# Laboratorní a výpočetní technika

## DETERMINATION OF THERMAL CONDUCTIVITY OF CERAMIC MIXES

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A method and conditions for reliable determination of the thermal conductivity of water-saturated ceramic nixes in plastic state are described. The moisture dependence of thermal conductivity, density and specific heat capacity of a water-saturated porcelain mix were determined experimentally. The moisture dependence of thermal conductivity of the porcelain mix was calculated and compared with experimental values.

#### INTRODUCTION

Mathematical modelling of the processes involved in the conduction of heat in an immobile medium requires, among others, also knowledge of the thermal conductivity of the system and its temperature dependence. If the medium is water-saturated ceramic mix in plastic state, one has also to know the dependence of the thermal conductivity on moisture content.

As indicated by the literature [1, 2, 3, 4], determination of these data is far from simple and the results published are frequently controversial. The experimental data are mostly obtained by the heated wire method and not verified, with respect to their reliability, by other methods.

The present study had therefore the aim to work out a method suitable for the determination of thermal conductivity of water-saturated ceramic mixes and its dependence on moisture content, and to verify the reliability of the results obtained

#### THEORETICAL

The conduction of heat in an immobile medium is described by heat balance in the form [5, 6]:

$$\rho c_p \delta_t T = -\operatorname{div} \boldsymbol{q},\tag{1}$$

where  $\rho$  is density,  $c_p$  is the specific heat capacity at constant pressure, T is temperature, t is time and q is the heat flux, for which in an isotropic medium it holds that

$$\mathbf{q} = -\lambda \operatorname{grad} T, \tag{2}$$

where  $\lambda$  is the coefficient of thermal conductivity. On the assumption that the material quantities  $\lambda$ ,  $\rho$ ,  $c_p$  are constant, equation (1) acquires the following form for a one-dimensional situation:

$$\delta_t T = (\lambda/\varrho c_p) \,\delta_{xx} T \tag{3}$$

and equation (2) has then the form

$$\boldsymbol{q} = -\lambda \, \delta_x T, \tag{4}$$

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where x is the coordinate in the direction of heat conduction.

There are two types of possible solutions of equations (3) and (4). The first includes solutions of non-stationary problems, and requires the knowledge of initial and boundary conditions describing the given experimental arrangement. The second type is concerned with a stationary situation which requires that just the boundary conditions are known.

## Method of planar heat source

If a method for thermal conductivity determination is to be worked out, it is convenient to base it on the known fact that the most plausible results are obtained by measurement in steady state, i.e.  $\mathbf{q} = \text{const.}$ ,  $\delta_t T = 0$ .

On considering a plate-shaped body L in length, then by resolving equation (3) for the boundary conditions

$$\begin{array}{ll} x = 0, & T = T_2, \\ x = L, & T = T_1, \end{array}$$
 (5)

one obtains the steady state temperature profile in the form

$$T = T_2 - (T_2 - T_1) x/L.$$
(6)

If the heat source is a thin planar electric element, the heat flux produced by the source is described by the equation

$$\mathbf{q} = RI^2/S,\tag{7}$$

where R is the source resistivity, I is the electric current and S is the source area while its thickness is neglected.

On joining equations (4), (6) and (7) one obtains the following equation for the calculation of thermal conductivity:

$$\lambda = RI^2 L/S(T_2 - T_1). \tag{8}$$

The equation shows that for the calculation of  $\lambda$  one has to know the temperature profile in the body T = T(x) and the heat flux q in steady state. The experimental arrangement of the method as well as the preparation of the bodies must take into account these conclusions and also the relationship  $\lambda = \lambda(W, T)$ , where W is the moisture content in the mix. The conditions which the methods have to conform to can be formulated as follows:

- reproducible preparation of the bodies with initial homogeneous moisture distribution which must be constant in terms of time,
- one-dimensional conduction of heat through the body, attainment of steady state, i.e. constancy of  $T_1$  and  $T_2$ ,
- negligible losses of heat from the source into the environment,
- the possibility of measuring T = T(x) and the electrical quantities.

### EXPERIMENTAL

The experimental arrangement shown in Fig. 1 was chosen to ensure the conditions given above. Ceramic bodies 1 ( $3 \times 3 \times 3$  cm in size), with uniform moisture content were prepared by drawing on a vacuum auger and coated with lacquer 2 to ensure

constant moisture content in the course of the measurement. An electric planar heating element 4 ( $3 \times 3 \times 0$ . lcm in size) was placed between the two bodies which were set on two hollow cubes 5 kept at constant temperature  $T_1$ . The temperature was kept constant by water heated in thermostat 6. The remaining body surfaces were thermally insulated with expanded polystyrene 3 to rule out dissipation of heat into the environment. Onedimensional conduction of heat through the bodies

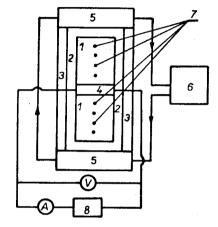


Fig. 1. Schematic diagram of the planar heat source method.

was ensured by dissipation of heat through the cubes and by thermal insulation of the bodies. The DC source 8 ensured a constant flux from the heating element into the two bodies. When the resistance is known, the required electrical quantities are measured by an ammeter and a voltmeter. The temperature profiles were measured by a system of copper-konstantan thermocouples in differential connection, situated at various distances on the body axis. The position of the thermocouples was determined by measuring after cutitng up the bodies. The symmetrical arrangement allowed two bodies to be measured in a single experiment. The thermal conductivity in one element is calculated from equation (8), which had the following form for the given arrangement:

$$\lambda = RI^2 L/2S(T_2 - T_1), \tag{9}$$

where the values of  $T_1$  and  $T_2$  were obtained by approximation of the profile measured in the body, according to equation (6), by the least squares method. Typical temperature profiles are shown in Fig. 2.

The thermal conductivity measurements were carried out on a water-saturated porcelain mix with a moisture content  $W = (24.5 \pm 0.2)$ %. The values obtained from 12 measurements are listed in Table I. The dependence of thermal conductivity on moisture content was measured over the range of  $W \in \langle 30\%, 14\% \rangle$  by steps of 2%. Each moisture content was measured at least four times. The resultant dependence is plotted in Fig. 3.

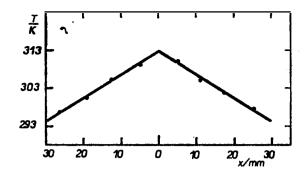


Fig. 2. Experimentally determined temperature profiles in the bodies.

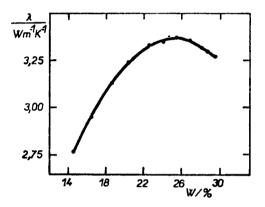


Fig. 3. Dependence of thermal conductivity on moisture content in the mix.

Table I

| λ/Wm <sup>-1</sup> K <sup>-1</sup>                       |
|--|
|  |
| 20   |
|  |
| 20<br>20   |
| 9  |
| $\hat{\lambda} = (3.4 \pm 0.2) \mathrm{Wm^{-1}  K^{-1}}$ |
| 6  |
| 57   |
| 52   |
| 59   |
| 60   |
| 60   |
|  |

#### DISCUSSION

The mean value of thermal conductivity of the porcelain mix with a moisture content of W = 24.5% was  $\lambda = (3.4 \pm 0.2)$  W m<sup>-1</sup> K<sup>-1</sup>. The dispersion of the mean value indicates that the method allows  $\lambda$  to be determined with an error of about 6%. The moisture dependence of thermal conductivity determined over the interval  $W \in \langle 14\%, 30\% \rangle$  was approximated by a parabolic dependence in the form

$$\lambda = A + BW + CW^2, \tag{10}$$

where the constants had the following values:

$$A = -1.275 \times 10^{-2} \text{ W m}^{-1} \text{ K}^{-1},$$
  

$$B = 2.683 \times 10^{-1} \text{ W m}^{-1} \text{ K}^{-1} \%^{-1},$$
  

$$C = -5.325 \times 10^{-3} \text{ W m}^{-1} \text{ K}^{-1} \%^{-2}.$$
(11)

The reliability of the experimentally established  $\lambda$  values can be verified by comparing the temperature diffusivity of the mix calculated for known  $\varrho$ ,  $c_p$  according to the equation

$$a = \lambda / \rho c_p, \tag{12}$$

with the value determined experimentally. The quantities  $\rho$  and  $c_p$  are likewise functions of the moisture content in the mix. The dependence of mix density on moisture content was determined by weighing the bodies and measuring their volume in kerosene:

$$\rho = (2,883 - 24.08 \text{ W}) \text{ kg m}^{-3}.$$
 (13)

The method of DSC calorimetry<sup>\*</sup>) was used to determine the dependence of  $c_p$  on moisture content:

$$c_p = (1,107.1 + 34.73 \text{ W}) \qquad \text{J kg}^{-1} \text{ K}^{-1}.$$
 (14)

The functional relationships (10), (11), (13) and (14) were used to calculate the dependence of thermal diffusivity on moisture content. The dependence was determined experimentally for the same porcelain mix by the two-source method in [7]. The calculated and the experimentally determined dependences are plotted in Fig. 4. The courses of the curves can be assessed by calculating the dispersion  $\sigma^2$  or standard deviation  $\sigma$  around the regression parabola according to equation (8):

$$\sigma = \sqrt{\left[\sum (a_v - a_{\exp})^2\right]/(n-3)},\tag{15}$$

where  $a_v$  is the calculated thermal diffusivity,  $a_{exp}$  is the thermal diffusivity determined experimentally and n is the number of data. If the experimental dependence  $a_{exp} = a_{exp}(W)$  is a regression curve, the value of the standard deviation is  $\sigma = 3 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$  and implies a small dispersion between the two relationships.

<sup>\*)</sup> The measurement was carried out by Ing. J. Endrýs, CSc., Depaartment of Thermal Analysis, Institute of Chemical Technology, Prague.

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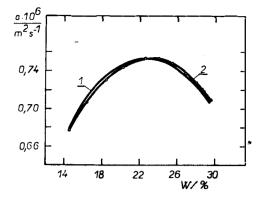


Fig. 4. Comparison of the dependence of thermal conductivity of the mix on its moisture content; 1 — calculated, 2 — measured.

#### CONCLUSION

Both the experimental and the calculated results showed that the planar heat source method allows the dependence of the thermal conductivity of a watersaturated ceramic mix in plastic state on moisture content to be determined reliably. For a porcelain mix, this relationship has the following form over the moisture content interval  $W \in \langle 14\%, 30\% \rangle$ :

$$\lambda = (-1.285 \times 10^{-2} + 2.683 \times 10^{-1} \text{ W} - 5.325 \times 10^{-3} \text{ W}^2)$$
 W m<sup>-1</sup> K<sup>-1</sup>.

Using the new method, supplemented with the determination of the dependence of density and specific heat content on moisture content, it is possible to obtain the dependence of thermal diffusivity of ceramic mixes on moisture content while the results are comparable to those of the two-thermal-sources method.

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#### STANOVENÍ TEPELNÉ VODIVOSTI KERAMICKÉ SMĚSI

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$$\begin{split} \lambda &= (-1,285 \, . \, 10^{-2} \, + \, 2,683 \, . \, 10^{-1} \, \mathrm{W} - \, 5,325 \, . \, 10^{-3} \, \mathrm{W}^2) \, \mathrm{Wm^{-1} \ K^{-1}}, \\ \varrho &= (2 \, 883 \, - \, 24,08 \, \mathrm{W}) \, \mathrm{kg \ m^{-3}}, \\ c_p &= (1 \, 107,1 \, + \, 34,73 \, \mathrm{W}) \, \mathrm{J \ kg^{-1} \ K^{-1}}. \end{split}$$

Závislosti jsou platné pro interval vlhkosti  $W \in (14 \%, 30 \%)$ . Ze získaných údajů vypočtená vlhkostní závislost teplotní vodivosti porcelánové směsi je porovnána s jejím průběhem, stanoveným metodou dvou tepelných zdrojů, a zjištěna dobrá vzájemná shoda.

Obr. 1. Schéma metody plošného zdroje tepla.

Obr. 2. Naměřené teplotní profily v tělesech.

Obr. 3. Závislost tepelné vodivosti na vlhkosti směsi.

Obr. 4. Porovnání vlhkostní závislosti teplotní vodivosti směsi; 1 – vypočtená, 2 – naměřená.

## ОПРЕДЕЛЕНИЕ ТЕПЛОПРОВОДНОСТИ КЕРАМИЧЕСКОЙ СМЕСИ

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Для определения теплопроводности насыщенной водой керамической смеси в пластическом состоянии разработали метод плоскотного источника тепла, в котором провод тепла считается одноразмерным. Определяли условия метода надежного определения теплопроводности смеси. Экспериментальным путем установили зависимость влагосодержания теплопроводности  $\lambda$ , плотности  $\varrho$  и удельного тепла  $c_p$  насыщенной водой фарфоровой смеси в виде:

 $\lambda = (-1,285, 10^{-2} + 2,683, 10^{-1} W - 5,325, 10^{-3} W^2) W_{M}^{-1} K^{-1},$ 

 $q = (2883 - 24,08 W) \text{ kgm}^{-3},$ 

 $c_p = (1\,107, 1\,+\,34, 73\,W)$  Джкг<sup>-1</sup>К<sup>-1</sup>.

Зависимости справедливы для интервала влажности  $W \in (14\%, 30\%)$ . Из полученных данных рассчитанная зависимость влагосодержания теплопроводности фарфоровой смеси сопоставляется с ее ходом, установленным с помощью метода вдух термических источников и установили хорошее взаимное согласие.

- Рис. 1. Схема метода плоскотного источника тепла.
- Рис. 2. Измеренные температурные профили в телах.
- Рис. 3. Зависимость теплопроводности от влажности смеси.
- Рис. 4. Сопостаеление зависимости влагосодержания теплопроводности смеси: 1 рассчитанная, 2 — измеренная.

#### 8. MEZINÁRODNÍ KONFERENCE O JÍLOVÉ HMOTĚ "AIPEA" v DEN-VERU, USA

Osmá mezinárodní konference AIPEA (= Mezinárodní asociace pro výzkum jílové hmoty) se konala v Denveru, Colorado, ve dnech 28. 7. až 2. 8. 1985 za organizačního zajištění americkou Společností pro výzkum jílových materiálů (The Clay Minerals Society) a filiálkou Geologického ústavu v Denveru (U. S. Geological Survey). Cílem AIPEA je podporovat mezinárodní spolupráci při výzkumu a technologii jílové hmoty. Jedním ze základních úkolů AIPEA je organizovat mezinárodní setkání odborníků všech zemí světa, na nichž se formou přednášek, kurzů, exkurzí, výstav a vzájemných diskusí seznamují odborníci se současným