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THE EFFECT OF SPECIMEN SIZE ON THE MECHANICAL PARAMETERS OF PORCELAIN MIXTURE

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On the basis of bending strength and Young's modulus experimental data of a porcelain mixture, evaluated by three-point bending in the course of firing, the basic mathematical model has been extended to include the effect of the increasing share of microstructure flaws with increasing test specimen volume.

INTRODUCTION

Optimizing the firing of ceramics is a way to achieve energy savings. Knowledge of the material mechanical parameters in the course of firing, particularly in the region of its brittle behaviour, is a necessary prerequisite of any analytical optimizing efforts. Such measurements were carried out on a porcelain mixture with an elevated content of alumina over the temperature interval of 200 to 700 °C, with the use of a suitable apparatus [1, 2].

The relationship of the Young's modulus E and bending strength σ_{P0} of the porcelain mixture on firing temperature as evaluated from our experimental data has shown a distinct systematic dependence on the specimen cross-sectional area. In order to achieve consistent interpretation of these results the basic mathematical model of the experiment has been modified to include the shear force effect, and gradually evolved into a model of bending of a ceramic body while respecting the effect of porosity. The measurements were carried out by the three-point bending method (Fig. 1), with constant span by a quasistatically increasing force F, on three sets of specimens with various diameters (cf. Table I). The variation coefficients with in the individual set of specimens did not exceed 30 % for the force and 11 % for the deflection.

QUANTIFICATION OF THE SHEARING FORCE EFFECT

The influence of the shearing force on the beam deflection can be determined according to the Castigliani's principle by comparing the derivatives of the state of stress potential energies due to bending and shear. In the experimental arrangement employed it is possible to use the simplified analytical expression for the state of stress potential energy due to the bending moment in the form

$$L_M = \frac{1}{2EJ} \int_0^l (M_0(x))^2 \, dx, \tag{1}$$

where $M_0(x)$ is the local value of the bending moment; for the state of stress potential energy due to the shearing force the following equation (3) for the interval $1/10 <$ $< d/l < 1/4$ can be employed:

$$L_T = \beta \int_0^l \frac{(T(x))^2}{GS} \, dx, \tag{2}$$

where $T(x)$ is the local value of the shearing force and the numerical value of the coefficient β has been evaluated according to Zhuravskii's rule for circular cross--section as $\beta = 1.18$.

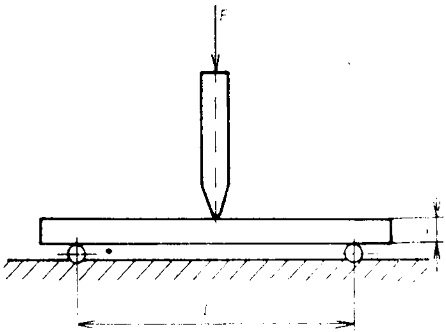

Fig. 1. Schematic diagram of the experimental arrangement employed for measuring the mechanical parameters by the threepoint bending method.

Table I

Mean values measured for various diameters of test specimens at the given temperatures and calculated according to the elementary model. Span of supports l = 125 mm

d / mm	t / °C	F_{max} / N	y / mm	E / GPa	σ_{Pt} / MPa
	200	32.31	0.270	5.68	6.91
	400	36.27	0.333	5.16	7.57
12	500	38.54	0.287	6.36	8.27
11.5	600	56.18	0.381	6.98	12.51
	700	62.06	0.412	7.14	13.51
	200	72.78	0.317	2.76	5.46
17	400	80.86	0.326	2.98	5.87
16.2	500	71.84	0.262	3.30	5.43
	600	114.84	0.298	4.63	8.64
	700	151.14	0.411	4.43	10.93
	200	136.70	0.382	1.20	3.91
23	400	153.02	0.359	1.43	4.39
22.3	500	153.45	0.275	1.87	4.40
	600	241.91	0.335	2.42	6.44
	700	265.13	0.363	2.45	7.61

Notice: The mean values of the mechanical parameters E and σ_{P0} were calculated from 9–15 specimens. σ_{Pt} is the limit stress calculated according to the linear elastic model of bending.

The total state of stress potential energy is given by the sum of individual states of stress energies,

$$L = L_M + L_T. \tag{3}$$

For the case of three-point bending of a prismatic beam, equations (1) and (2) can be analytically integrated. Following rearrangement, one obtains the expressions

$$L_M = \frac{F^2 l^3}{96EJ}, \tag{4}$$

$$L_T = \beta \frac{F^2 l}{8GS}, \tag{5}$$

where F is the loading force, l is the span, G is the modulus of elasticity in shearing, S is the specimen sectional area.

According to Castigliani's theorem, the deflection under the force F can be expressed as partial derivative of the state of stess potential energy according to the acting force, which in this case leads to the expression

$$y_F = \frac{Fl^3}{48EJ} + \beta \frac{Fl}{4GS}, \tag{6}$$

in which the first term represents the well-known elementary relationship for lateral deflection of a beam loaded by force F, derived while considering only the normal stress in the beam. The other term can be regarded as a correction factor describing the actual state of stress (σ, τ). To estimate the quantitative relation of the two strain components, requation (6) for a circular cross section can be adjusted by using the relationship between the moduli in tension and in shear

$$E/G = 2(1 + \mu) \tag{7}$$

to the form

$$y_F = \frac{Fl^3}{48EJ} \left(1 + \frac{3}{2} \beta (1 + \mu) \left(\frac{d}{l}\right)^2\right) \tag{8}$$

where μ is the Poisson's ratio, d is the specimen diameter.

On substituting $\beta = 1.18$, $\mu = 0.3$ into this equation, one obtains

$$y_F = \frac{Fl^3}{48EJ} (1 + 2.3) \left(\frac{d}{l}\right)^2\Bigg). \tag{9}$$

In the case of using equation (9) for calculating the modulus of elasticity for various specimen diameters the correction for the shearing force affects the results in the desired sense (i.e. increasing values of the ratio d/l increase the value of E with respect to the value calculated from the elementary equation). However the correction value of the order of up to 10 % is quite inadequate in view of the experimental results (e.g. for $d = 23$ mm, the value of the correction is approximately 8 %).

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THE IMPACT OF THE POROSITY REDUCTION OF THE BEAM CROSS SECTIONAL AREA UNDER TENSION ON THE STRESS AND STRAIN DISTRIBUTIONS

The pores of ceramic materials are assumed to be unable to transmit tensile stresses, whereas in the region of compressive stresses no difference will be observed between a cross sectional area with pores or defects and one completely free of them, when viewed from the standpoint of macrostresses and macrodeformations. In consequence of this assumption, the geometric-form of the transvese sectional area will change as a result of the effective sectional area reduction in the tensional region. In view of the possibility of ready analytical description, further calculations apply to a beam cross section of square form. The calculation procedure is quite analogous for circular specimen cross sections which were in fact employed; however, the analytical procedure is significantly more difficult. Uniform distribution or porosity over the sectional area is assumed in the first approximation. From the point of view of planar bending, this situation is equivalent to a reduction of the transverse sectional area of the beam according to Fig. 2. The amount of pores over the area is represented by the quantity ω which is a function (obviously a non-decreasing one) of the characteristic dimension a of the body.

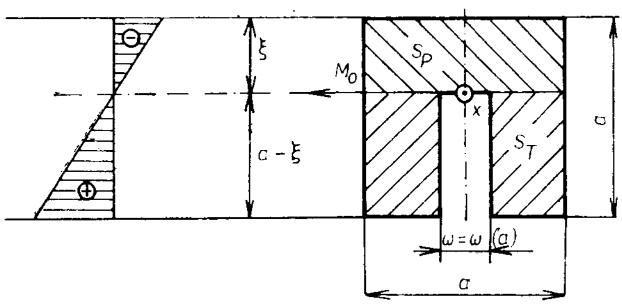

Fig. 2. Model of the porosity effect on the load carrying sectional area.

The position of the neutral axis, given by the ordinate ξ, is determined from the condition of equilibrium of forces into the x axis: according to the Bernoulli's hypothesis, the following equation holds for the deformation:

$$\varepsilon = \alpha\eta, \tag{10}$$

where α is a proportionality coefficient, and according to the Hooke's law, the stress is given by the equation

$$\sigma = E\alpha\eta, \tag{11}$$

where η is the distance from the neutral axis.

From the condition of equilibrium of static forces with respect to axis x,

$$E\alpha \iint_{(s)} \eta \, \mathrm{d}s = 0, \tag{12}$$

the following quadratic equation can be writen for the position parameter of the neutral axis (after rearrangement):

$$-\omega\xi^2 + 2a(\omega - a)\,\xi + a^2(a - \omega) = 0, \tag{13}$$

and its physically significant root ($\omega/a < 1$) provides the expression for the position parameter of the neutral axis in terms of the reduction parameter of the sectional area stressed by tension:

$$\xi = \frac{\sqrt{1 - \frac{\omega}{a}} - \left(1 - \frac{\omega}{a}\right)}{\frac{a}{\omega}} a . \tag{14}$$

The moment of inertia of the form in Fig. 2 with respect to the neutral axis of bending can be regarded as the reduced moment of inertia whose value is given by the expression

$$J' = \frac{1}{3} [a\xi^3 + (a - \omega)(a - \xi)^3]. \tag{15}$$

As the further treatment is aimed at finding a mathematical model describing the experimental values, the expressions need some additional adjustment. On designating the coefficient expressing the measure of reducing the area stressed by tension as

$$\frac{\sqrt{1 - \frac{\omega}{a}} - \left(1 - \frac{\omega}{a}\right)}{\frac{\omega}{a}} = A, \tag{16}$$

the position of the neutral axis of bending can be expressed by the product

$$\xi = Aa \tag{17}$$

and for the cross section moment of intertia with respect to this axis the following expression is valid

$$J' = \frac{1}{3} a^4 \left[A^3 + \left(1 - \frac{\omega}{a}\right)(1 - A)^3\right]. \tag{18}$$

Using the expression for the moment of inertia the full sectional area (free of pores) with respect to the central axis of inertia, equation (18) can be adjusted to the form

$$J' = \varrho J, \tag{19}$$

where the reduction coefficient is defined as

$$\varrho = 4 \left[A^3 + \left(1 - \frac{\omega}{a}\right)(1 - A)^3\right]. \tag{20}$$

Comparison with the original expression of Young's modulus according to the elementary theory of elasticity,

$$E = \frac{Fl^3}{48 J y_F}, \tag{21}$$

where y_F is the deflection under the loading force and the other quantities are designated according to Fig. 1, shows that on respecting the assumed porosity over the beam cross section the reduced values of the moment of inertia have to be substituted into (21)

$$E = \frac{Fl^3}{48y_F J'} = \frac{Fl^3}{48y_F J} \cdot \frac{1}{\varrho} = \bar{E}\frac{1}{\varrho}, \tag{22}$$

in eq. (22) $\bar{E}$ represents the modulus of elasticity evaluated according to the original uncorrected equation.

By simultaneous use of the experimentally established equation $\varrho = \varrho(a)$ (derived from the values in Table I) and the analytical equation $\varrho = \varrho(\omega/a)$ (expression of the moment of inertia reduction with the material measure of imperfection) the dependence of the characteristic dimension (dimension a in the present simplified case) on the measure of microstructural inhomogeneity (here represented by ω/a) can be estimated. The procedure for the chosen temperature of 200 °C is shown in Fig. 3.

The actual course of the dependence $\varrho = \varrho(\omega/a)$ for the given arrangement is given in Table II.

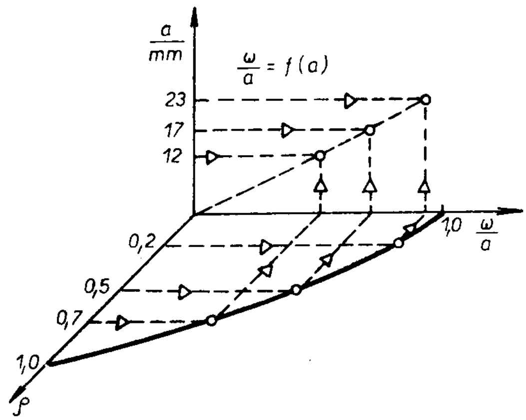

Fig. 3. The procedure for determining the dependence of porosity on the characteristic dimension of the specimen.

Table II

Correction of the cross sectional moment of inertia vs. the microstructural inhomogeneity parameter of the material

$\frac{\omega}{A}$	A	Q
0.1	0.4868	0.9480
0.2	0.4721	0.8916
0.3	0.4555	0.8300
0.4	0.4365	0.7621
0.5	0.4142	0.6863
0.6	0.3874	0.6003
0.7	0.3539	0.5009
0.8	0.3090	0.3819
0.9	0.2402	0.2309
0.95	0.1827	0.1336

The change in the effective beam sectional area considered is also reflected on the state of stress of the beam. The maximum tensile stresses (i.e. those in the fibres most distant from the neutral axis of bending), which are regarded as criteria of possible failure in the case of ceramic materials, are given by the following expression for the case when reduction of the area is considered:

$$\sigma' = \frac{M_0}{W'_0} . \tag{23}$$

For the reduced bending resistance modulus of the cross section holds the relationship

$$W'_0 = \frac{J'}{e_2} , \tag{24}$$

where e_2 is the maximum distance in the sectional plane between the neutral axis and the tensional section border line. In the given case, equation (24) can be written in the form

$$W'_0 = \frac{J}{a} \frac{\varrho}{1 - A} ; \tag{25}$$

using comparison with the cross section bending resistance modulus for a material free of microstructural defects W_0, equation (25) can be expressed as

$$W'_0 = W_0 \frac{\varrho}{2(1 - A)} . \tag{26}$$

The normal stresses in a beam can then be calculated by the following equation for the given tensional area reduction:

$$\sigma' = \frac{M_0}{W_0} \frac{2(1 - A)}{\varrho} = \sigma\varphi , \tag{27}$$

where the term M_0/W_0 is the stress evaluated according to the elementary theory of beam bending. Experimental values of this stress are listed in Table III, together with the respective correction and corrected stresses.

Table III

The values of experimentally determined stress σ, the respective corrections and the corrected stress σ'

$\frac{d\text{-specimen}}{a \text{ (mm)}}$	$\frac{\sigma}{\text{MPa}}$	φ	$\frac{\sigma'}{\text{MPa}}$
12	6.9	1.31	9.03
17	5.4	2.58	13.9
23	3.9	6.58	25.6

The values of stress σ given in Table III indicate that a feasible explanation of the strain state beams has been presented within the framework of the working hypothesis formulated in this chapter; however, this is not the case for the state of stress.

To eliminate this discrepancy between the experimental data and the theoretical model, let us further generalize the model by considering the dependence of the macroscopic Young's modulus on the specimen size.

MODIFICATION OF THE STRESS AND STRAIN STATES THROUGH CONSIDERING THE DEPENDENCE OF GLOBAL ELASTICITY MODULUS ON THE SPECIMEN SIZE

In view of the fact that for an arbitrary non-zero pore area V_{def}, $\sigma_m > \sigma_{nom}$, the mean deformation in the local cross section zone containing pores is greater than its nominal value over the validity range of the Hooke's law. An additional increase in the state of stress (and thus deformation) in the crack tip region is due to a local increase in stress above the σ_m value. In consequence of the joint effects of the two factors the actual macrodeformation of a relatively porous ceramic body can be expected to exceed the one corresponding to the Hooke's law with the modulus of elasticity of the pore-free material.

For the present case of bending of a prismatic ceramic beam the original working hypothesis has to be supplemented with the assumption of decreasing macroscopic Young's modulus with increasing pore content (corresponding in its turn to the increase in the characteristic body dimension) in order to achieve agreement between the mathematical model and experimental data for both the stress and strain states.

Such an agreement is attained by comparing the fictive state of stress according to the original basic model to the limit real state of stress for which the tensile strength obtained by the bending test of $d = 12$ mm specimens is regarded as the failure criterion. Using the condition of equality of limit deformations (28) for the model derived earlier and for the expanded model respecting the variability of Young's modulus in tension with changing pores content in the body,

$$\frac{\sigma'}{E} = \frac{\sigma_{Pt}}{E'}, \tag{28}$$

where σ_{Pt} is the ultimate tensile strength, the following relationship for the tensional modulus of elasticity of porous materials can be obtained:

$$E' = E\,\frac{\sigma_{Pt}}{\sigma'}, \tag{29}$$

In view of the shift of the neutral axis following from the establishment of the forces equilibrium the result will have to be determinated by iteration for the top iteration limit.

On retaining the assumptions of both the deformation of a body element and the linear distribution of strees (Fig. 4), it is possible to introduce the term' reduced modulus of elasticity in bending' ,E_{red}.

On the basis of the analogy of the expressions (cf. Fig. 4),

$$\tan \psi_0 = \frac{\sigma_2}{E(a - \xi')} = \frac{M_0}{J'}\,\frac{1}{E} \tag{30}$$

and

$$\tan \psi = \frac{M_0}{J'}\,\frac{1}{E}\left(\xi' + \frac{E_0}{E'}\,(a - \xi')\right)\frac{1}{a}, \tag{31}$$

the reduced modulus of elasticity in bending can be expressed as follows:

$$\frac{1}{E_{\text{red}}} = \frac{1}{E}\left(\xi' + \frac{E}{E'}(a - \xi')\right)\frac{1}{a}. \tag{32}$$

Using equation (32), E_{red} can be expressed explicitly as

$$E_{\text{red}} = E \frac{a}{\xi' + \frac{E}{E'}(a - \xi')}. \tag{33}$$

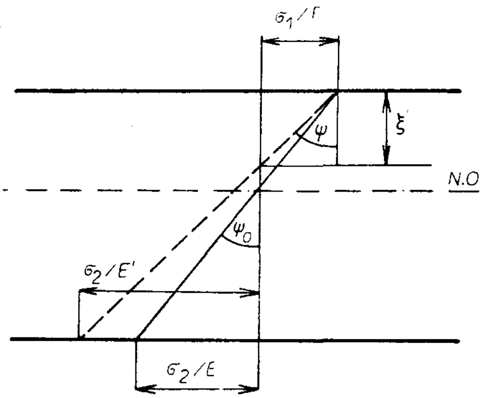

Fig. 4. Shift of the neutral axis due to reduction of the Young's modulus.

On introducing the term ξ for the ratio of the moduli,

$$\frac{E}{E'} = \xi \tag{34}$$

where $\xi = \xi(\omega/a)$, the final form of the expression for the calculation of Young's modulus of a pore-free material from measuring the deflection of a beam is obtained

$$E = \frac{Fl^3}{48Jy_F}\frac{1}{\varrho\nu} = E_{\text{fict}}\frac{1}{\varrho\nu}, \tag{35}$$

where E_{fict} is the value of the modulus calculated according to the elementary uncorrected equation; ϱ is the correction for the tensional section area reduction; ν is the correction for the decrease of the Young's modulus in tension due to the presence of pores, for which it holds that

$$\nu = \frac{a}{\xi' + \xi(a - \xi')}.$$

For the simplified case of a beam of square cross section, the correction coefficient can be expressed in the form

$$\nu = \frac{1}{A + \xi(1 - A)}. \tag{36}$$

It follows from equation (29) that the sought for relationship (34) can in our case be established from the relationship

$$\xi = \xi(a) = \frac{\sigma'(a)}{\sigma_{Pt}} . \tag{37}$$

On the assumption of a constant modulus of elasticity of pore-free ceramic material under constant conditions of firing and of constant limit stress, it is possible to use experimental data (cf. Table I) in determining the numerical values of corrections ϱ and ν for the individual measured diameters of test specimens. The calculation procedure is shown schematically in Fig. 5.

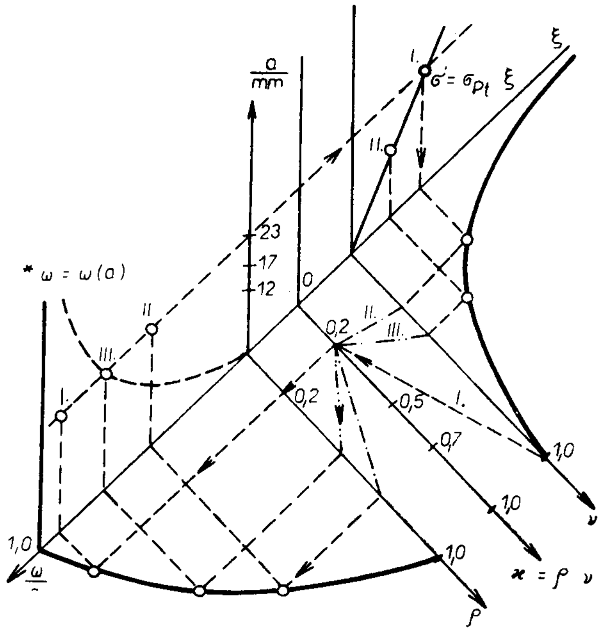

Fig. 5. Iteration procedure for determining the dependence of porosity on the body size. I, II and III are the iteration steps, $^*\omega = \omega(a)$ is the sought for relationship.

CONCLUSION

The presented mathematical model which explicitly takes into account the microstructural character of ceramics — particularly the porosity — indicates the way how to explain consistently the dependence of the experimentaly established values of the Young's modulus and bending strength on the test specimens diameter. The proposed procedure represents a feasible method for correlating the information obtained for test specimens to the conditions of actual products. The subject matter is highly topical both in the region of mathematical modelling of technological and / or operational strains of ceramic products and in the field of numerical evaluation of their resistance to these loads.

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VLIV VELIKOSTI VZORKU NA MECHANICKÉ PARAMETRY PORCELÁNOVÉ SMĚSI

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U souboru dat mechanické pevnosti a Youngova modulu pružnosti stanovených metodou tříbodového ohybu během výpalu válcových vzorků připravených z porcelánové směsi se zvýšeným obsahem oxidu hlinitého byla zjištěna výrazná závislost vypočtených parametrů na jejich průměru. K vysvětlení tohoto jevu byl rozšířen základní model tříbodového ohybu pro zahrnutí rostoucího podílu defektů mikrostruktury keramického materiálu s průměrem vzorků. Výsledkem je iterační postup, který s využitím vztahu (35) umožňuje konzistentní interpretaci experimentálních dat.

Obr. 1. Schéma uspořádání pro měření mechanických parametrů metodou tříbodového ohybu.
Obr. 2. Model vlivu pórovitosti na nosnou plochu průřezu.
Obr. 3. Postup určení závislosti podílu pórů na charakteristickém rozměru vzorku.
Obr. 4. Posun neutrálné osy při redukci Youngova modulu.
Obr. 5. Iterační postup určení závislosti podílu pórů na velikosti tělesa. I., II., III. jsou iterační kroky $^*\omega = \omega(a)$ *— hledaná závislost.*

ВЛИЯНИЕ РАЗМЕРА ОБРАЗЦА НА МЕХАНИЧЕСКИЕ ПАРАМЕТРЫ ФАРФОРОВОЙ СМЕСИ

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В наборе данных механическкой прочности и модуля Юнга, полученных с помощью метода трехточечного изгиба во время обжига цилиндрических образцов, приготовленных из фарфоровой смеси с повышенным содержанием оксида трехвалентного алюминия установили резкую зависимость рассчитанных параметров от их диаметра. Для объяснения данного явления расширили основную модель трехточечного изгиба на растущую долю дефектов микроструктуры каремического материала со средним размером образцов. Результатом того является итерационный прием, который с использованием отношения (35) представляет возможность консистентного объяснения экспериментальных данных.

Рис. 1. Схема упорядочения установки для измерения механических параметров с помощью метода трехточечного изгиба.
Рис. 2. Модель влияния пористости на несущую поверхность диаметра.
Рис. 3. Последовательность определения зависимости доли пор от характеристического размера образца.
Рис. 4. Смещение нейтральной оси при восстановлении модуля Юнга.
Рис. 5. Итерационная последовательность определения зависимости доли пор от размера тела; I, II, III *— итерационные шаги,* $^*\omega = \omega(a)$ x *— искомая зависимость.*