Silikáty *30*, s. 199—204 (1986)

MOISTURE CONTENT HOMOGENIZATION IN A CERAMIC MIX

Jiří Havrda, František Oujiří

Institute of Chemical Technology, Department of the Technology of Silicates, Suchbátarova 5
166 28 Prague 6

Received 19. 9. 1985

The suggested mathematical model of moisture content homogenization in a ceramic mix regards this process as diffusion of water in a binary mixture of incompressible components. Suitability of the choice of the model and conditions of solving was verified experimentally. The model was used to calculate the temperature dependence of the time of homogenization.

INTRODUCTION

Homogenization of the water content in ceramic mixes is an essential technological operation included in various stages of the production of ceramics based on the plastic process. Its aim is to achieve uniform distribution of moisture throughout the mix while maintaining the required mean moisture content. The system thus exchanges just heat with the environment whereas the mass or moisture transfer proceeds inside the system.

If water-saturated ceramic mix is defined as a binary mixture of incompressible components, the equalization of the initial moisture content field in the mix under isothermal conditions can be described by the respective equations for diffusion. The mathematical model obtained in this way will describe quantitatively the process in question, and at the same time provide considerable information with a minimum number of experiments; the information can be used to control or optimize the process.

THEORETICAL

On the assumption that one-dimensional moisture transfer under isothermally isobaric conditions [1, 2, 3] proceeds by diffusion, the moisture flow by volume is given by the equation

$$\boldsymbol{h} = -D\,\partial_x C, \tag{1}$$

where D is the diffusion coefficient and C is the moisture content by volume. The moisture balance has the form

$$\partial_\tau C = \partial_x(D\,\partial_x C). \tag{2}$$

where τ is time and x is the coordinate in the direction of the diffusion. On the assumption that D is constant, equation (2) acquires the form

$$\partial_\tau C = D\,\partial_{xx} C. \tag{3}$$

Solution of equation (3) yields the time developement of the moisture profile in a plate-shaped body. Using suitable initial and boundary conditions one can then model the course of any operation based on water transfer in the mix.

Let us consider an insulated body L_1 in length, in which at time $\tau = 0$ the initial moisture distribution is a general continuous function of position, as illustrated by Fig. 1.

The following parameters can be determined in the body in the beginning:

- the mean moisture content in the body, defined by the equation

$$\bar{C} = \frac{1}{L_1} \int_0^{L_1} C(x)\,\mathrm{d}x, \tag{4}$$

- the maximum moisture content $C_{\max}$,
- the maximum deviation, i.e. the difference between the maximum and the mean value

$$\Delta C_{\max} = C_{\max} - \bar{C}, \tag{5}$$

- the extent of inhomogeneity L pertaining to maximum moisture content $C_{\max}$.

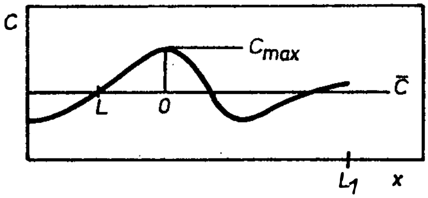

Fig. 1. Schematic diagram of initial moisture distribution in mix.

If attainment of uniform moisture content distribution in the mix is required, then when the maximum deviation has equalized within time τ_1, the smaller deviations have certainly equalized during the same time. On this assumption it is thus possible to simplify the homogenization of moisture content in a mix to equalization of the moisture content in the maximum inhomogeneity as illustrated by Fig. 2, and for the course of the process it holds that

$$\bar{C} = \text{const}\,\vartheta, \qquad C_{\max} = C_{\max}(\tau), \tag{6}$$

while it further holds that

$$\lim_{\tau \to \infty} C_{\max}(\tau) = \bar{C}. \tag{7}$$

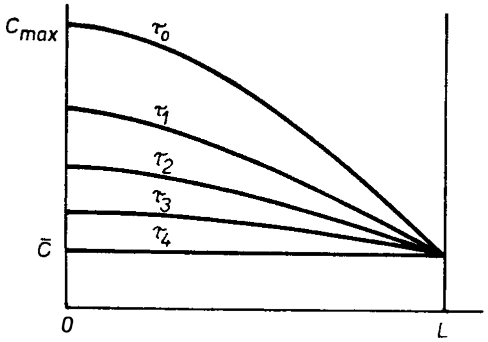

Fig. 2. Schematic diagram of moisture content equalization in the maximum inhomogeneity.

The initial moisture distribution in the maximum inhomogeneity can be defined by a cosine function. The initial condition then has the form

$$\tau = 0, \qquad x \in \langle 0, L \rangle \; \frac{C - \bar{C}}{C_{\max} - \bar{C}} = \cos\left(\frac{\pi}{2}\,\frac{x}{L}\right) \tag{8}$$

and the boundary conditions are

$$\begin{aligned} \tau > 0, \qquad x &= 0, \qquad \partial C/\partial x = 0, \\ x &= L, \qquad C = \bar{C}. \end{aligned} \tag{9}$$

The solution of equation (3) for conditions (8) and (9) yields the time dependence of the decrease of the maximum moisture content in the form [3]:

$$C_{\max}(\tau) = \bar{C} + (C_{\max} - \bar{C}) \exp\left(\frac{\pi^2}{4}\,\frac{D\tau}{L^2}\right). \tag{10}$$

The time of homogenization can then be calculated from the equation

$$\tau = -\frac{4L^2}{\pi^2 D} \ln \frac{C_{\max}(\tau) - \bar{C}}{C_{\max} - \bar{C}}. \tag{11}$$

Equation (11) indicates that the time of homogenization will be the longer the greater the deviation from the mean moisture content, the larger the inhomogeneity degree and the lower the diffusion coefficient value.

EXPERIMENTAL

The homogenization of moisture content was studied experimentally on a porcelain mix. Bodies with an inhomogeneous initial moisture distribution were prepared by drawing a mix (of two moisture contents) on a vacuum auger. Following the manufacture of the bodies, the initial moisture distribution was determined by cutting up the specimens [4], and also the mean moisture content $\bar{C}$, the maximum deviation $C_{\max}$ and the corresponding inhomogeneity degree L. The remaining part of the bodies was insulated and left alone for moisture content homogenization at $T = 294$ K. After times $\tau = 34$, 43 and 60 hours the same values were determined as in the beginning, i.e. $\Delta C_{\max}(\tau)$, $\bar{C}$ and L. Typical moisture profiles determined by these measurements are shown in Figs. 3, 4, 5 and 6.

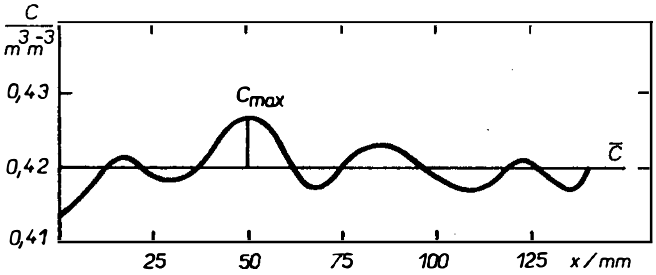

Fig. 3. Initial moisture profile in the body.

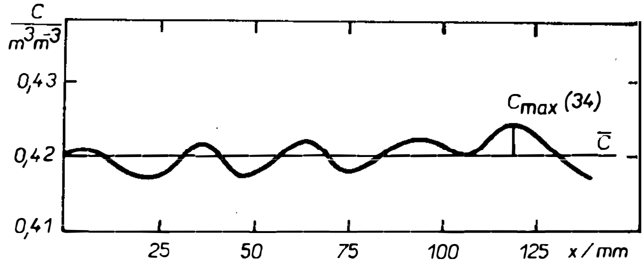

Fig. 4. Moisture profile after 34 hours.

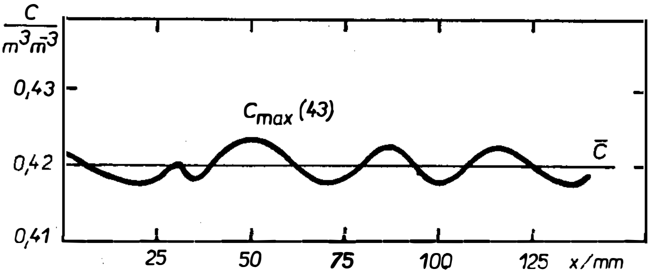

Fig. 5. Moisture profile after 43 hours.

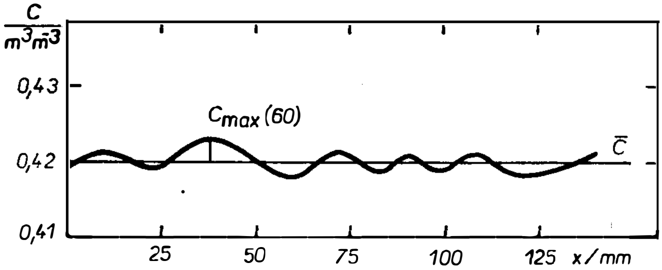

Fig. 6. Moisture profile after 60 hours.

DISCUSSION

To calculate and compare a model with an experiment one has to know the temperature dependence of the diffusion coefficient of water in the mix. This dependence, determined by the diffusion couple method [4], has the following form for the porcelain mix in question:

$$D = 5.52\times 10^{11} \exp(-14\,484\ \mathrm{K/T})\ \mathrm{m^2\,s^{-1}}. \qquad (12)$$

The following values are given by the initial moisture distribution in the body according to Fig. 3:

$$\bar{C} = 0.420\,06\ \mathrm{m^3\,m^{-3}}, \qquad C_{\max} = 0.426\,67\ \mathrm{m^3\,m^{-3}},$$
$$\Delta C_{\max} = 0.006\,61\ \mathrm{m^3\,m^{-3}}, \qquad L = 0.012\ \mathrm{m}. \qquad (13)$$

The diffusion coefficient value calculated from equation (12) for $T = 294$ K is $D = 2.22 \times 10^{-10}$ m^2 s^{-1}. The corresponding maximum moisture contents $C_{\max}(\tau)$ and the maximum deviations $\Delta C_{\max}(\tau)$ were calculated for the times of homogenization $\tau = 34$, 43 and 60 hours respectively by substituting the value for D and the values (13) into equation (10):

$$\begin{array}{ll} C_{\max}(34) = 0.424\,21 \text{ m}^3\text{ m}^{-3}, & C_{\max}(34) = 0.041\,50 \text{ m}^3\text{ m}^{-3}, \\ C_{\max}(43) = 0.423\,73 \text{ m}^3\text{ m}^{-3}, & C_{\max}(43) = 0.003\,67 \text{ m}^3\text{ m}^{-3}, \\ C_{\max}(60) = 0.422\,97 \text{ m}^3\text{ m}^{-3}, & C_{\max}(60) = 0.002\,91 \text{ m}^3\text{ m}^{-3}. \end{array} \tag{14}$$

The following values are obtained from the experimentally established moisture profiles shown in Figs. 4, 5 and 6:

$$\begin{array}{ll} C_{\max}(34) = 0.424\,29 \text{ m}^3\text{ m}^{-3}, & C_{\max}(34) = 0.004\,23 \text{ m}^3\text{ m}^{-3}, \\ C_{\max}(43) = 0.423\,81 \text{ m}^3\text{ m}^{-3}, & C_{\max}(43) = 0.003\,75 \text{ m}^3\text{ m}^{-3}, \\ C_{\max}(60) = 0.422\,86 \text{ m}^3\text{ m}^{-3}, & C_{\max}(60) = 0.002\,80 \text{ m}^3\text{ m}^{-3}. \end{array} \tag{15}$$

The calculated values (14) show a satisfactory agreement with the experimentally determined ones (15), which indicates that the simplifying assumptions adopted in the formulation of the mathematical model were correct, and that the initial and boundary conditions of the problem have been suitably chosen.

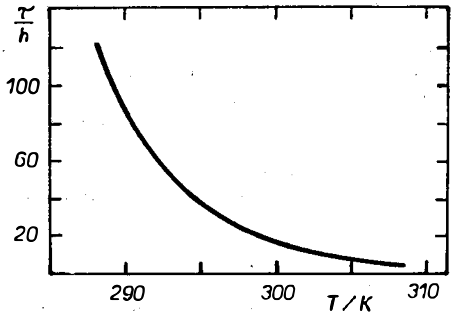

Fig. 7. Temperature dependence of the time of moisture content homogenization.

In view of the reliability of the model and knowledge of the temperature dependence of the diffusion coefficient it is possible to predict quentitatively the effect of temperature on the time of moisture homogenization in a given mix. On the basis of experimentally determined initial values (13) for the calculation of the temperature dependence of the time of homogenization required for reducing the maximum moisture deviation by $\Delta C_{\max} - \Delta C_{\max}(\tau) = 0.003$ m^3 m^{-3}, i.e. to the value $\Delta C_{\max}(\tau) = 0.003\,61$ m^3 m^{-3} and $C_{\max}(\tau) = 0.423\,67$ m^3 m^{-3}, it is possible to calculate from equations (11) and (12), for the temperature interval $T \in \langle 288$ K; 303 K$\rangle$ the relationship shown in Fig. 7. This indicates that when the same degree of inhomogeneity is maintained, temperature will affect significantly the time of homogenization. This finding allows us to control the process as required by technology.

CONCLUSION

If water transfer in a ceramic mix proceeds solely in liquid phase, a mathematical model can be formulated for moisture homogenization which is described by diffusion equations. When the temperature dependence of the diffusion coefficient is known, solution of the model for the maximum moisture inhomogeneity permits the time of homogenization to be calculated together with the temperature dependence of homogenization required for the attainment of a moisture deviation in the mix.

References

[1] Havrda J.: Silikáty *25*, 127 (1981).
[2] Crank J.: *The Mathematics of Diffusion*, Clarendon Press, Oxford 1965.
[3] Hrma P.: Sklář a keramik *27*, 105 (1977).
[4] Havrda L., Oujiří F.: Silikáty *26*, 203 (1982).

ГОМОГЕНИЗАЦИЯ ВЛАЖНОСТИ В КЕРАМИЧЕСКОЙ СМЕСИ

Иржи Гаврда, Франтишек Оуиржи

кафедра технологии силикатов Химико-технологического института
166 28 Прага 6

Ход гомогенизации влажности в керамической смеси можно предполагать на основании математического моделирования данной операции. Модель можно основывать на предположении, что перенос воды протекает согласно механизму изотермически изобарической диффузии в бинарной смеси несжимаемых компонентов. Решение модели для условий (8) и (9) приводит временную зависимость понижения влажности в неоднородном теле с помощью приводимого уравнения (10). Подставляя установленные экспериментальным путем величины в модель, можно рассмотреть ее надежность. Таким образом установили пригодность подбора упрощающих предположений, вводимых при формулировке задания. Зная термическую зависимость коэффициента диффузии воды, можно проводить расчет влияния температуры на приводимую операцию.

Рис. 1. Схема исходного распределения влажности в теле.
Рис. 2. Схема выравнивания влажности в максимально неоднородном теле.
Рис. 3. Исходный профиль влаги в теле.
Рис. 4. Профиль влаги после 34 часов.
Рис. 5. Профиль влаги после 43 часов.
Рис. 6. Профиль влаги после 60 часов.
Рис. 7. Температурная зависимость времени гомогенизации влажности.

HOMOGENIZACE VLHKOSTI V KERAMICKÉ SMĚSI

Jiří Havrda, František Oujiří

Vysoká škola chemicko-technologická, katedra technologie silikátů, 166 28 Praha 6

Průběh homogenizace vlhkosti v keramické směsi lze předpovídat na základě matematického modelování této operace. Model lze založit na předpokladu, že přenos vody probíhá mechanismem izotermně izobarické difúze v binární směsi nestlačitelných složek. Řešení modelu pro podmínky (8) a (9) dává časovou závislost poklesu vlhkosti v nehomogenitě, popsanou rovnicí (10). Dosazením experimentálně zjištěných hodnot do modelu lze vyhodnotit jeho spolehlivost. Takto byla zjištěna vhodnost volby zjednodušujících předpokladů zavedených při formulaci úlohy. Znalost teplotní závislosti difúzního koeficientu vody umožňuje výpočet vlivu teploty na uvedenou operaci.

Obr. 1. Schéma počátečního rozložení vlhkosti v tělese.
Obr. 2. Schéma vyrovnávání vlhkosti v maximální nehomogenitě.
Obr. 3. Počáteční vlhkostní profil v tělese.
Obr. 4. Vlhkostní profil po 34 hodinách.
Obr. 5. Vlhkostní profil po 43 hodinách.
Obr. 6. Vlhkostní profil po 60 hodinách.
Obr. 7. Teplotní závislost doby homogenizace vlhkosti.