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CONVECTIVE DRYING OF CERAMIC BODY, PART I — MATHEMATICAL MODEL

František Oujiří, Jiří Havrda

Department of the Technology of Silicates, Institute of Chemical Technology Suchbátarova 5, 166 28 Prague 6

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Mathematical modelling was used to describe the course of convective drying of a ceramic body. The procedure is based on the assumption that the ceramic body comprises a binary mixture of incompressible components in which the moisture transfer is due to diffusion and heat conduction. The respective balance and constitutive equations are solved for the initial and boundary conditions defining the operation of convective body drying. Solutions of the equations provided models of convective body drying both involving and not involving thermodiffusion.

INTRODUCTION

Operations involving water transfer in a saturated ceramic mix are typical of classical ceramic technology. The course of these operations can be studied and described in two ways. The first is the experimental method based on empirical determination of parameters and their dependence on various mutually variable quantities. This procedure is demanding as regards the number of experiments and does not always lead to optimum parameters of the operation in question. The second procedure, that of mathematical modelling, is based on determining the principle of the operation being described and on mathematical description of its course with defined materials. In a simplified way, the procedure can be summarized into the following points [1]:

1. Derivation of elementary balance and constitutive equations and determination of their material constants provide a general model of the description.
2. On the basis of an analysis of the technological operation one obtains the initial and boundary conditions necessary for the resolving of the transfer equations.
3. Resolving of the equations provides a mathematical description of the course of the technological operation.
4. A comparison of the model with experiment will show whether the simplifying assumptions and the choice of initial and boundary conditions were suitable.

The present study has the aim to demonstrate application of the procedure mentioned above for obtaining and verifying a mathematical model of convective drying of plate-shaped ceramic bodies in a medium of constant parameters, and to determine the effect of thermodiffusion on water transfer in the course of drying.

Balance and constitutive equations

The saturated ceramic mix is defined as a binary isotropic mixture of incompressible components (ceramic material and water) involving simultaneous transfer of moisture and heat. The moisture and heat balance can be expressed by the following equations [2]:

$$\frac{DC}{Dt} = -\mathrm{div}\,\boldsymbol{h}, \tag{1}$$

$$\frac{DT}{Dt} = -\mathrm{div}\,\boldsymbol{q}, \tag{2}$$

where $\frac{\mathrm{D}}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{u}$ grad represents substantial differentiation of the quantity being balanced, $\boldsymbol{h}$ is the moisture flux by volume, $\boldsymbol{q}$ is the heat flux density, C is the moisture content by volume, T is temperature and $\boldsymbol{u}$ is the mean mixture relaxation flux rate. To describe the water and heat transfer in ceramics use is made of the situation in which the individual components in the mix are in motion, whereas the medium proper remains stationary, i.e. $\boldsymbol{u} = \boldsymbol{0}$. Equations (1) and (2) then have the forms:

$$\partial_t C = -\mathrm{div}\,\boldsymbol{h}, \tag{3}$$

$$\varrho c_p\, \partial_t T = -\mathrm{div}\,\boldsymbol{q}, \tag{4}$$

where ∂_t represents partial derivative in terms of time. The moisture and heat flux in saturated ceramic mix can then be described by linear constitutive equations for moisture and heat flux in the forms:

$$\boldsymbol{h} = -D\,\mathrm{grad}\,C - D_T\,\mathrm{grad}\,T - D_P\,\mathrm{grad}\,P, \tag{5}$$

$$\boldsymbol{q} = -\lambda\,\mathrm{grad}\,\mathrm{T}, \tag{6}$$

where P is pressure, D, D_T, D_P are the coefficients of diffusion, thermodiffusion and barodiffusion respectively, and λ is the coefficient of thermal conductivity. On introducing the term of effective diffusion coefficient, i.e. a coefficient whose value includes the effect of capillary barodiffusion [3]:

$$D^+ = D + D_P\,\partial_C P, \tag{7}$$

equation (3) will have the form:

$$\boldsymbol{h} = -D^+\,\mathrm{grad}\,C - D_T\,\mathrm{grad}\,T. \tag{8}$$

If the moisture and heat transfer are unidimensional, equations (3), (4), (6) and (8) can be expressed in the forms:

$$\partial_t C = \partial_x\,(D^+\,\partial_x C + D_T\,\partial_x T), \tag{9}$$

$$\varrho c_p\,\partial_t T = \partial_x\,(\lambda\,\partial_x T), \tag{10}$$

$$\boldsymbol{h} = -D^+\,\partial_x C - D_T\,\partial_x T, \tag{11}$$

$$\boldsymbol{q} = -\lambda\,\partial_x T. \tag{12}$$

On the assumption that the material variables in equations (9 and 10) are constants, the equations can be written as follows:

$$\partial_t C = D^{+} \partial_{xx} C + D_T \partial_{xx} T, \tag{13}$$

$$\partial_t T = a \partial_{xx} T, \tag{14}$$

where $a = \lambda / c_p \varrho$ is the thermal conductivity of the mix.

Initial and boundary conditions

To resolve equations (13) and (14), one has to know the initial and boundary conditions which follow from the analysis of the given technological operation [4]. The aim is to obtain a mathematical model of convective drying of ceramic bodies.

The course of convective drying of ceramic body in a medium of constant parameters, i.e. temperature T, rate of air flow v and relative air humidity φ can be expressed by the schematic diagram in Fig. 1. Curve 1 represents the time

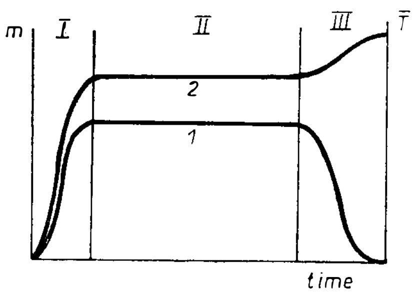

Fig. 1. Curve of body drying in a mediaum of constant parameters.

dependence of drying rate m and curve 2 the time dependence of the mean body temperature $\bar{T}$. The diagram indicates that three characteristic periods of time can be distinguished [5]:

I — the period of heating through,
II — the period of a constant drying rate,
III — the period of a decreasing drying rate.

From the standpoint of a safe course of drying, the quality of green ware isdecisively affected by the course of periods *I* and *II*. Moreover, these two periods comply with the assumption of the binary mix introduced in the formulation of the problem. Let us therefore pay attention to these two characteristic periods.

Period *I* involves an increase in the drying rate and the rate of heating the body, which results in the creation of moisture and temperature gradients in the body. The end of the first period can be characterized by attainment of a constant rate of drying and the body temperature, which is equal to the temperature of adiabatically saturated air. Period II is characteri zed by a constant drying rate and a constant body temperature. The end of this period is indicated by the critical point.

In the course of drying, the water evaporates from the body surface into the ambient atmosphere, which results in water transfer from the body interior towards the surface. As a consequence of this water transfer, a moisture gradient arises in the body and is responsible for internal stress. This situation is schematically represented in a simplified way in Fig. 2. The given findings show that knowledge of the time development of moisture and temperature profiles in the body in terms of the size of the surface moisture and temperature fluxes (drying rate and heating rate respectively) will be decisive for a correct description of the convective body drying operation.

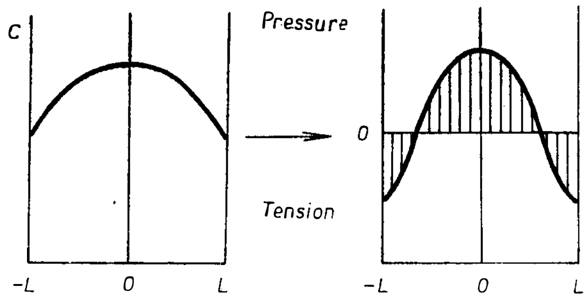

Fig. 2. Schematic diagram of the development of stress in a body.

To formulate the initial and boundary conditions, let us consider a body 2L in thickness, with a generally defined moisture and temperature distribution at time $t = 0$. This condition can be mathematically expressed by the initial condition:

$$t = 0 \qquad x \in (0, L) \qquad C(x) = f_1(x) \tag{15}$$

$$T(x) = f_2(x), \tag{16}$$

where L is the half body thickness. The boundary conditions can then be formulated by means of assumed knowledge of the surface moisture and heat fluxes and the symmetry conditions, i.e.:

$$t > 0 \qquad x = 0 \qquad \partial_x C = \partial_x T = 0 \tag{17}$$

$$x = \mathrm{L} \qquad h_L(t) = -D^{+}(\partial_x C)_L - D_T(\partial_x T)_L \tag{18}$$

$$q_L(t) = \lambda(\partial_x T)_L + r \,.\, h_L(t), \tag{19}$$

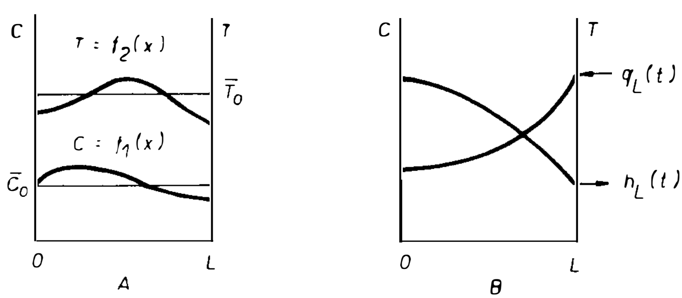

Fig. 3. Schematic diagram of initial and boundary conditions in a body.

where the second term on the right-hand side of equation (**19**) expresses the consumption of heat due to evaporation of water from the body surface. The initial and boundary conditions defined in this way are represented aby the simplifying schematic diagram in Fig. **3**. A shows that the initial mean moisture distribution $\bar{C}_0$ and the initial mean temperature distribution $\bar{T}_0$, for which it holds that $\partial_x \bar{C}_0 = \partial_x \bar{T}_0 = 0$, i.e. $\bar{C}_0$ and $\bar{T}_0 =$ const., can be defined in the body at time $t = 0$; these are defined as follows:

$$\bar{C}_0 = L^{-1} \int_0^L f_1(x)\,\mathrm{d}x, \qquad \bar{T}_0 = L^{-1} \int_0^L f_2(x)\,\mathrm{d}y. \tag{20}$$

To simplify the expression and control the model, it is convenient to convert the transfer equations into dimensionless forms. On introducting

$$\tilde{C} = (\bar{C}_0 - C(x))/\bar{C}_0, \tag{21}$$

$$\tilde{T} = (T(x) - T_0)/T_0, \tag{22}$$

$$\tau = at/L^2, \tag{23}$$

$$\xi = x/L, \tag{24}$$

$$F_2(\xi) = (\bar{C}_0 - f_1(x))/\bar{C}_0, \tag{25}$$

$$F_1(\xi) = (f_2(\xi) - \bar{T}_0))/T_0, \tag{26}$$

$$C_0 = \int_0^1 F_1(\xi)\,\mathrm{d}\xi, \tag{27}$$

$$T_0 = \int_0^1 F_2(\xi)\,\mathrm{d}\xi, \tag{28}$$

equations (**13**) and (**14**) will have the forms:

$$\partial_\tau \tilde{C} = \eta\, \partial_{\xi\xi} \tilde{C} - \eta\gamma\, \partial_{\xi\xi} \tilde{T}, \tag{29}$$

$$\partial_\tau \tilde{T} = \partial_{\xi\xi} \tilde{T}, \tag{30}$$

where

$$\eta = D^+/a, \tag{31}$$

$$\delta = D_T/D^+, \tag{32}$$

$$\gamma = \delta T_0/C_0, \tag{33}$$

and the initial and boundary conditions will have the forms

$$\tau = 0, \qquad \xi \in (0,1) \qquad \tilde{C} = F_1(\xi), \tag{34}$$

$$\tilde{T} = F_2(\xi), \tag{35}$$

$$\tau < 0, \qquad \xi = 0 \qquad \partial_\xi C = \partial_\xi \tilde{T} = 0, \tag{36}$$

$$\xi = 1 \qquad \beta_2 = \partial_\xi \tilde{C} - \gamma\, \partial_\xi T, \tag{37}$$

$$\beta_1 = \partial_\xi T - \eta\mu\beta_2, \tag{38}$$

where

$$\beta_1 = q_L(t)\, L/\lambda \bar{T}_0, \tag{39}$$

$$\beta_2 = h_L(t)\, L/D^+ \bar{C}_0, \tag{40}$$

$$\mu = rC_0/\varrho c_p T_0. \tag{41}$$

The solutions of equations (29) and (30) for conditions (34 through 38) can be obtained by Laplace's transformation in the following form [6]:

$$\tilde{C} = \sum_{k=1}^{2} \sum_{i=1}^{2} A^m_{ki} P_{ki} + B^m_{ki} Q_{ki}, \tag{42}$$

$$\tilde{T} = \sum_{k=1}^{2} \sum_{i=1}^{2} A^q_{ki} P_{ki} + B^q_{ki} Q_{ki}, \tag{43}$$

where

$$A^q_{1i} = (-1)^i (1 - \nu_i^2)/(\nu_1^2 - \nu_2^2), \tag{44}$$

$$A^m_{1i} = B^m_{1i} = (-1)^i \gamma/(\nu_1^2 - \nu_2^2), \tag{45}$$

$$B^q_{1i} = (-1)^i (1 - \nu_i^2)(\nu_1^2 - \nu_2^2), \tag{46}$$

$$A^{qi}_2 = 0, \tag{47}$$

$$A^m_{2i} = (-1)^i (1/\eta - \nu_i^2)/(\nu_1^2 - \nu_2^2), \tag{48}$$

$$B^q_{2i} = (-1)^i \mu\eta(\nu_i^2 - 1)/(\nu_1^2 - \nu_2^2), \tag{49}$$

$$B^m_{2i} = (-1)^i \eta(1/\eta - \nu_i^2) - \mu\gamma\eta/(\nu_1^2 - \nu_2^2), \tag{50}$$

$$\nu_i^2 = ((1 + 1/\eta) + (-1)^i ((1 + 1/\eta)^2 - 4/\eta)^{1/2})/2, \tag{51}$$

$$P_{ki} = \int_0^1 F_k(\xi)\, d\xi + 2 \sum_{n=1}^{\infty} \cos(n\pi\xi) \exp(-n^2\pi^2\nu_i^2) \times$$
$$\times \int_0^1 F_k(\xi) \cos(n\pi\xi)\, \mathrm{d}\xi \quad \text{and} \tag{52}$$

$$Q_{ki} = \int_0^{\tau} \beta_k(\tau')\, \mathrm{d}\tau' + \sum_{n=1}^{\infty} (-1)^n \cos(n\pi\xi) \times$$
$$\times \exp(-n^2\pi^2\nu_i^2\eta\tau) \int_0^{\tau} \beta_k(\tau') \exp(n^2\pi^2\nu_i^2\eta\tau')\, \mathrm{d}\tau). \tag{53}$$

If the initial moisture and temperature distribution in the body is homogeneous, conditions (15) and (16) have the form

$$t = 0 \qquad x(0, L) \qquad C(x) = C_0,$$
$$T(x) = T_0. \tag{54}$$

Rearrangement of equations (42) and (43) yields the moisture and temperature profiles in the body:

$$\tilde{C} = \sum_{k=1}^{2} \sum_{i=1}^{2} B^m_{ki} Q_{ki}, \tag{55}$$

$$\tilde{T} = \sum_{k=1}^{2} \sum_{i=1}^{2} B^q_{ki} Q_{ki}. \tag{56}$$

All the symbols in these equations have the same significance as those in the previous instances.

THE MODEL OF CONVECTIVE BODY DRYING BY NEGLECTING THERMODIFFUSSION

Thermodiffusion is a water transfer process whose driving force is the temperature gradient arising in the ceramic body. Determination of the effect of this phenomenon on water transfer in the drying of ceramic materials is so far not a quite clear problem. The determination can be based on comparing models of drying involving thermodiffusion with those neglecting thermodiffusion. The latter can be obtained by solving equations (29) and (30) while neglecting the cross terms for conditions (34 through 38). The corresponding time developments of temperature and moisture profiles will have the forms:

$$\begin{aligned}\tilde{T} = {} & \int_0^1 F_1(\xi)\,\mathrm{d}\xi + \int_0^\tau [\beta_1(\tau') - \mu\eta\beta_2(\tau')]\,\mathrm{d}\tau' + \\ & + 2\sum_{n=1}^{\infty} \cos(n\pi\xi)\,\{\exp(-n^2\pi^2\tau) \int_0^1 F_1(\xi)\cos(n\pi\xi)\,\mathrm{d}\xi\} + \\ & + 2\sum_{n=1}^{\infty} (-1)^n \cos(n\pi\xi)\,\{\exp(n^2\pi^2\tau) \int_0^\tau [\beta(\tau') - \\ & - \mu\beta_2(\tau')] \exp(n^2\pi^2\tau')\,\mathrm{d}\tau'\},\end{aligned} \tag{57}$$

$$\begin{aligned}\tilde{C} = {} & \int_0^1 F_2(\xi)\,\mathrm{d}\xi + \eta \int_0^1 \beta_2(\tau')\,\mathrm{d}\tau' + s\sum_{n=1}^{\infty} \cos(n\pi\xi) \times \\ & \times \exp(-n^2\pi^2\tau)\,\{\int_0^1 F_2(\xi)\cos(n\pi\xi)\,\mathrm{d}\xi + \\ & + \eta(-1)^n \int_0^\tau \beta_2(\tau') \exp(n^2\pi^2\eta\tau')\,\mathrm{d}\tau'\},\end{aligned} \tag{58}$$

where the symbols used in the equations have the same significance as those used in the previous chapter. Adjustments identical with those used in the previous chapter will yield solutions corresponding to the initial homogeneous moisture and temperature distribution in the body.

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КОНВЕКТИВНАЯ СУШКА КЕРАМИЧЕСКОГО ТЕЛА, ЧАСТЬ I — МАТЕМАТИЧЕСКАЯ МОДЕЛЬ

Франтишек Оуиржи, Иржи Гаврда

кафедра технологии силикатов, Химико-технологический институт
166 28 Прага

Модель конвективной сушки насыщенного керамического тела основывается на предположении, что в теле происходит общая передача влаги диффузией и тепла теплопроводностью. При данных предположениях были получены балансевые и конститутивные уравнения обоих процессов. Путем их решения для начальных и контурных условий, определяющих операцию конвективной сушки насыщенного керамического тела, было получено математическое описание хода операции включая влияние термодиффузии на передачу влажности в смеси. Модель определяется уравнениями (42—53). Пренебрежением влияния термодиффузии на передачу влажности при сушке и решением уравнений передачи была получена модель без термодиффузии, определяемая уравнениями (57) и (58).

Рис. 1. Кривая сушки тела в среде с постоянными параметрами.
Рис. 2. Схема возникновения напряжения в теле.
Рис. 3. Схема начальных и контурных условий в теле.

KONVEKČNÍ SUŠENÍ KERAMICKÉHO TĚLESA, ČÁST I — MATEMATICKÝ MODEL

František Oujiří, Jiří Havrda

Katedra technologie silikátů, Vysoká škola chemickotechnologická, 166 28 Praha 6

Model konvekčního sušení nasyceného keramického tělesa je založen na předpokladu, že v tělese dochází ke společnému sdílení vlhkosti difúzí a tepla vedením. Za těchto předpokladů byly získány základní bilanční a konstitutivní rovnice obou procesů. Jejich řešení pro počáteční a okrajové podmínky definující operaci konvekčního sušení nasyceného keramického tělesa byl získán matematický popis průběhu operace zahrnující vliv termodifúze na přenos vlhkosti ve směsi. Model je definován rovnicemi (42—53). Zanedbáním vlivu termodifúze na přenos vlhkosti při sušení a řešením přenosových rovnic byl získán model bez termodifúze, definovaný rovnicemi (57) a (58).

Obr. 1. Křivka sušení tělesa v prostředí s konstantními parametry.
Obr. 2. Schéma vzniku pnutí v tělese.
Obr. 3. Schéma počátečních a okrajových podmínek v tělese.

O. G. MARTYČENKO, A. G. SEMENOV, JU. A. SOKOVIŠIN: PARAMETRIČESKIE METODY V SVOBODNOJ KONVEKCII (Parametrické metody ve volné konvekci). Nauka i technika, Minsk 1984, 239 stran, cena 1 r. 70 k., 22 Kčs.

Kniha se zabývá přenosem tepla volnou konvekcí mezi vertikálním povrchem a okolní tekutinou, a to pro nekonstantní teplotu povrchu. Uvažuje se pouze stacionární stav a úloha se řeší ve dvou rozměrech pro polonekonečnou stěnu s počátkem na dolní hraně. Pro řešení hraniční vrstvy se aplikují různé varianty parametrických metod, které se postupně zobecňují. Je odvozena univerzální rovnice pro výpočet parametrických funkcí a jsou diskutována různá přiblížení, jejich přesnost a rychlost konvergence. Z numerických výpočtů plyne vysoká přesnost uvedené metody.

Zajímavým rozšířením je řešení úloh volné konvekce spojené s kondukcí v pevné stěně. To umožňuje řadu konkrétních aplikací, např. výpočet svislého žebra ohřívaného zespodu, svislé desky s vnitřními tepelnými zdroji, pevné stěny rozdělující dvě oblasti s volnou konvekcí, apod.

V závěru knihy jsou příklady inženýrských výpočtů týkajících se vysokoteplotních a kryogenních armatur a různých částí elektrických zařízení.

Schill