is imperative to employ a method with a sufficient number of kernel functions that do not worsen the problem. Thus, regularization is frequently required in parameter estimation (Tikhonov and Arsenin, 1977). In geodetic research, L2-norm regularization, which is also known as Tikhonov regularization, is widely employed, as evidenced by previous studies, see e.g. (Chang et al., 2018; Chen et al., 2001; Kusche and Klees, 2002). In some literature, Tikhonov regularization is also referred to as ridge estimation (Hoerl and Kennard, 2000).

L2-norm regularization, which is also known as Tikhonov regularization, addresses the pathological problem by reducing the estimated parameter vector. However, the parameter vector is still nonzero, meaning that the parameter solution is dense (Yu et al., 2022). In contrast, if the accuracy of the model remains unaffected or slightly decreases when the parameters are shrunk to zero, then a sparse model can improve the efficiency of variable assignment. Among various approaches, L1-norm regularization (Lasso) is a special solution that can automatically select the model without loss of convexity of the problem. The sole difference between Lasso and Tikhonov regularization is the substitution of L2 constraints with L1 constraints. As such, Lasso can be better interpreted in the regularization framework. Notably, when $p \geq 1$, all Lp-norm regularizations transform into a convex optimization problem; while $0 \leq p \leq 1$, all Lp-norm regularizations can produce sparse models. This phenomenon demonstrates the unmatched advantages of L1-norm regularization and highlights the significance of its widespread use in various fields under different names, such as total variation denoising in image processing (Rudin et al., 1992), soft threshold denoising in wavelet analysis (Donoho, 1995), basis pursuit in dictionary learning (Chen et al., 2001), and compressed sensing in signal processing(Candes et al., 2006; Donoho, 2006). L1-norm regularization was initially applied in geoscience research in the 1980s, e.g. (Oldenburg et al., 1983; Santosa and Symes, 1986). In recent times, L1-norm regularization has regained attention from geodetic scholars, as evidenced by recent publications, e.g. (Qian et al., 2020; Yu et al., 2022; Qian et al., 2022; Candès et al., 2006).

In this study, SRBF is applied to the DWC of airborne gravity data. The main contributions of this work are: 1) to our knowledge, the first application of Lasso-regularized SRBFs to the DWC problem; 2) a comprehensive evaluation of the sparsity and accuracy of the proposed method under various flight heights, noise levels, and data resolutions; and 3) a demonstration that sparse models can maintain accuracy while significantly reducing model complexity.

The paper is structured as follows. In Section 2, the theoretical background and methodology are presented, encompassing a comprehensive introduction to spherical radial basis functions, the Lasso problem, and a method for choosing the optimal regularization parameters. Section 3 is dedicated to data analysis, where we describe the simulation experiment setup and compare the results obtained from varying experimental conditions. Finally, Section 4 offers a conclusion and outlook for future research.

2. THEORY AND METHOD

This section is split into three parts, namely the spherical radial basis function model, the Lasso method, and 10-fold cross-validation. The details are introduced in the following three parts respectively.

2.1. *SPHERICAL RADIAL BASIS FUNCTION MODEL*

In this study, we utilize the point mass kernel function and gravity anomaly observations to conduct the DWC via the remove-restore technique. The model, which is commonly applied in the development of regional gravity field models, is of significant interest in geophysical research. Denoting the residual gravity anomaly as Δg, the point mass model can be represented as follows:

$$\Delta g = \sum_{j=1}^{N} \alpha_j \frac{1}{\|x - z_j\|_2} \tag{1}$$

where x, z_j represent the center of observation point and SRBF respectively. The location of point mass is distributed on a regular grid which is selected empirically. When the radial basis function (RBF) has u layers and the number of grid nodes has V, the number of parameters is $V \times u$, and the corresponding observation equation is as follows:

$$y = A\alpha + e \tag{2}$$

where $\boldsymbol{y}$ denotes the measurement vector; $\boldsymbol{\alpha}$ denotes the parameter vector; $\boldsymbol{e}$ represents the observation error with zero mean and assumes that its covariance matrix $\boldsymbol{Q} = \mathrm{cov}[\boldsymbol{e}]$ is known. For the gravity anomaly observations, suppose that A_{ij} is an element of matrix $\boldsymbol{A}$, then A_{ij} can be expressed as (Lin et al., 2014):

$$A_{ij} = \frac{r_i - R_j cos\phi_{ij}}{l_{ij}^3} - \frac{2}{r_i l_{ij}} \tag{3}$$

where $\begin{cases} cos\phi_{ij} = \frac{x_i^T z_j}{\|x_i\|_2 \|z_j\|_2} \\ l_{ij} = \|x_i - z_j\|_2 \end{cases}$ and denote the radial distance of the ith observation point by r_i. Eventually, the parameter vectors $\boldsymbol{\alpha}$ are estimated by building SRBF model using the observation vector $\boldsymbol{y}$ and the covariance matrix $\boldsymbol{Q}$.

2.2. THE LASSO METHOD

In Tikhonov regularization, the ill-conditioning of the original matrix is improved by the regularization parameter and regularization matrix, and the parameter can be estimated by the following formula:

$$\hat{\alpha} = argmin[(y - A\alpha)^T Q^{-1}(y - A\alpha) + \mu\|\alpha\|_2^2] \quad (4)$$

The following is the analytical form of the above parameter estimation:

$$\alpha = (A^T PA + \mu N)^{-1} A^T Py \quad (5)$$

where $\boldsymbol{A}$ is the parameter matrix of radial basis functions; $\boldsymbol{P} = \boldsymbol{Q}^{-1}$ is the weight matrix of the observation; μ is the regularization coefficient; $\boldsymbol{y}$ is the measurement vector. In this work, the identity matrix of $\boldsymbol{I}$ is denoted as $\boldsymbol{N}$ (Kusche and Klees, 2002). In addition, iterative algorithms such as preprocessing conjugate gradients would still be used in preference to analytical solutions, in order to avoid the explicit matrix inversion involved in the formula, see e.g. (Friedman et al., 2010).

For the Lasso method, parameter estimation is defined as follows:

$$\hat{\alpha} = argmin[(y - A\alpha)^T Q^{-1}(y - A\alpha) + 2\mu\|\alpha\|_1] \quad (6)$$

The key difference lies in the regularization term: Lasso uses the L1-norm $2\mu\|\alpha\|_1$, whereas Tikhonov uses the L2-norm $\mu\|\boldsymbol{\alpha}\|_2^2$. While the regularization solution for Tikhonov regularization can be computed using its analytic form with a set of regularization coefficients, the case of Lasso is different due to the non-differentiability of the L1-norm, and the only option is to use an iterative algorithm since there is no general analytic formula. Such iterative algorithm includes alternating direction method of multipliers (ADMM) (Boyd et al., 2011) and iterative reweighted least squares (IRLS) (Daubechies et al., 2010). In this work, we adopt an efficient method called coordinate descent to solve the problem of L1-norm regularization (Friedman et al., 2010). Coordinate descent is an iterative algorithm, which aims to optimize one coordinate direction at each iteration while fixing the values of other coordinates, thus transforming a multivariate optimization problem into a univariate one. This method is advantageous in high-dimensional optimization spaces and does not require gradient calculations, making it suitable for solving certain large-scale optimization problems. Unlike gradient descent, coordinate descent only considers a single dimension at each iteration, potentially leading to faster convergence to the global optimal solution. In coordinate descent method, we calculate iteratively by the following formula:

$$\begin{cases} \boldsymbol{\alpha}_k^* = \dfrac{1}{\boldsymbol{Z}_k} S(r_k, 2\mu), \quad k = 1, 2, ..., n \\ Z_k = \sum_{i=1}^{n} A_{ik}^2 \\ r_k = \sum_{i=1}^{n} (y_i - \sum_{j \neq k} A_{ij}\alpha_j) A_{ik} \end{cases} \quad (7)$$

In the above, $S(r_k, 2\mu) = \mathrm{sign}(r_k)\cdot\max(|r_k| - 2\mu, 0)$ is the soft thresholding function and sign(·) is the sign function. $\boldsymbol{\alpha}_k^*$ denotes minimal value point on the kth coordinate axis. The termination condition for iteration is set as $\|\boldsymbol{\alpha}_k - \boldsymbol{\alpha}_{k-1}\|_1 / \|\boldsymbol{\alpha}_{k-1}\|_1 \leq \xi$, where ξ is an empirically threshold value and set to 0.001. There are many zero elements in the parameter vector, which means that the model is sparse. That is the computational effort to find the parameter vector will be greatly reduced, which makes it easier to apply.

2.3. 10-FOLD CROSS-VALIDATION

There are various methods for selecting the number and depth of radial basis functions, such as the generalized cross-validation (GCV) (Klees et al., 2008) and multipole analysis based on fitting the basis function to the signal covariance function (Marchenko et al., 2001). Sometimes there are difficulties in selecting the minimum value of the GCV, so it is not always possible to obtain the best parameters for the radial basis functions by this method. As for the multipole analysis method, it requires the determination of the optimal 3D spatial position of each basis function, which increases the time complexity of the algorithm. To avoid the drawbacks of the above methods, in this work, the k-fold cross-validation method is employed. For better estimation of the hyperparameters, $k = 10$ is taken, i.e., the airborne gravity observations are divided into 10 groups.

The specific calculation process is described as follows:

(a) The hyperparameter μ is chosen from a predefined set of candidates, which is determined empirically. For instance, μ is selected from the range [$\mu_1, \mu_2, \mu_3, \mu_4, \mu_5 \ldots$].

(b) The gravity anomaly observations are randomly split into ten equal parts, with nine parts being used as the training set and one as the validation set.

(c) For each hyperparameter, the model is trained using the training set and tested using the validation set to obtain the root mean square error (RMSE) of the model on the current validation set.

(d) Steps (c) are repeated until the model has been tested on all validation sets and 10 prediction errors have been obtained.

(e) The 10 prediction errors are averaged to obtain the final RMSE of the model for that hyperparameter.

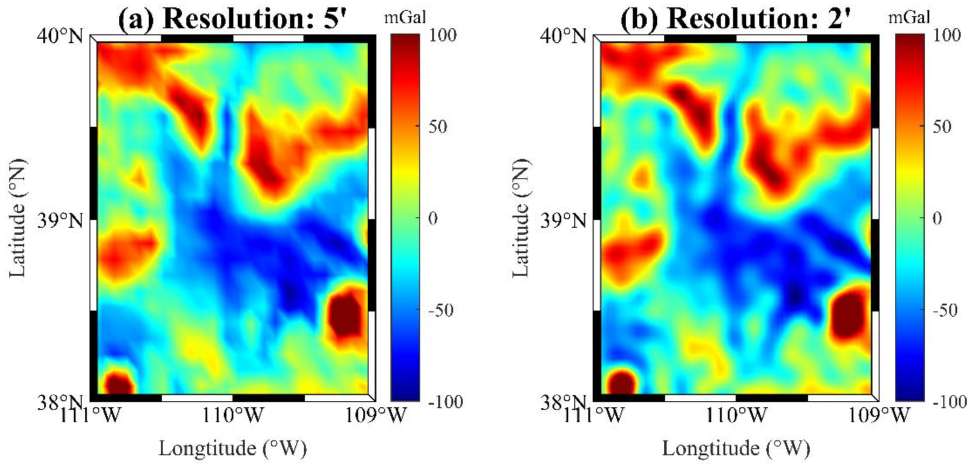

Fig. 1 The distribution of the ground truth gravity anomalies (0 km height) in the test area at (a) 5′ and (b) 2′ resolutions.

Table 1 Statistics of simulated gravity anomalies (mGal).

Resolution	Height/km	Noise' STD	Max	Mean	Min	STD
5′	0 (error free)	0	185.63	0.47	-102.49	37.48
	2	2	154.91	0.24	-71.50	29.85
	3	3	132.25	0.22	-62.76	26.87
	4	4	111.97	0.14	-53.12	24.22
2′	0 (error free)	0	223.39	0.55	-102.05	37.83
	2	2	164.72	0.10	-73.52	29.78
	3	3	141.27	0.10	-65.60	26.75
	4	4	124.96	0.16	-57.12	24.19

(f) Steps (c)-(e) are performed using the remaining hyperparameters in the candidate set to obtain the final RMSE for each model.

(g) The model with the best RMSE is selected as the final model.

3. EXPERIMENT RESULTS

3.1. SETUP OF SIMULATIONS

To verify the effectiveness of the two regularization methods proposed earlier, we used EGM2008 to obtain airborne gravity anomalies at different flight heights *H* (height values relative to the mean Earth radius of 6371.393 km), with EGM2008 degrees ranging from 360 to 2160 degrees. We selected a 2°×2° block (latitude: 38°N-40°N; longitude: 249°E-251°E) in the western United States as the test area (Fig.1), which is a mountainous area with a complex gravity field and where EGM2008 has higher accuracy in the continental United States (Pavlis et al., 2013). To reduce edge effects, according to experience, the radial basis function area is usually extended by 0.2° in longitude and latitude directions compared to the data area, and the data area is extended by 0.3° in longitude and latitude directions compared to the test area (Lin et al., 2019). To simulate the actual situation of airborne gravity measurement, we chose three different flight heights, namely 2 km, 3 km, and 4 km, and added Gaussian white noise with means of 0 and standard deviations of 2 mGal, 3 mGal, and 4 mGal, respectively. As ground gravity data in the experimental area could not be obtained, we used gravity anomaly data with a height of 0 km and no noise pollution as the true value of ground gravity data.

As is well known, DWC height and observation error are important factors affecting model performance. Vanicek et al. (2017) found that when using the inverse Poisson method for DWC, it is relatively easy for gravity data with a resolution of 5′. When the resolution is greater than 2′, instability in DWC begins to occur. Therefore, we simulated two datasets with resolutions of 5′ and 2′ to test the sensitivity of the SRBF model to the grid resolution of the data points. The statistical information of the simulated gravity anomalies can be found in Table 1.

3.2. RESULTS AND ANALYSIS

We downward continued the airborne gravity data to 0 km and compared the DWC values with the true values to obtain the RMSE of the prediction error. The depth of each layer was adjusted through a large number of experiments, and the smaller the RMSE of the prediction error, the better the depth of the layer. A single-layer depth was employed in this study. This design is supported by the Runge-Krarup theorem,

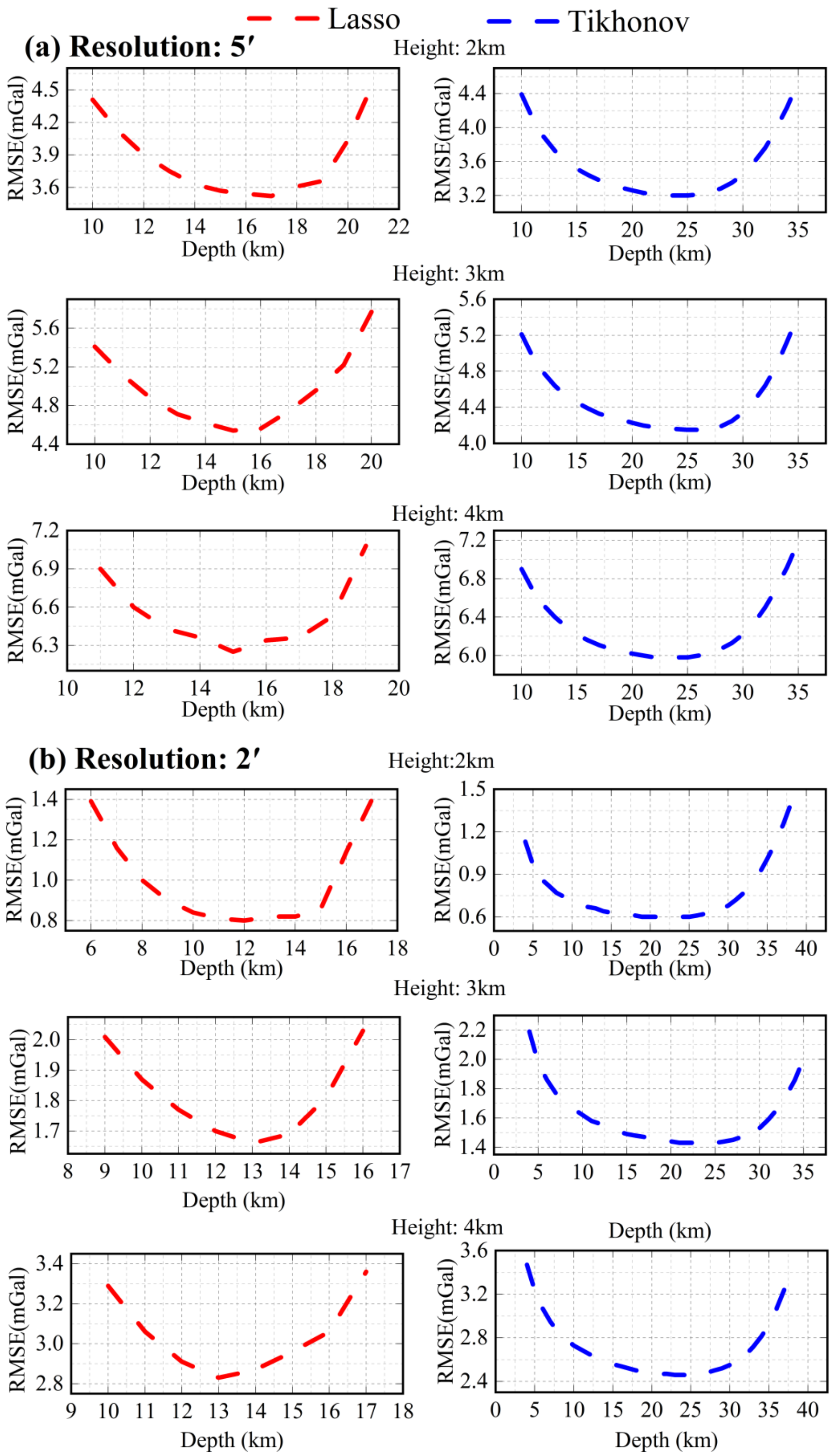

Fig. 2 Relationship between predicted RMSE and SRBF depth for three DWC heights under different regularization methods.

Table 2 Statistics of prediction errors for three DWC heights under different regularization methods. SR: sparsity ratio; NZP: number of zero parameters.

Height /km	Method	α	Max	Mean	Min	RMSE	SR	NZP
Resolution: 5′								
2	Tikhonov	1.81×10^{-23}	16.31	0.35	-12.89	3.20	0	0
	Lasso	4.60×10^{-4}	16.44	-0.12	-12.71	3.52	12%	111
3	Tikhonov	6.14×10^{-24}	17.47	0.45	-17.28	4.15	0	0
	Lasso	5.18×10^{-4}	16.58	-0.49	-16.72	4.56	16%	150
4	Tikhonov	9.34×10^{-23}	17.46	0.00	-20.10	5.98	0	0
	Lasso	9.97×10^{-4}	18.60	0.46	-19.51	6.25	22%	216
Resolution: 2′								
2	Tikhonov	1.04×10^{-21}	2.40	0.02	-2.19	0.60	0	0
	Lasso	4.93×10^{-4}	3.14	-0.04	-3.18	0.80	56%	3418
3	Tikhonov	7.24×10^{-22}	5.04	0.00	-5.01	1.43	0	0
	Lasso	6.31×10^{-4}	5.19	0.09	-5.11	1.66	65%	3955
4	Tikhonov	4.71×10^{-22}	8.20	0.14	-8.78	2.46	0	0
	Lasso	8.74×10^{-4}	10.39	-0.28	-9.23	2.83	70%	4276

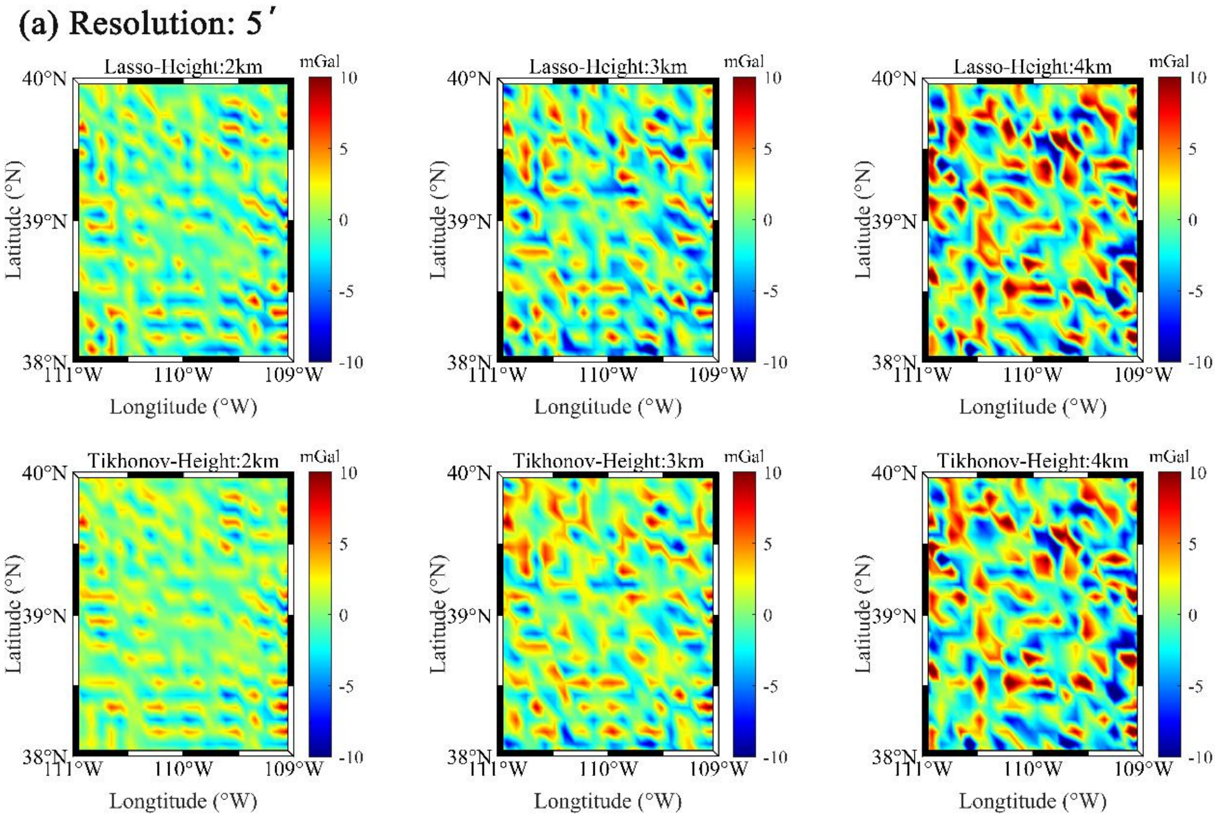

Fig. 3 Distribution of the differences between the DWC results and the true values for three heights under different regularization methods.

which states that a single layer of basis functions with a sufficiently dense grid can approximate the gravitational potential arbitrarily well. This approach simplifies the parameter tuning process while maintaining theoretical rigor. Figure 2 shows the process of adjusting the depth, revealing that the optimal depth for Tikhonov regularization is consistently deeper than that for Lasso, and that Lasso's performance is more sensitive to depth changes. Table 2 shows the experimental data under different schemes. When the height is 2 km, 3 km, and 4 km, we respectively added Gaussian white noise with standard deviation of 2 mGal, 3 mGal, and 4 mGal.

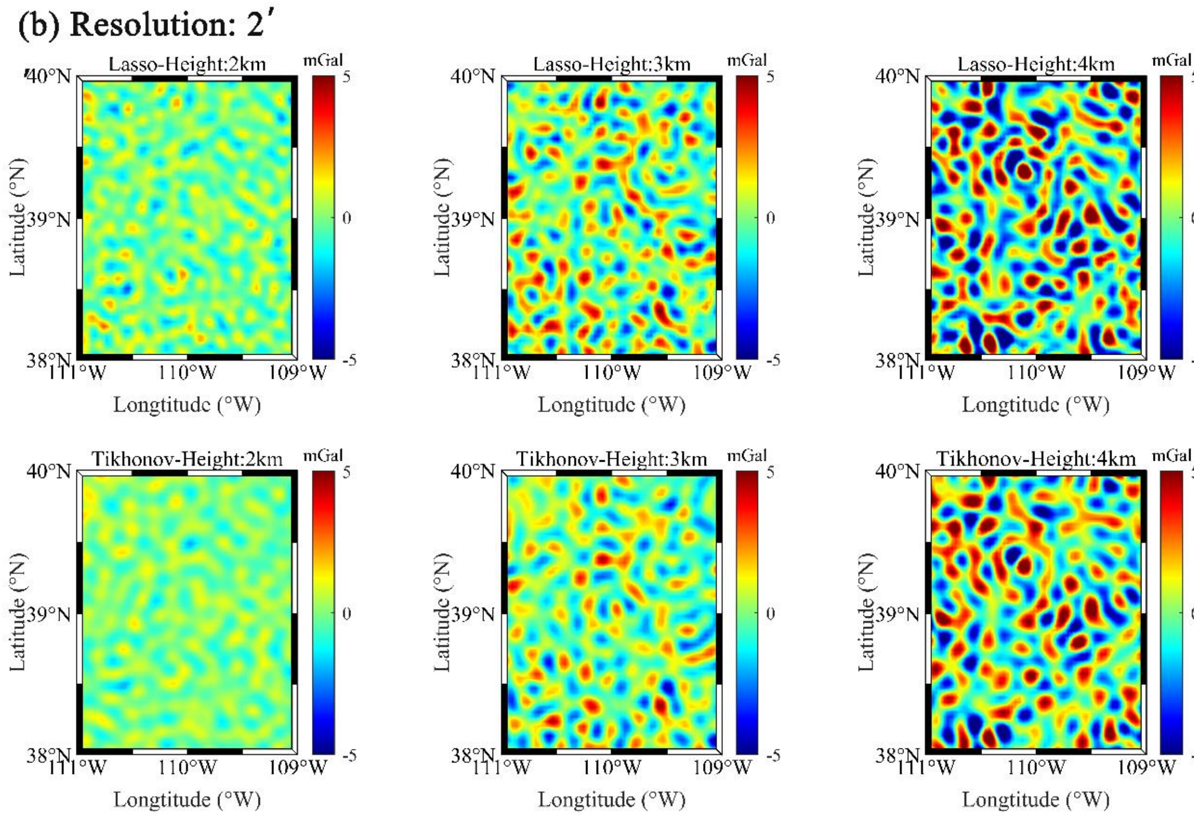

Fig. 3 Continued.

From the perspective of prediction accuracy, the SRBF model based on regularization filtering has a good filtering effect. For the sparse model, as the DWC height increases, the regularization coefficient becomes larger, producing a sparser model. As a result, experiments show that Lasso serves as a data-driven model selection method. Figure 3 shows the spatial distribution of prediction errors, intuitively demonstrating the accuracy of the two regularizations. Combining Table 2 and Figure 3, we found that the accuracy of the Lasso model is comparable to the Tikhonov regularization, but the model complexity is reduced. Figures 4 and 5 respectively show the frequency distribution and spatial distribution of the coefficient solution in the sparse model. We want to emphasize that the purpose of this study is not to replace the Tikhonov regularization, but to propose a very promising solution. The sparse model compresses the parameters of the model and improves the generalization ability of the model.

In this paper, the RMSE is used as the evaluation standard for hyperparameter selection. One of the most distinguishing features of Lasso is its sparse property. By taking sparse indicators as a constraint for hyperparameter selection, we can increase the prediction error appropriately to obtain a sparser model. Whether to choose a sparser model or a more accurate model depends on the user's preference.

4. CONCLUSIONS

Recently, SRBF has received wide attention in the field of gravity field modeling. Regularization is usually necessary in solving overfitting problems, and Tikhonov regularization is almost universally employed. However, in this study, we used SRBF based on Lasso regularization to establish DWC models. The applicability of Lasso regularization was verified by comparison experiments at different data grid resolutions, different flying heights, and different noise pollution conditions. In terms of precision, the Lasso model is comparable to the Tikhonov model; and when the observation data grid resolution is 2′, the sparsity rates of the Lasso models at the three heights are 56 %, 65 %, and 70 %, respectively. This phenomenon indicates that Lasso regularization can significantly reduce the number of parameters of the SRBF model, thereby reducing the complexity of the SRBF model. And according to the Occam's razor criterion, the simplest model should be chosen to explain the data (Haykin, 2009; Anderson and Burnham, 2004; Bishop, 1995). Sparse models have better generalization ability because they can better avoid overfitting phenomenon.

In summary, sparse SRBF models are an alternative solution for DWC, at least in some cases. The primary contributions of this study are threefold: (1) the novel application of Lasso-regularized SRBFs

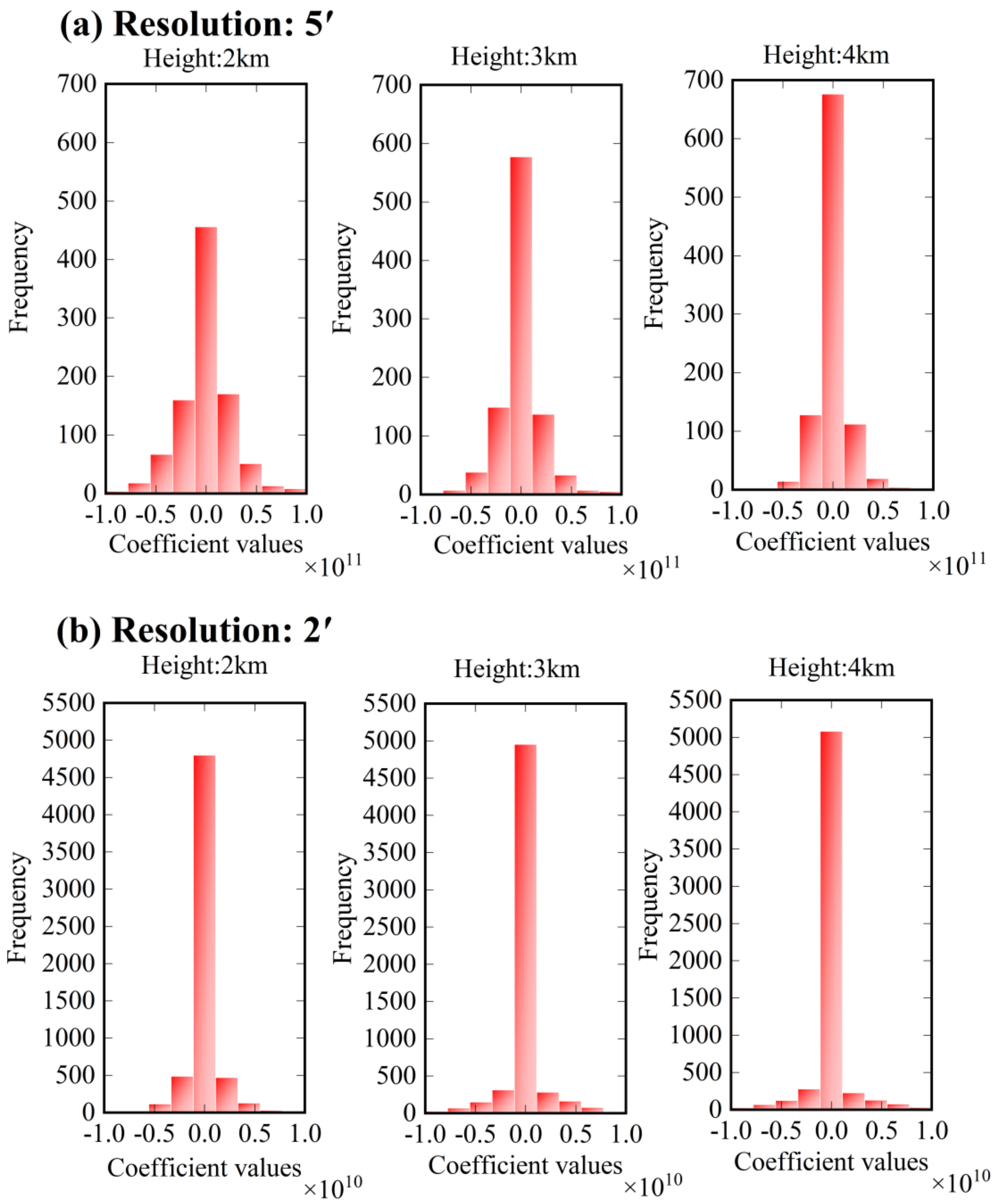

Fig. 4 Frequency distributions of the coefficient values with sparse models.

to the airborne gravity DWC problem; (2) a systematic demonstration that Lasso achieves model sparsity of up to 70 % with comparable accuracy to Tikhonov regularization; and (3) the provision of a data-driven framework that enhances model interpretability and computational efficiency. Future work will mainly focus on further testing and improving the sparse SRBF model with real airborne gravity data from mountainous and offshore areas.

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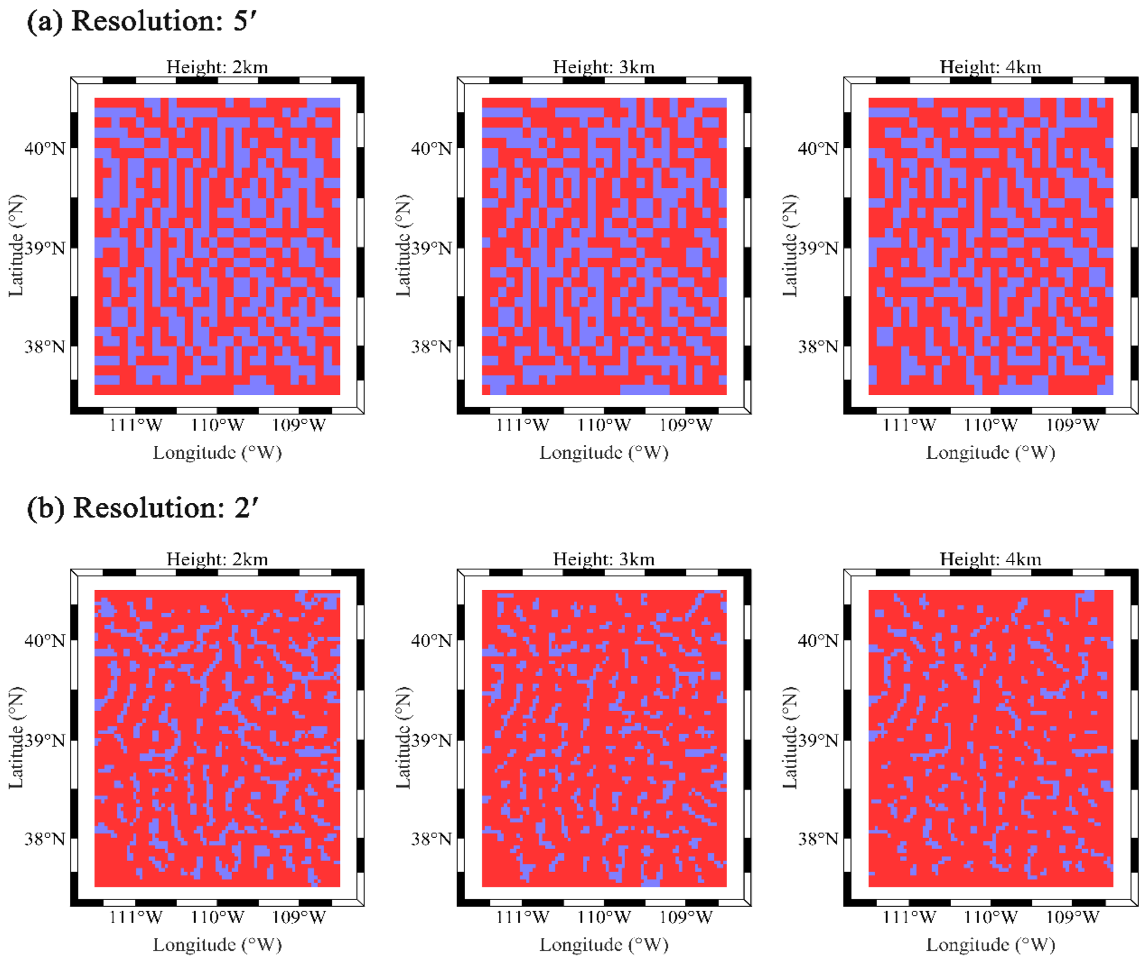

Fig. 5 The spatial distribution of the coefficient with six sparse models. Red point: zero coefficient point; Blue point: nonzero coefficient point.

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