# CORROSION OF GLASS FURNACE SIDEWALLS AND HEAT LOSSES

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An equation for the corrosion profile evolution of horizontal glass furnace sidewalls has been established. The non-uniform vertical temperature distribution and strong temperature dependence of viscosity, diffusion coefficient and refractory solubility in glass melt are taken into account. The effect of cooling on the corrosion rate as well as on the life-time of sidewall blocks is also analysed, and the cooling regime minimizing the energy loss during a given furnace campaign is being discussed.

#### 1. INTRODUCTION

The flux-line corrosion of current refractories is considerably faster at glass-melting temperatures than that below the glass level. Accordingly, the corrosion below the glass level would not seem to be paid special attention to. However, this is not the case when the cooling is taken into account. If the wall thickness has been considerably reduced owing to the developed corrosion, the cooling may essentially diminish the temperature of the inner wall surface. But from a certain temperature down, the corrosion at the glass level (flux-line corrosion) becomes slower than that below it, as it is apparent from a number of experimental observations. An excellent survey of experimental results is given by A. Smrček [1], who found out that the rate of corrosion is related to the viscosity of glass as follows:

$$j\mu^a = \text{const.},\tag{1.1}$$

where j is the corrosion rate,  $\mu$  the dynamic viscosity and a the corrosion index. For the flux-line corrosion a = 3, whereas for the corrosion below the glass level a = 3/2. The consequence of the greater index of the flux-line corrosion is that it is damped more effectively by cooling, as it is seen from Fig. 1 and 2. In case of zirconia-alumina-silica refractory and the Float glass both types of corrosion reach the same rate at 1390 °C (Fig.2). The significance of the corrosion below the glass level cannot be thus underestimated, for it is this corrosion that determines the life-time of the cooled walls.

The first works analysing the corrosion process by the transport phenomena theory were published by Barret [2] and Cooper [3], who pointed out that the dissolution of refractories in a glass melt is governed by the diffusion in the moving concentration layer. The motion of a liquid in the concentration layer may be driven either by the outer flow\*) or by the buoyancy forces resulting from the density

<sup>\*)</sup> The outer flow is meant to be the flow of a glass melt out of the boundary layer (i.e., the flow driven by the pull current and temperature buyoancy forces). Smrček [4] calls it the technological flow.



Fig. 1. The time-dependence of the cooled wall thickness for the flux-line corrosion (s) and the corrosion below the glass level (d). At the beginning the flux line corrosion is faster, but, at a certain wall thickness, the rate of both types of corrosion becomes equal due to damping by cooling. Finally, the wall is corroded through owing to the effect of the corrosion below the glass level.



Fig. 2. The temperature-dependence of the rate of the flux-line corrosion (s) and the corrosion below the glass level (d) of a zirconia-alumina-silica refractory by the Float glass melt. At temperatures beneath 1390 °C the flux-line corrosion is slower, above this temperature the ratio of both corrosion rates is reverse.

differences due to the concentration gradient. Both driving machanisms operate in case of the corrosion of vertical glassf urnace walls. By Smrček [4], the buoyancy flow is supressed as the glass melt with higher content of the dissolved defractory material (concentration layer) is being beld on the wall by its surface roughness and the dissolution is mainly controlled by the outer flow. However, on the places with less intensive outer flow the buoyancy flow can prevail. This case has been recently analysed by Hrma and Tesař [5]. Though their model does not take into account the effect of the technological flow, it makes possible, owing to its simplicity, an easy analysis of various factors effecting both the corrosion rate and the shape of corrosion profile. Besides the temperature distribution inside the furnace, it is the regime of cooling the walls, the homogeneity of blocks, etc.

The relations representing the mathematical model for the simulation of the corrosion profile evolution of glass furnace sidewalls during the whole furnace campaign are briefly given in the following section of this contribution. They start from the boundary layer theory and rest on the assumptions that the direct effect of the technological flow upon the corrosion is negligible and that the upward-drilling corrosion does not arise. They express both the effect of local conditions (temperature) and the protective effect of corrosin products flowing down the wall upon the corrosion rate.

The predictions of corrosion is possible, if necessary data are known such as material properties of glass melt, transport coefficients, etc. The simple approximation relations for estimating such data (Arrhenius equation for the viscosity, Stokes-Einstein relation for the diffusion coefficient, Smrček's relation for the flux-line corrosion rate) are dealing with in Sect. 3. The next two sections discuss the applications to simple situations such as the initial corrosion rate (when the inner surface of the wall can be considered as vertical—Sect. 4) and the region closely below the glass level (where the temperature is usually highest and so the walls are most likely to be corroded through-Sect. 5). In Sect. 5, the effect of cooling upon the lifetime of wall is discussed in greater detail. Sect. 6 is confined to the question of energy losses, which is a very important standpoint for the cooling optimization.

#### 2. BASIC RELATIONS

In 1881 Lorenz published the analysis of the heat transfer from the vertical surface with the temperature  $T_0$  into the quiescent liquid with the temperature  $T_{\rm eo}$ . At a steady state the heat transfer is controlled by the buoyancy convection along the surface, and Lorenz described it by the relation that can be briefly written in the form  $N_{\rm Nu_x} = 0.411 N_{\rm Ra_x}^{1.4}$ , where  $N_{\rm Nu_x}$  is the local Nusselt number and  $N_{\rm Ra_z}$  is the local Rayleigh number. Later on, it was found out by more exact analysis that the original Lorenz relation should be substituted by the expression  $N_{\rm Xu_z} =$  $= F(N_{\rm Pr})N_{\rm Ra_x}^{1/4}$ , where the coefficient F is a function of the Prandtl number,  $N_{\rm Pr}$ . If  $N_{\rm Pr} \to \infty$ , then [6] F = 0.5028. Substituting the Nusselt number by the Sherwood number, the Rayleigh number for the heat transfer by that for the mass transfer and the Prandtl number by the Schmidt number, we get an analogous expression for the mass transfer from the isothermal vertical surface. Particularly, for  $N_{\rm Sc} = \nu/D \to \infty$ 

where

$$N_{\rm Sh_z} = 0.5028 N_{\rm Ra_z}^{1/4},\tag{2.1}$$

$$N_{\rm Sh_{z}} = (\varrho_{W}/\varrho_{w}) \ (1 - C_{w})xj/[(C_{w} - C_{\infty}) D], \tag{2.2}$$

$$N_{\mathrm{Ra}_x} = \beta (C_w - C_\infty) g x^3 / \nu D, \qquad (2.3)$$

 $\varrho_W$  is the density of the dissolved wall,  $\varrho_w$  — the density of the ambient liquid,  $C_w$  — the equilibrium volume fraction of the solid in the liquid,  $C_{\infty}$  — the volume fraction of the solid in the bulk, x — the distance from the leading edge, j — the velocity of the interface related to the wall (the dissolution rate),  $\nu$  — the kinematic viscosity, D — the diffusion coefficient,  $\beta$  — the concentration expansion coefficient, and g — the gravity acceleration.

The relation (2.1) fits satisfactorily the situation from  $N_{\rm Sc} = 10^2$  up, so it can be applied to the dissolution of refractories in glass melts, where the Schmidt number reaches the values  $10^{10}$  and even more. When  $\nu$  and D depend on the volume fraction of the solid, C, the relation (2.1) leads to acceptable results, if the values of  $\nu$  and Dfor the mean concentration are used.

The dissolution of glass furnace side walls occurs in nonuniform temperature fields. The temperature differences between the bath surface and the bottom usually reaches over 100 K. Among the physical properties of the system, the temperature mainly effects v, D and  $C_w$ . The case of constant v and D and temperature-dependent  $C_w$  can be easily derived from the analysis of nonisothermal heat transfer that was published by Kao, Domoto and Elrod [7]. After a small arrangement of their original relation we have

$$N_{\mathrm{Sh}_x} = A(\lambda N_{\mathrm{Ra}_x})^{1/4},\tag{2.4}$$

where A is a constant and

$$\lambda = (C_w - C_{\infty}) / \langle C_w - C_{\infty} \rangle.$$
(2.5)

The brackets  $\langle \rangle$  denote the mean value defined as

$$\langle \psi \rangle = x^{-1} \int_{0}^{x} \psi \mathrm{d}x.$$

In a more general case, when both  $C_w$ ,  $\nu$ , and D are temperature-dependent, we have to introduce into (2.4) the expression [8]

$$\lambda = (C_w - C) D \nu^{-1/3} / \langle (C_w - C) D \nu^{-1/3} \rangle.$$
(2.6)

Owing to the flux-line corrosion, the distance from the leading edge, x, is difficult to define. Taking the origin of the x-coordinate arbitrarily, (2.1) becomes

$$N_{\rm Sh_x} = 0.5028(1 + x_r/x)^{-1/4} N_{\rm Ra_x}^{1/4}, \tag{2.7}$$

where  $x_r$  is the effective distance of the origin from the hypothetical leading edge. If we know the volume of the dissolved substance,  $Q_r$ , passing from the region above the origin to that below it in a time unit, then  $x_r$  can be expressed as follows [8]:

$$x_r = 1.7046 Q_r^{4/3} D_r^{-1} (\nu_r / \beta g)^{1/3} (C_{wr} - C_{\infty})^{-5/3}, \qquad (2.8)$$

where  $C_{wr}$ ,  $v_r$  and  $D_r$  are the values of  $C_w$ , v and D at x = 0.

The equation (2.7) can be generalized for the case of temperature-dependent transport coefficients. Then the following expression must ve introduced into the equation (2.4):

$$\lambda = (C_w - C_{\infty}) D \nu^{-1/3} x [\langle (C_w - C_{\infty}) D \nu^{-1/3} \rangle x + (C_{wr} - C_{\infty}) D_r \nu_r^{-1/3} x_r]^{-1}.$$
(2.9)

For  $x_r = 0$ , (2.9) turns into (2.6), and for  $D = D_r$ ,  $\nu = \nu_r$  and  $C_w = C_{wr}$ , (2.9) reduces to  $\lambda = (1 + x_r/x)^{-1}$ , which is the coefficient in the equation (2.7).

Introducing (2.2) and (2.3) into (2.4) we get for j the expression:

$$j = 0.5028(\varrho_w/\varrho_w) (C_w - C_{\infty})^{5/4} (1 - C_w)^{-1} D^{3/4} (\lambda \beta g/x_v)^{1/4}.$$
(2.10)

This equation may be further arranged substituting for  $\lambda$  and  $x_r$  from (2.8) and (2.9):

$$j = 0.4400(\varrho_w/\varrho_W) (C_w - C_{\infty})^{3/2} (1 - C_w)^{-1} (C_{wr} - C_{\infty})^{1/6} \times D(\beta g/\nu Q_r)^{1/3} (1 + r)^{-1/4},$$
(2.11)

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where

$$r = x_r^{-1} \int_0^x \frac{(C_w - C_\infty) D \nu^{-1/3}}{(C_{wr} - C_\infty) D_r \nu_r^{-1/3}} \, \mathrm{d}x =$$
  
= 0.5866 Q\_r^{-4/3} (\beta g)^{1/3} (C\_{wr} - C\_\infty)^{2/3} \int\_0^x (C\_w - C\_\infty) D \nu^{-1/3} \, \mathrm{d}x. (2.12)

Let us denote the corrosion, i.e. the distance of the interface from its original position, as Y. The relations (2.10) up to (2.12) are analogous with those for heat transfer, where the interface is assumed to be horizontal at any time. As a consequence



Fig. 3. The interface s between the glass melt G and the wall W is represented by the function Y(x). The tangent to the interface at the point A makes an angle  $\alpha$  with the gravity acceleration vector. The horizontal rate of wall dissolution is denoted  $Y_t = dY/dt = j/\cos \alpha$ , where j is the dissolution rate in the normal direction.

of the dissolution process, the interface, however, is being deformed, and hence the angle  $\alpha = \arctan y_x$ , where  $Y_x = \partial Y/\partial x$ , differs from zero. Therefore g in (2.11) and (2.12) should be substituted by the expression  $g \cos \alpha = g(1 + Y_x^2)^{-1/2}$ . Then j in (2.11) means the dissolution rate in the direction perpendicularly to the interface (Fig. 3). The dissolution rate in the horizontal direction is  $Y_t = \partial Y/\partial t = j/\cos \alpha$ , and so

$$\begin{split} Y_t &= 0.4400 (\varrho_w/\varrho_w) (C_w - C_\infty)^{3/2} (1 - C_w)^{-1} (C_{wr} - C_\infty)^{1/6} \times \\ &\times D(\beta g/\nu Q_r)^{1/3} (1 + Y_x)^{1/3} (1 + r)^{-1/4}, \end{split} \tag{2.13}$$

where

$$r = 0.5866Q_r^{-4/3}(\beta g)^{1/3} (C_{wr} - [C_{\infty})^{2/3} (1 + Y_x)^{-1/6} \int_{0}^{\beta} (C_w - C_{\infty}) D \nu^{-1/3} \mathrm{d}x.$$
(2.14)

The equation (2.13) with regard to (2.14) is an integrodifferential equation for the time evolution of the corrosion profile, i.e., for the function Y(x, t). In equations

(2.13) and (2.14) all the quantities, except for  $\varrho_W$ , are functions of the temperature  $T_i(x, t) = T(x, Y)$  at the interface between the wall and glass melt. If this temperature and the wall density distribution,  $\varrho_W(x, y)$ , where y is the horizontal coordinate, are known, the equation (2.13) can be solved. The necessary assumption is, of course, that all the pertinent material parameters for the relevant temperature interval are available.

## 3. CONSTITUTIVE RELATIONS

The temperature functions of the viscosity, diffusivity, and equilibrium concentration are expressed by the constitutive (material) relations.

In a relatively wide temperature range, the viscosity satisfies the Arrhenius equation

$$\nu = K_{\mathbf{y}} \mathrm{e}^{\Theta/T},\tag{3.1}$$

where  $K_{\bullet}$  and  $\Theta$  are constants. We shall call the constant  $\Theta$  as the characteristic temperature of glass melt for viscosity (its product with the gas constant is the activation energy).

There is a lack of experimental data for the temperature dependence of the diffusion coefficient corresponding to the mutual penetration of glass melt and dissolved refractory. Oishi, Terai and Ueda [9] found out that for the self-diffusion coefficient of oxygen ions in glass melts the Stokes—Einstein relation

$$\nu D = \bar{k}T/l\rho, \tag{3.2}$$

is valid at higher temperatures, where  $\bar{k}$  is the Boltzmann constant and  $l = 2.8 \cdot 10^{-10}$  m is the effective size of the diffusing particle. On the assumption that for the mutual penetration of glass melt and refractory the diffusion of oxygen ions is rate-controlling, (3.2) may be used as an estimation of the diffusion coefficient in equations (2.13) and (2.14). Connecting (3.2) with (3.1) and substituting the temperature in (3.2) by its mean value for the pertinent interval, we obtain

$$D = K_D \mathrm{e}^{-\Theta/T},\tag{3.3}$$

where  $K_D = \bar{k} T_{\text{mean}} / K_{\nu} \varrho l$ .

The liquidus curve of perfect binary mixtures can be described by the following thermodynamic equation

$$x_{\rm e} = \exp(-(\Delta H_{\rm M}/R)(T^{-1} - T_{\rm M}^{-1})), \qquad (3.4)$$

where  $x_e$  is the equilibrium molar fraction of a constituent, the latent heat of melting and melting temperature of which are  $\Delta H_M$  and  $T_M$ , respectively. This relation suggests that the temperature dependence of  $C_w$  may be approximated by

$$C_w = K_{\rm C} e^{-b\Theta/T},\tag{3.5}$$

where b and  $K_{\rm C}$  are constants.

Locating the origin of the x-coordinate closely below the glass level, the dependence of  $Q_r$  of the temperature may be expressed by (1.1), for  $Q_r = 2Rj_s$ , where R is the half height of meniscus. So we get the equation

$$Q_r = K_Q \mathrm{e}^{-a\Theta/T_r},\tag{3.6}$$

where  $K_Q = 2Rj_{0s}v_o^a$ ,  $j_{0s}$  is the flux-line corrosion rate corresponding to the arbitrarily chosen viscosity of glass melt,  $\nu_0$ . The rate of flux-line corrosion may be also calculated from the relation [10]

$$j_s = 0.38(C_{wr} - C_{\infty})(D^2(\sigma_r - \sigma_{\infty})/R^2\mu)^{1/3}, \qquad (3.7)$$

where  $\sigma$  is the surface tension. The concentration dependence of  $\sigma$  may be approximated by the linear function

$$\sigma_r = \sigma_{\infty} [1 + \gamma (C_{wr} - C_{\infty})], \qquad (3.8)$$

where  $\gamma$  is the concentration coefficient for surface tension. Then

$$Q_{\rm r} = 0.76 (C_{wr} - C_{\infty})^{4/3} (D^2 \gamma \sigma_{\infty} R/\mu)^{1/3}.$$
(3.9)

Neglecting the temperature dependence of R and  $\sigma_{\infty}$  and using the relation  $\gamma = K_{\gamma} e^{-d\Theta/T}$ , where  $K_{\gamma}$  and d are constants, then, by (3.1), (3.3), and (3.5) and on the assumption that  $C_{\infty} = 0$ , the equation (3.6) is valid again with  $K_Q = 0.76 \times K_C^4 K_D^2 K_\gamma K_{\nu}^{-1} (\sigma_{\infty} R/\varrho_w)^{1/3}$  and

$$a = \frac{4}{3}b + \frac{1}{3}d + 1. \tag{3.10}$$

Denoting  $j_1 = 0.4400 K_C^{5/3} K_D (\beta g/K_Q K_r)^{1/3}$  and  $r_1 = 1.333 j_1/K_Q$ , the equations (2.13) and (2.14) can be rewritten as

$$Y_{t} = j_{1}(1 + Y_{x})^{1/3}(1 - K_{C}e^{-b\Theta/T_{i}})^{-1}\exp(-\Theta(n_{1}/T_{i} - n_{2}/T_{ir}))(1 + r)^{-1/4}, \quad (3.11)$$

$$r = r_{1}(1 + Y_{x})^{-1/6} \int_{0}^{x} \exp(-\Theta(n_{2}/T_{i} - n_{2}/T_{ir}))dx \quad (3.12)$$

$$Y = r_1 (1 + Y_x)^{-1/6} \int_0^\infty \exp(-\Theta(n_3/T_i - n_4/T_{ir})) \mathrm{d}x, \qquad (3.12)$$

where

$$n_{1} = \frac{3}{2}b + \frac{4}{3}, \qquad n_{2} = -\frac{1}{6}b + \frac{1}{3}a, \qquad n_{3} = b + \frac{4}{3},$$
$$n_{4} = -\frac{2}{3}b + \frac{4}{3}a. \qquad (3.13)$$

When  $T_i = T_{ir}$ , i.e., x = 0, (3.10) has to turn into (1.1), so  $n_1 + n_2 = 3/2$ . As a = 3, we get from (3.10) and (3.13) b = 0.7 and d = 3.2. Thus we have the following values for  $n_i : n_1 = 2.383$ ,  $n_2 = 0.883$ ,  $n_3 = 2.03$ , and  $n_4 = 3.53$ .

## 4. INITIAL CORROSION

At the beginning, i.e., at t = 0, the wall is vertical everywhere, so  $Y_x = 0$ . The value of the corrosion rate for t = 0 and x = 0 is given by the relation (2.11), if we take r = 0,  $C_w = C_{wr} = C_{w0}$ ,  $D = D_0$ ,  $v = v_0$ , and  $Q_r = Q_0$ , the index o being related to x = 0 and t = 0:

$$\begin{split} j_0 &= Y_t(0, 0) \\ &= 0.4400(\varrho_{w0}/\varrho_{w0}) (C_{w0} - C_{\infty})^{5/3} (1 - C_{w0})^{-1} D_0(\beta g/\nu_0 Q_0)^{1/3}. \end{split}$$

Assuming that  $\varrho_w \approx \varrho_{w0}$ ,  $C_{w0} \ll 1$  (low refractory solubility) and  $\varrho_W = \varrho_{W0}$  (homogeneous wall), then by connecting (2.11) and (4.1) we get for t = 0

$$j(x, 0) = j_0 \Gamma^{3/2} \varphi (1 + x_r^{-1} \int_0^x \Gamma \varphi dx)^{-1/4},$$
(4.2)

where the following notation was introduced:

$$T = (C_w - C_{\infty})/(C_{w0} - C_{\infty}),$$
  

$$\varphi = (\nu/\nu_0)^{-1/3} (D/D_0).$$
(4.3)

Using the relations (3.1), (3.3) and (3.5), then, for  $C_{\infty} = 0$ , we have

$$\Gamma = \exp[-b\Theta(T_i^{-1} - T_{i0}^{-1})]$$

and

$$\varphi = \exp\left[-\frac{4}{3} \Theta(T_i^{-1} - T_{i0}^{-1})\right].$$

Introducing (3.1), (3.3), (3.5) and (3.6) into (4.1) yields

$$j_0 = j_1 (1 - C_{w0})^{-1} \exp(-m\Theta/T_{i0}), \qquad (4.4)$$

where

$$m = \frac{1}{3} (5b - a + 4). \tag{4.5}$$

The relations (4.2) up to (4.5) prescribe an initial corrosion rate profile to an arbitrary temperature profile  $T_i(x, 0)$ .\*)

#### 5. CORROSION EVOLUTION AT X = CONST.

Let us assume that the wall is homogeneous and the interface is nearly vertical during the whole process, so  $Y_x$  can be omitted in (2.13). Since the origin of x-coordinate can be chosen arbitrarily, we take x = 0, hence r = 0, and thus the expression with the integral disappears. The equation (2.13) then reduces to

$$j_r = Y_t(0, t) = 0.4400(\varrho_{wr}/\varrho_{W0}) (1 - C_{wr})^{-1} (C_{wr} - C_{\infty})^{5/3} D_r (\beta g/\nu_r Q_r)^{1/3}.$$
(5.1)

This equation can be put into the form

$$Y_t = j_0 \hat{j}(Y), \tag{5.2}$$

where  $j_0 = Y_t(0, 0)$  is defined by (4.1) and

$$\hat{j} = j_r/j_0 
= (\varrho_{wr}/\varrho_{w0})[(C_{wr} - C_{\infty})/(C_{w0} - C_{\infty})]^{5/3}[(1 - C_{w0})/(1 - C_{wr})] \times 
\times (D_r/D_0)(\nu_r Q_r/\nu_0 Q_0)^{-1/3}.$$
(5.3)

If  $C_{\infty} = 0$ ,  $\varrho_{wr_{a}}^{r} \approx \varrho_{w0}$ ,  $C_{w} \ll 1$  and the relations (3.1), (3.3), (3.5) and (3.6) operate,  $j_{r} = Y_{t} = j_{0} \exp(-m\Theta(T_{ir}^{-1} - T_{i0}^{-1})),$  (5.4)

where m is given by (4.5).

Note that the relation (3.6) with a = 3 is valid only on the assumption that the point x = 0 is located closely below the glass level. This point is, fortunately, the

<sup>\*)</sup> Initial corrosion rates were calculated by (4.2) in [5], where the assumption was used that  $C_{so}$  does not depended on the temperature, so b = 0, m = 1/3 and I' = 1. Relation (4.2) was rewritten by means of the dimensionless flux-line source  $q = \left(\frac{4}{3} x_r/L\right)^{3/4}$ , where L is the basin depth.

most interesting for our purposes in most cases, as the temperature reaches here its maximum value and thus the time of the complete corrosion of the wall is minimal.

The temperature of the inner wall surface is a function of both position and time, i.e.,

$$T_i = T(x, Y(x, t)),$$
 (5.5)

so  $T_{ir} = T[0, Y(0, t)]$  and  $T_{i0} = T[0, Y(0, 0)] = T(0, Y_0)$ . The function (5.5) can be obtained from the equation for steady one dimensional heat flow through a slab:

$$q = \alpha_i (T_g - T_i) = (k/Y_w) (T_i - T_e) = \alpha_e (T_e - T_m),$$
(5.6)

where  $T_g$  is the temperature of the glass melt far from the wall,  $T_e$  the temperature of the outer wall surface,  $T_m$  the temperature of cooling medium,  $\alpha_i$  and  $\alpha_e$  are the heat transfer coefficients, k the mean thermal conductivity of the wall and  $Y_w =$  $= Y_0 - Y$  the wall thickness. From the equations (5.6) we get after the elimination of  $T_e$  and q (see also [4] and [11]):

$$T_{i} = T_{g} - (T_{g} - T_{m})k\alpha_{i}^{-1}[(\alpha_{i}^{-1} + \alpha_{e}^{-1})k + Y_{0} - Y]^{-1}.$$
 (5.7)

In this equation the temperature of glass melt may be regarded as a function of the depth below the glass level (at a steady state  $T_g$  does not depend on t), while the ambient temperature, the glass/wall heat transfer coefficient, the thermal conductivity of the wall and its initial thickness may be considered constant. The important parameter, which can be controlled by cooling or insulation of the outer surface, is the heat transfer coefficient  $\alpha_e$ , that can be a function of position and time, i.e.,

$$\alpha_e = \alpha_e(x, t). \tag{5.8}$$

Assuming  $\alpha_e$  independent of time (which means that the cooling does not change during the furnace performance) the equation (5.4) can be integrated. To this end the following auxiliary quantities are useful:

$$a_{1} = Y_{0} + k(\alpha_{e}^{-1} + \alpha_{i}^{-1}T_{m}/T_{g}),$$
  

$$z_{0} = m\Theta k(\alpha_{i}a_{1})^{-1}T_{g}^{-1}(1 - T_{m}/T_{g}),$$
(5.9)

$$z = z_0 / (1 - Y/a_1). \tag{5.10}$$

Using (5.1) the equation (5.4) turns into the form

$$Y_t = j_0 \exp(-z_0/(a_1/Y - 1)).$$
(5.11)

Introducing the substitutions (5.9) and intergating we get

$$t = t_0 \int_{z_0}^{z} z^{-2} e^z dz, \qquad (5.12)$$

where

$$t_0 = m\Theta T_g^{-1} (1 - T_m/T_g) k \alpha_i^{-1} j_1^{-1} \exp(m\Theta/T_g).$$
 (5.13)

The equation (5.12) expresses the time necessary for corrosion to reach the value  $Y = a_1(1 - z_0/z)$ . The complete corrosion of the wall comes about when  $Y = Y_0$ . It  $\alpha_e = 0$ , (5.4) can be integrated with the result

$$Y = j_0 t. \tag{5.14}$$

Example. The glass level temperature  $T_g(0) = 1760$  K, the ambient temperature  $T_m = 300$  K, the characteristic glass melt temperature for the viscosity  $\Theta = 2.10^4$  K, the initial wall thickness  $Y_0 = 0.3$  m, its thermal conductivity k = 4.70 W/mK and the glass/wall heat transfer coefficient  $\alpha_l = 815$  W/m<sup>2</sup>K. For the initial corrosion the relation (4.4) is valied with  $C_{w0} \ll 1$ . In case of the flux-line corrosion m = 3 and  $j_1 = 1.97 \times 10^{13}$  m/month and in case of the corrosion below the glass level m = 3/2 and  $j_1 = 2.88 \times 10^5$  m/month. Our task is to find out the time for the complete corrosion of the wall at the glass level and closely below it in dependence of the cooling intensity,  $\alpha_e$ , being independent of time.

If the wall is insulated,  $\alpha_e = 0$ , then, by (5.7)  $T_i = T_g$  and from (4.4) and (5.14) we get for the complete corrosion time  $t_{s,in} = 0.75$  months<sup>\*</sup>) at the glass level and  $t_{d,in} = 26.33$  months below it. For  $\alpha_e \neq 0$  it is necessary to carry out the integration of (5.12). From (5.13) we have

$$t_{0s} = 5.301 \text{ months}, \qquad t_{0d} = 7.157 \text{ months}$$

and from (5.9) and (5.10) we obtain the following expressions for the integration limits:

$$z_{0s} = b_s/(c_1 + \alpha_e^{-1}), \qquad z_{1s} = b_s/(c_2 + \alpha_e^{-1}), \\ z_{0d} = b_d/(c_1 + \alpha_e^{-1}), \qquad z_{1d} = b_d/(c_2 + \alpha_e^{-1}),$$

where  $b_s = 3.47 \times 10^{-2}$ ,  $b_d = 1.74 \times 10^{-2}$ ,  $c_1 = 6.40 \times 10^{-2}$ ,  $c_2 = 2.09 \times 10^{-4}$  and the upper limit of integration is denoted by the index 1.



Fig. 4. The time of the complete corrosion of the wall by the flux-line corrosion (s) and the corrosion below the glass level (d) in dependence of the wall/ambient heat transfer coefficient; if  $\alpha_e < 126 W/m^2 K$ , the wall is corroded through at the flux line, whereas if  $\alpha_e > 126 W/m^2 K$ , the corrosion below the glass level ultimately prevails.

<sup>\*)</sup> This result is not realistic, since the flux-line corrosion slows down as a result of accumulated corrosion products when the flux line cut becomes deep enough.

The result is drawn in Fig. 4. It is apparent that the life-time of wall increases with the intensity of cooling. When  $\alpha_e < 126 \text{ W/m}^2\text{K}$ , the life-time of the wall is determined by the flux-line corrosion, whereas when  $\alpha_e > 126 \text{ W/m}^2\text{K}$  it is limited by the corrosion below the glass level.

## 6. RELATION BETWEEN THE CORROSION RATE AND HEAT LOSSES

The equation (5.4) may be put into the form

$$j_r = j_g \exp[-m\Theta(T_{ir}^{-1} - T_g^{-1})],$$
 (6.1)

where  $j_g$  is the corrosion rate at the temperature  $T_{tr} = T_g$ , i.e., the corrosion rate of the insulated wall (or the adiabatic corrosion rate). By introducing the dimen-

sionless variables  $\tilde{j} = j_r/j_g$  and  $\tilde{\Theta} = m\Theta/T_g$  we get (6.1) in the form

$$\tilde{j} = \exp[\tilde{\Theta}(1 - T_g/T_{ir})].$$
(5.2)

The heat flux through the wall is expressed by  $(5.6)_1$ . Using the dimensionless variable  $\tilde{q} = q/\alpha_i T_g$ , it can be rewritten as

$$\tilde{q} = 1 - T_{ir}/T_g.$$
 (6.3)

Joining (6.2) and (6.3) we have the relation

$$\widetilde{q} = (1 - \widetilde{\Theta}/\ln \widetilde{j})^{-1}, \tag{6.4}$$

from which it is apparent (see Fig. 5) that a substantial reduction of the corrosion rate can be achieved, but is always accompanied by considerable losses of heat.

Eliminating  $T_i$  and  $T_e$  from  $(5.6)_{1,2,3}$  we get

$$q = U(T_g - T_m), \tag{0.5}$$



Fig. 5. The heat losses by cooling vs. the corrosion reduction ratio. The curve numbers correspond to the following values of  $\Theta/T_g$ : 16(1), 18(2), 20(3). An effective reduction of corrosion rate (in comparison with the adiabatic case) is accompaned by substantional heat losses.

(рис. 1 и 4). При анализе коррозии под уровнем с помощью метода пограничного слоя оказывается необходимым учитывать то, что: а) температура стекломассы зависит от вертикальной координаты, от толщины стены и от интенсивности охлаждения и б) коэффициент диффузии, вязкость стекломассы и растворимость огнеупорного материала сильно зависят от температуры. Если эти зависимости имеют вид функции Аррения (уравиение (3.1) и (3.5)) и если справедниво для скорости коррозии на уровне отношение Смрчека (3.6), то развитие профиля коррозии описывается интегродифференциальным уравнением (3.11), выражающим отношение между скоростю коррозии в определениом месте стены от температуры в этом месте и от распределения температур выше этого места (упомянутое распределение действует посредством защитного влияния стекающих продуктов коррозии). В уравнение (3.11) входит также коррекция искривления стены при развитой коррозии (рис. 3). Внимание уделяется двум специальным случаям. Первым является определение исходного профиля коррозии — отношение (4.2), вторым — ход коррозни в области под самым мениском уровия — отношетие (5.4). Температуру предела между стекломассой и стеной расчитали с помощью баланса (5.6). Если скорость охлаждения по все время хода нечи постояниа, то зависимость толщины от времени дана отношением (5.9)—(5.13). Срок службы печи в зависимости от интеисивности охлаждения приводится на рис. 4, из которого видио, что охлаждением можно время прокорродирования значительно увеличить, но только за счет больших затрат тепла (рис. 5). Потери тепла при данном сроке службы печи можно минимизировать в том случае, если удается предотвратить резкое понижение толщины стены коррозией (рис. 6).

- Рис. 1. Зависимость толщины охлаждаемой стены от времени; s коррозия на уровне, d — коррозия под уровнем. Коррозия на уровне оказывается сначала более быстрой по сравнению с коррозией под уровнем, но в результате охлаждения замедляется настолько, что при определенной толщине стены оба вида коррозии равны друг другу по величине. Прокорродирование стены вызывается действием коррозии под уровнем.
- Рис. 2. Зависимость скорости коррозии на уровне (s) и под уровнем (d) алюмоцирконсиликатного огнеупорного материала стекломассой Float от температуры. Ниже температуры 1390 °С коррозия на уровне протекает медленнее, выше ее отношение скоростей обоих видов коррозии обратно.
- Рис. 3. Стекломасса С от стены W отделена пределом s, выраженным функцией V(x). Касательная к пределу образовывает в точке A с вектором гравитационного ускорения угол  $\alpha$  = arctg dV/dx. Горизонтальная скорость убыли стены  $V_i = dV/dt = j/\cos \alpha \ (j - скорость убыли стены в направлении нормалы).$
- Рис. 4. Время прокорродирования стены коррозией на уровне (s) и под уровнем (d) в зависимости от козффициента теплопередачи между стеной и средой. При  $\alpha_e = 126 \text{ sm}/m^2 K$  стена прокорродируется коррозией на уровне, при  $\alpha_e > 126 \text{ sm}/m^2 K$ коррозией под уровнем.
- Рис. 5. Потери тепла охлаждением в зависимости от степени понижения скорости коррозии (по сравнению а адиабатической коррозией) для следующих величин  $\Theta/T_g$ : 16(1), 18(2), 20(3). Эффективное понижение коррозии можно получить только из-за больших потер тепла.
- Рис. 6. Зависимость U от Y<sub>w</sub> (к для разных  $\alpha_e$ ) в вт (м<sup>2</sup>К): 10(1), 10<sup>2</sup>(2), 10-(3). Интенсивное охлаждение при небольшой толщине стены вызывает высокие величин U, а следовательно, высокие потери тепла.